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Hedge ratios in Greek stock index futures market

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This paper examines hedging in Greek stock index futures market. The focus is on various techniques to estimate constant or time-varying hedge ratios. For both available stock index futures contracts of the Athens Derivatives Exchange (ADEX), a variety of econometric models are employed to derive and estimate underlying hedge ratios. Standard OLS regressions, simple and vector error correction models, as well as multivariate generalized autoregressive heteroscedasticity (M-GARCH) models are employed to estimate corresponding hedge ratios that can be employed in hedging (viewed as risk management). In both cases for Greek stock index futures, M-GARCH models (capturing time-variation) provide best hedging ratios, in line with similar findings in the literature. These models are strongly recommended to risk managers dealing with Greek stock index futures.

I. INTRODUCTION

The main objective of hedging (i.e. risk management[1]) is controversial, and there is no clear view on the purpose of hedging. Nevertheless hedging is the most important function of futures markets (that is trading in index futures), the origin of the term is unclear. Hedging uses futures markets to reduce risk of a cash market position, see Working (1953). In general, hedge is the action taken by a buyer or seller to protect his/her business or assets against a change in prices. It is the act of reducing uncertain in value fluctuations of financial portfolios by combining a portfolio of risky assets with a position in a financial instrument, which is highly negatively correlated with the portfolio. Thus, the objective is that positive (negative) in value fluctuations of the portfolio will be off-set by negative (positive) in value fluctuations of the hedging instrument.

Early investigations of hedging include Working (1953), Johnson (1960), Stein (1961) and Ederington (1979). There are three goals of hedging:[2] risk minimization, profit maximization, and the portfolio approach, see Rutledge (1972). Risk minimization is the traditional view of hedging, where hedgers are risk averse and want to eliminate all price risk incurred in their portfolios. It refers to risk reduction. In other words, initial asset and security, used to off-set the risk of the asset, are of equal magnitude. An alternative view of risk minimization is profit maximization. Working (1953) argues that the objective of a hedge is to make profit (or to maximize profit) from movements of futures and spot contracts. That is, a profit can be made by speculation on the basis.[3] He also explains that hedgers function like speculators and argues that hedging is done in expectation of a change in spot–futures relation. In addition, he argues that a hedge may be viewed as a spread

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[1] Hedging is concerned with the management of risk, see Johnson (1960).

[2] Sharda and Musser (1986) argue that there are four goals of hedging, namely: risk, transaction costs, margin payments, and margin opportunity costs.

[3] Hedging is carried out to eliminate/reduce the risks associated with price fluctuations and profit from movements in the basis. The basis is the key point for hedgers to 'watch'.

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between the futures contract (standard unit of trading for futures) and the corresponding spot asset. In addition, Houthakker (1968) argues that traders hedge not to reduce risk, but to increase profit. Furthermore, Johnson (1960), Stein (1961), and Ederington (1979) argue that the classic role of futures markets is that they facilitate hedging. In other words, '*[futures markets] allow those who deal in a commodity to transfer the risk of price changes in that commodity to speculators more willing to bear such risks*' (Ederington, 1979, p. 157). The previously mentioned authors also argue that the main objective of hedging is to minimize the risk (variance) of the portfolio (overall position), and point out that a portfolio approach to hedging is superior to both the traditional view of hedging (i.e. risk minimization) and Working's profit maximization. Also Wu *et al.* (1990) discuss that the objective of hedging is to create a portfolio with reduced risk. Furthermore, Howard and D' Antonio (1984) argue that the hedger's objective is to optimize the risk-return trade-off. Ghosh (1993, p. 743) states that '*the objective of hedging is to minimise the risk of the portfolio for a given level of return*', while Duffie (1989, p. 201) argues that the essence of futures hedging is '*the adoption of a futures position that, on average, generates profits when the market value of the given commitment is lower than expected, and generates losses when the market value of the commitment is higher than expected*'. Also, Moschini and Myers (2002, p. 1) point out that '*hedging reduces risk because cash and futures prices for the same commodity tend to move together, so that changes in the value of a cash position are offset by changes in the value of an opposite futures position*'.

Stock index futures contracts can be used to hedge the risk. According to Hull (2000, p. 66), an index futures hedge leads to the value of the hedged position growing at close to the risk-free interest rate. Hedging with index futures removes the risk arising from market moves and leaves the hedger exposed only to the performance of the portfolio relative to the market. When the relationship between the cash price and the price of a futures contract is very close, the hedge is more effective. However, because this relationship is usually not perfect (spot and futures positions do not match perfectly), the hedge is a cross-hedge. In this case, the hedger should trade the right number of futures contract to control the risk.

In general, hedging reduces risk and allows greater flexibility in planning. It does not interface with normal business operations, and permits easier and greater financing. Hedging is carried out to (i) eliminate risk due to adverse price fluctuation, (ii) reduce risk due to adverse price moves, (iii) profit from changes in the basis, and (iv) maximize expected return for a given risk and minimize risk for a stated return. Numerous studies have addressed the efficiency of stock index futures markets in risk management and hedging. As mentioned above, when futures prices tend to move together, hedging reduces risk. However, because cash and futures prices are not perfectly correlated, determination of the optimal hedge ratio[4] is required. First, Ederington (1979), Johnson (1960) and Stein (1961) suggest that the hedge ratio should be the optimal hedge ratio (i.e. the minimum variance hedge ratio). That is, the optimal amount of futures bought or sold expressed as a proportion of the cash position. Also, the hedge ratio is referred to as the naive hedge ratio (when the cash and futures position have similar characteristics). The naive hedge ratio reduces the risk optimally only in the absence of risk (i.e. when the change of the basis is zero), and it is a static risk management strategy. In addition, it is important for the hedger to be able to identify the number of contracts needed to hedge his/her portfolio. For this, the hedge ratio will be used, so the right number of futures contracts minimizing risk can be chosen. Therefore, the hedge ratio[5] is the number of futures contracts bought, or sold, divided by the number of spot contracts whose risk is being hedged. For a commodity, the hedge ratio is the number of futures contracts to hold for a given position. For a portfolio, the use of Minimum Variance Hedge Ratio (MVHR) is required, as it aims to minimize portfolio risk by finding the value of futures position that reduces the variability of price changes of a hedged position, see Johnson (1960). The MVHR assumes that hedgers have a mean-variance utility function with infinite risk-aversion. Ederington (1979) assumes constant (time-independent) covariance between prices to show that the MVHR is the ratio of the covariance of futures price (or price change) and spot price (or price change) to the variance of futures price (or price change). This ratio is the optimal hedge ratio for any unbiased futures market. So, if the futures market is unbiased, the MVHR is the optimal hedge ratio for any risk averse producer regardless of the degree of risk aversion.

Several measures have been proposed for the hedge ratio. First, the hedge ratio (HR) is estimated from an OLS regression of cash on futures prices. The method is introduced by Ederington (1979), and Anderson and Danthine (1980). It is proved that the MVHR is the slope coefficient of this OLS regression, see Ederington (1979). More recent studies have extended the OLS regression specification, see Myers and Thompson (1989), and Howard and D' Antonio (1991). Howard and D' Antonio (1984, 1987) derive a hedge ratio, which is

[4] The portfolio approach to hedging permits a wide range of hedge ratios to be efficient, see Sutcliffe (1993).
[5] Hedge ratios can also be applied to spread and exploit the correlation structure of the underlying series for profitable advantage.

based on rates of return for situations where the spot position is fixed. Hammer (1988) derives a hedge ratio assuming that the trader wishes to maximize the ratio of the expected return on the hedged portfolio to its variance, while Chang and Fang (1990) derive a utility-maximizing hedge ratio.

Alternative estimation of optimal hedge ratio supports the phenomenon that cash and futures prices display time-varying volatility, and hence GARCH models are to be preferred in view of this. These models are used for estimating time-varying optimal hedge ratios, see Cecchetti, *et al*. (1988), and Baillie and Myers (1991). The differences between the constant and time varying hedge ratios are discussed later on.

Early studies on optimal hedge ratios assume constant hedge ratios (see Ederington, 1979; Anderson and Danthine, 1981) and obtain the optimal hedge ratios as the slope coefficients of simple OLS regressions. However, several researchers argue that this method is inappropriate because it is clear that optimal hedge ratios depend on price movements and that hedge ratios vary over time. In addition, these studies ignore serial correlation and (conditional) heteroscedasticity. Clearly, there are a number of econometric problems when applying OLS regressions to compute risk-minimizing hedge ratios, rendering OLS techniques for estimating optimal hedge ratios unsatisfactory. A preferable approach is to use stochastic volatility models.

Optimal hedge ratios can be estimated by modelling stock index futures price changes within a GARCH framework, allowing dynamic comparison of hedge ratios. Cecchetti *et al*. (1988) use a univariate ARCH model instead of an OLS model, and they assume that the conditional correlation between cash and futures prices is constant. On the other hand, Baillie and Myers (1991) and Kroner and Sultan (1993) use bivariate GARCH (BGARCH) models to estimate time-varying hedge ratios in commodity futures and foreign exchange futures. *A priori*, GARCH hedge ratio is expected to provide greater reduction of risk with stock index futures.

Another problem in modelling a spot–futures relationship arises from cointegration between these financial markets. Ghosh (1993) and Lien (1996) argue that when cointegration is neglected, it can result in an under-hedged position due to the misspecification of spot and futures prices. In particular, Ghosh (1993) analyses stock index futures and underlying stock price index and finds that minimum hedge ratio is biased downwards due to misspecification if both prices are cointegrated. Also, Lien (1996) points out that when a hedger omits cointegration will adopt smaller than optimal futures position, resulting in a poor hedging performance. From an econometric point of view, Ghosh (1993) shows that MVHRs are biased downwards due to misspecification, if spot and futures are cointegrated and the error correction term (ECT) is not included in the regression, while Lien (1996) discusses that the ECT reduces the problem of under-hedging.

Furthermore, several papers use error correction models (ECM) to estimate hedge ratios, see Chou *et al*. (1996), and Lien (1996). Other papers use error correction terms with a time-varying risk structure when analysing the spot-futures relationship, see Kroner and Sultan (1993), and Lien and Tse (1999). Notice that the conventional regression model ignores the error correction term, as well as lagged changes in variables (futures and spot values).

In this paper, the focus is on model specification and empirical comparison of several models for hedge ratio estimation, using data from Greek futures markets (FTSE/ASE-20 index and FTSE/ASE Mid 40 index). In particular, various econometric methods are employed, including the traditional regression model (OLS), ECM, VECM, and the multivariate GARCH model (M-GARCH).[6]

The paper is organized as follows: Section II provides a detailed literature review, while Section III shows an overview of econometric models employed for estimating hedge ratios. Section IV describes the data, and Section V presents empirical results from various econometric models. Finally, Section VI concludes the paper and summarizes the findings.

II. DETAILED LITERATURE REVIEW

There is much empirical research on optimal hedge ratio calculation. Optimal hedge ratio can be estimated by modelling stock index futures prices within several frameworks. First, the hedge ratio is estimated by OLS in regression of the cash price against the futures price. Ederington (1979) examines the hedging performance of the New futures markets (GNMA and T-Bill), concluding that the risk minimizing hedge ratio is less than one. This finding is in accordance with Malliaris and Urrutia (1991), who also find evidence that hedge ratio follows a random walk.

Nevertheless, OLS method has several problems. First, the assumption that the risk in spot and futures markets is constant is incorrect[7] because asset prices are characterized by time-varying distributions. Hence, it is clear that

[6] A natural framework to take cross-sectional information into account is a multivariate model. Many problems in finance like hedging and *Value-at-Risk* require multivariate volatility measures, see Cecchetti *et al*. (1988).

[7] Lence (1995) shows that the simple OLS model is incorrect because the optimal hedge ratio depends on agent's utility function.

risk-minimizing hedge ratios should be time-varying as well. Second, the OLS method ignores the existence of a long-run cointegrating relationship between spot and futures prices. To avoid these problems, a number of papers measure optimal hedge ratios by modelling stock index and index futures prices using GARCH processes, where conditional variances of prices vary over time. Also, several papers use error correction models for estimating hedge ratios. Next, the findings of empirical studies examining futures hedge ratios are reviewed.

Cecchetti *et al*. (1988) and Myers and Thomson (1989) suggest that the standard OLS technique of regressing cash prices on futures prices is unsatisfactory. They apply a univariate ARCH model for estimating an optimal futures hedge. Notice that Myers and Thomson (1989) argue that the hedge ratio should be adjusted continuously based on conditional information. They point out that the hedging decision should be made using a hedge ratio conditioned on the information available when the hedge is placed, and not from the traditional regression of cash price on future price, leading to unconditional hedge ratio. Hence, hedge ratio should be calculated from conditional variance and covariance.

Baillie and Myers (1991) examine commodity futures using a GARCH framework, and Kroner and Sultan (1991, 1993) use currency futures data. In particular, Baillie and Myers (1991) examine six different commodities in accordance to optimal hedge ratios using bivariate GARCH models (GARCH framework with a conditional t-distribution). Their results show that optimal hedge ratios contain unit roots, and therefore, they are not constant (i.e. they vary over time). Baillie and Myers (1991) also compare the hedge ratios estimated from OLS and GARCH models, and find that time-varying hedge ratios perform better to reduce risk (time-varying hedge ratios exhibit significant variations). They obtain a good fit in almost all cases. However, Lien *et al*. (2002)[8] show that the GARCH models incur 20% more risk than the OLS regression, indicating that the OLS hedge ratio performs better that the Vector GARCH hedge ratio. Furthermore, Kroner and Sultan (1993) estimate futures hedge ratios for foreign currency futures using a bivariate error correction model with a GARCH error structure. The proposed model provides greater risk reduction (and hence more effective hedges) than the conventional (OLS) model. Park and Switzer (1995) investigate the risk-minimizing futures hedge ratios for S&P 500 index futures, MMI futures, and Toronto 35 index futures. They estimate the optimal hedge ratios using a bivariate cointegration model with a GARCH error. The GARCH-based hedge ratios are found to be varying over time. Park and Switzer (1995) also find that a constant GARCH model with an error-correction leads into improvements of variance reduction for index futures contracts.

Ghosh (1995) uses cointegration theory to estimate the hedge ratios for European Currency Unit (ECU) futures contracts. His results confirm that hedge ratio, estimated from ECM, is superior to the hedge ratio obtained from the OLS method. In addition, Ghosh and Clayton (1996) use an intertemporal error correction model to estimate hedge ratios, using data from stock index futures contracts for France (CAC 40), the UK (FTSE 100), Germany (DAX), and Japan (Nikkei). Their results indicate that hedge ratios obtained from ECMs are superior to those obtained from the traditional hedging techniques.

Chou *et al*. (1996)[9] examine and compare conventional and ECM hedge ratios for Japan's NSA index and the NSA index futures. They show that the ECM outperforms the conventional model. Thus, hedge ratios obtained by ECM reduce financial risk of the cash position more. Also, Sim and Zurbruegg (2001) find that a cointegrating time-varying hedge ratio performs better than a simple constant hedge ratio. They show that hedge ratios have significantly dropped after the Asian financial market crisis and that risk has increased.

Bera *et al*. (1997) estimate time-varying hedge ratios for corn and soybeans using the BGARCH model and the random coefficient autoregressive (RCAR) model. They show that BGARCH model provides the largest reduction in the variance of the return portfolio for corn and soybeans. Furthermore, Kavussanos and Nomikos (2000) estimate time-varying hedge ratios using a BGARCH model and an augmented GARCH (GARCH-X) model for the BIFFEX market. They suggest that time-varying hedge ratios estimated from GARCH-X models outperform the ones from BGARCH models in reducing market risk.

Recently, Lafuente and Novales (2002) further discuss optimal hedging under departures from the cost-of-carry valuation, using data from the Spanish stock index futures market. To estimate the optimal hedge ratio, they employ a bivariate error correction model with GARCH innovations. Their empirical results and *ex ante* simulations indicate that hedge ratios lead into using a lower number of futures contracts than the one under a systematic unit ratio.

Yang (2001) estimates hedge ratios using various econometric models for both All Ordinary Index and SPI futures traded in the Australian Futures Market. He employs the OLS model, the bivariate vector autoregressive model, the error-correction model and the multivariate

[8] Lien *et al*. (2002) review some recent developments in futures hedging, and discuss the econometric implementation of various methods.
[9] This paper's finds that temporal aggregation has important effects on the hedge ratio.

diagonal Vector GARCH model. These results show that the GARCH hedge ratios exhibit the greatest portfolio risk reduction.

Moschini and Myers (2001) provide a new GARCH approach for estimating time-varying optimal hedge ratios, and they test whether optimal hedge ratios are constant over time, using *BEKK* parameterizations (Baba *et al.*, 1990) for the bivariate GARCH (BGARCH) model.

Seerljaroen (2000) examines hedge ratios of the SPI futures contract, and finds that the variance minimization model performs better than the naive model. Finally, Sercu and Wu (2000) compare the performances of various hedge ratios for three-month currency exposures, and find that the price-based hedge ratios perform better than the regression-based hedge ratios. According to Sercu and Wu (2000), price-based methods perform better, because they provide immediate adjustment to break in the data.

III. THEORY AND METHODOLOGY

Consider the general case of selling b units of asset 2 to finance the purchase of one unit of asset 1. Let the price of asset 1, over a given period, be the random variable S_1 and the price of asset 2 be S_2 (also random). The value of the underlying portfolio is given by $S_P = S_1 - bS_2$ and has variance $\mathrm{Var}(S_P) = \mathrm{Var}(S_1 - bS_2)$. Differentiate portfolio variance with respect to b to find minimum hedge ratio $\mathrm{Cov}(S_1, S_2)/\mathrm{Var}(S_2)$, which is the regression coefficient of regressing S_1 against S_2, also known as the OLS hedge ratio. Furthermore, consider a hedged portfolio of a long position in a security and a short position in a future on an asset, which is highly correlated with the security. Using returns, rather than prices, appears to give a different value for the hedge ratio, but it can be shown that it is in fact the same.

For this, let b denote the hedge ratio, $S(t)$ the price of the security at time t, and $F(t)$ the price of the future at time t. (Notice that in the case of the future, the price means the contracted future price.) The contract acquires value as the futures price changes over time. Thus, the initial value of the portfolio is $S(0)$, and the value at time t is $S(t)-b\ (F(t)-F(0))$. Let $R_S = [S(t) - S(0)]/S(0)$ be the security return, $R_F = [F(t) - F(0)]/F(0)$ the futures return, and $R_P = [P(t) - P(0)]/P(0)$ the portfolio return. It is true that

$$R_P = \frac{P(t) - P(0)}{P(0)} = \frac{S(t) - b(F(t) - F(0)) - S(0)}{S(0)} = \frac{S(t) - S(0) - b(F(t) - F(0))}{S(0)} = R_S - b\frac{F(0)}{S(0)}R_F$$

As above, the variance R_P can be differentiated to find the optimal hedge ratio. The optimal value is given by $(S(0)/F(0))(\mathrm{Cov}(R_S,R_F)/\mathrm{Var}(R_F))$. However, $\mathrm{Cov}(R_S,R_F) = \mathrm{Cov}(S(t), F(t))/[S(0)F(0)]$, and $\mathrm{Var}(R_F) = \mathrm{Var}(F(t))/F(0)^2$, so the new value coincides with the OLS value derived earlier.

An alternative approach determining the hedge ratio is to consider the investor's utility. An appropriate utility function is

$$U(R_P) = E(R_P) - c\mathrm{Var}(R_P)$$

where c is known as the risk aversion parameter. It holds that

$$\begin{aligned} U(R_P) &= E(R_S - b\frac{F(0)}{S(0)}R_F) - c\mathrm{Var}(R_S - b\frac{F(0)}{S(0)}R_F) \\ &= E(R_S) - b\frac{F(0)}{S(0)}E(R_F) - c\mathrm{Var}(R_S) \\ &\quad - 2b\frac{F(0)}{S(0)}\mathrm{Cov}(R_S, R_F) + b^2\frac{F(0)^2}{S(0)^2}\mathrm{Var}(R_F) \end{aligned}$$

which implies that

$$\begin{aligned} \frac{\mathrm{d}(U(R_P)}{\mathrm{d}b} &= -\frac{F(0)}{S(0)}E(R_F) + 2c\frac{F(0)}{S(0)}\mathrm{Cov}(R_S, R_F) \\ &\quad - 2cb\frac{F(0)^2}{S(0)^2}\mathrm{Var}(R_F) \end{aligned}$$

The future is priced so that expected price changes of the underlying security are discounted. Thus, the return to the future is a martingale, that is $E(R_F) = 0$. The derivative is zero, when $b = (S(0)/F(0))(\mathrm{Cov}(R_S, R_F)/\mathrm{Var}(R_F))$ as before.

In view of the expression for the minimum variance hedge ratio (MVHR), which is equivalent to the regression slope (conventional hedge ratio), it is common practice amongst fund managers to calculate futures hedge ratios using OLS regressions, and to periodically recalculate and rebalance the hedge over time. Typically, they will use data taken from the previous 100 trading days. Some studies derive hedge ratios that minimize the variance of price changes in the hedged portfolio, see Butterworth and Holmes (2000) for details. The variance–minimizing hedge ratio is the ratio of the unconditional covariance between cash and futures price changes to the variance of futures price changes. In addition, Butterworth and Holmes (2000) estimate the (*ex post*) MVHR using OLS, by regressing the log of the change (approximate return) in the spot price against the log of the change in the futures price. Hence,

$$\Delta S_t = c + b\Delta F_t + u_t \qquad u_t \sim \text{i.i.d.}(0, \sigma^2) \tag{1}$$

Nevertheless, this assumes that there is no serial correlation in returns as well as homoscedasticity. There is substantial evidence to suggest that financial time series do not comply with these assumptions. There is evidence that the returns to financial instruments exhibit heteroscedasticity, with time varying conditional variances or volatility. Herbst *et al.* (1993) argue that the above estimation of the MVHR suffers from the problem of serial correlation in the OLS residuals, while Park and Bera (1987) show that the OLS model is not appropriate to estimate hedge ratios, because it ignores potential heteroscedasticity. Also notice that, since conditional moments change, as new information arrives to the market, the hedge ratio (and optimal hedge ratio) changes over time. In view of this, Myers and Thompson (1989) argue that the hedge ratio should be adjusted continuously, based on conditional information and thus calculated from conditional variance and covariance.

To derive variances and covariances, required to compute the hedge ratio, a model accommodating time-varying conditional second moment is employed. One such model is the Bivariate GARCH (BGARCH) model. BGARCH models have been used to estimate time-varying hedge ratios in foreign exchange futures, see Kroner and Sultan (1993), in interest rate futures, see Gagnon and Lypny (1995), in commodity futures, see Baillie and Myers (1991), and in stock index futures, see Park and Switzer (1995). In BGARCH modelling, it is assumed that conditional mean equations are somehow modelled, no details are given now. When the variables are returns to the index S_t and the returns to the future F_t:

$$\Delta S_t = \text{Model}(\Delta S_t) + \varepsilon_{S,t} \tag{2}$$

$$\Delta F_t = \text{Model}(\Delta F_t) + \varepsilon_{F,t} \tag{3}$$

The error terms are then used in the building of conditional variance and covariance equations. In the simplest version, these equations take the form

$$\text{Var}(\Delta S_t) = c_1 + a_1(\varepsilon_{S,t-1})^2 + b_1\text{Var}(\Delta S_{t-1}) \tag{4.1}$$

$$\text{Var}(\Delta F_t) = c_2 + a_2(\varepsilon_{F,t-1})^2 + b_2\text{Var}(\Delta F_{t-1}) \tag{4.2}$$

$$\text{Cov}(\Delta S_t, \Delta F_t) = c_3 + a_3\varepsilon_{S,t-1}\varepsilon_{F,t-1} + b_3\text{Cov}(\Delta S_{t-1}, \Delta F_{t-1}) \tag{4.3}$$

In building this model nine GARCH parameters plus any parameters from the mean models are used. Estimation is by maximum likelihood. Having estimated the model parameters, the conditional variance of the future and the conditional covariance can be used in the computation of hedge ratio (b).

More specifically, hedge ratio is estimated using daily future and spot prices by the following model in returns form:

$$R_t^S = \mu_S + \varepsilon_{St}$$

$$R_t^F = \mu_F + \varepsilon_{Ft}$$

$$\varepsilon_t|\Psi_{t-1} \sim BN(0, H_t) \tag{5.0}$$

$\varepsilon_t = [\varepsilon_{St}, \varepsilon_{Ft}]'$. The form of H_t for a BGARCH(p,q) model is written as

$$\text{vech}(H_t) = \text{vech}(C) + \sum_{i=1}^{q} \text{A}_i\text{vech}(\varepsilon_{t-1}\varepsilon'_{t-1}) + \sum_{i=1}^{p} B_i\text{vech}(H_{t-1}) \tag{5.1}$$

where C is a 2×2 positive definite symmetric matrix and A_i and B_i are 3×3 matrices. However, the parameterization in Equation 5.1 is difficult to estimate, since positive definiteness of H_t is not guaranteed. Also, the model contains too many parameters. The VECH model allows for a very general dynamic structure of the multivariate volatility process. This specification suffers from high dimensionality of the relevant parameter space, which makes it intractable for empirical work. In the diagonal VECH formulation, it is assumed that the conditional variance of the index is not affected by the errors or by the conditional variance of the index, nor by the conditional covariance. A similar assumption is made for the conditional variance of the future. It is also assumed that the conditional covariance is not affected by the conditional variances. For this, 12 extra parameters are required for the full VECH formulation:

$$\begin{bmatrix} \text{Var}(\Delta S_t) \\ \text{Cov}(\Delta S_t, \Delta F_t) \\ \text{Var}(\Delta F_t) \end{bmatrix} = \begin{bmatrix} c_1 \\ c_3 \\ c_2 \end{bmatrix} + A \begin{bmatrix} (\varepsilon_{S,t-1})^2 \\ (\varepsilon_{S,t-1}\varepsilon_{F,t-1}) \\ (\varepsilon_{F,t-1})^2 \end{bmatrix} + B \begin{bmatrix} \text{Var}(\Delta S_{t-1}) \\ \text{Cov}(\Delta S_{t-1}, \Delta F_{t-1}) \\ \text{Var}(\Delta F_{t-1}) \end{bmatrix} \tag{5.2}$$

where A and B are 3×3 matrices.

The computational burden, introduced by more than doubling the number of parameters to be estimated, is very significant. Moreover, neither the diagonal VECH Bollerslev *et al.* (1988), nor the full VECH formulation of bivariate GARCH enforce positive definiteness of the covariance matrix. This can be remedied, without using too many parameters, by the *BEKK formulation*, see Engle and Kroner (1995). Positive definiteness is easily guaranteed by the BEKK model (named after Baba, Engle, Kraft, and Kroner 1990). The generic version of the model is

$$H_t = C^T C + A^T E_{t-1} E_{t-1}^T A + B^T H_{t-1} B \tag{6.0}$$

$$H_t = \begin{bmatrix} \text{Var}(S_t) & \text{Cov}(S_t, F_t) \\ \text{Cov}(S_t, F_t) & \text{Var}(F_t) \end{bmatrix} \tag{6.1}$$

$$E_t = \begin{bmatrix} \varepsilon_{S,t} \\ \varepsilon_{F,t} \end{bmatrix} \tag{6.2}$$

and where A, B and C are matrices of parameters:

$$C = \begin{bmatrix} C_{11} & C_{12} \\ 0 & C_{22} \end{bmatrix} \tag{6.3}$$

The *BEKK parameterization* requires estimation of only 11 parameters in the conditional–covariance structure, and also, guarantees H_t to be positive definite. Compared to the diagonal model, the BEKK model allows for convenient cross-dynamics of conditional variances.

In addition, Bollerslev (1990) introduces another way to simplify H_t. He presented the '*constant-correlation specification*' by assuming that the conditional correlation between ε_{St} and ε_{Ft} is constant over time. He defines H_t as

$$\begin{bmatrix} h^2_{ss,t} & h^2_{sf,t} \\ h^2_{fs,t} & h^2_{ff,t} \end{bmatrix} = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \begin{bmatrix} 1 & \rho_{St} \\ \rho_{St} & 1 \end{bmatrix} \begin{bmatrix} h_{S,t} & 0 \\ 0 & h_{F,t} \end{bmatrix} \tag{7}$$

In this case, positive definiteness is assured if $h_{S,t} > 0$ and $h_{F,t} > 0$. The above (constant correlation) model contains only seven parameters compared to 21 parameters encountered in the full VECH model.

On the other hand, Park and Switzer (1995) use the above methodology to compute hedge ratios. They employ a bivariate cointegration model with simple GARCH errors to estimate the optimal hedge ratio. The bivariate cointegration GARCH(1,1) distributions of spot and futures are given by:

$$\begin{aligned} \Delta S_t &= a_0 + a_1(S_{t-1} - \gamma F_{t-1}) + \varepsilon_{st} \\ \Delta F_t &= \beta_0 + \beta_1(S_{t-1} - \gamma F_{t-1}) + \varepsilon_{ft} \\ \varepsilon_t &= \begin{bmatrix} \varepsilon_{st} \\ \varepsilon_{ft} \end{bmatrix} | \Psi_{t-1} \sim N(0, H_t) \\ h^2_{st} &= c_s + a_s\varepsilon^2_{s,t-1} + b_s h^2_{s,t-1} \\ h^2_{ft} &= c_f + a_f\varepsilon^2_{f,t-1} + b_f h^2_{f,t-1} \end{aligned} \tag{8}$$

where $\Psi_{t-1}$is the information at time $t-1$, and $S_{t-1} - \gamma F_{t-1}$ is the error term obtained from the equation $S_t = \delta + \gamma F_t + e_t$. The time-varying hedge ratio provided by the BGARCH models is expressed as

$$b_t = \frac{h_{sf,t}}{h_{ff,t}} = \left(\frac{\mathrm{Cov}(\varepsilon_{st}, \varepsilon_{ft})}{\mathrm{Var}(\varepsilon_{ft})}\right) \tag{9}$$

Alternatively, BGARCH models provide the time-varying conditional variances and covariances of S and F, and calculate the time-varying hedge ratio at time $t-1$ as

$$b_{t-1} = \frac{h^2_{sf,t}}{h^2_{ff,t}} = HR_t \tag{10}$$

Notice that estimation of all multivariate GARCH models above is carried out by using conditional quasi maximum likelihood estimation. The conditional log-likelihood function for a single observation can be written as

$$\begin{aligned} L_t(\theta) = &-(n/2)\log(2\pi) - (1/2)\log(|H_t(\theta)|) \\ &- (1/2)\varepsilon_t(\theta)' H_t^{-1}(\theta)\varepsilon_t(\theta) \end{aligned} \tag{11}$$

where θ represents a vector of parameters, n is the sample size, and t is the time index.

Furthermore, Chou *et al*. (1996), following the method proposed by Engle and Granger (1987), estimate the hedge ratio, using an error correction model (ECM). Assuming the series are cointegrated, there exists an ECM of the form

$$\Delta S_t = c + a\varepsilon_{t-1} + b\Delta F_t + \sum_{i=1}^{n} \theta_i \Delta F_{t-i} + \sum_{j=1}^{k} \phi_j \Delta S_{t-j} + u_t \tag{12}$$

where $\Delta S_t = S_t - S_{t-1}$, $\Delta F_t = F_t - F_{t-1}$ and $\varepsilon_{t-1} = S_{t-1} - (a + bF_{t-1})$. In this ECM, the coefficient b is the hedge ratio. In addition, Ghosh (1993), and Lien (1996) calculate the optimal hedge ratio using a VECM:

$$\Delta S_t = \sum_{i=1}^{n} \beta \Delta S_{t-i} + \sum_{j=1}^{k} \delta \Delta F_{t-j} - a_s z_{t-1} + \varepsilon_{S,t} \tag{13}$$

$$\Delta F_t = \sum_{i=1}^{n} \gamma \Delta S_{t-i} + \sum_{j=1}^{k} \phi \Delta F_{t-j} - a_f z_{t-1} + \varepsilon_{F,t} \tag{14}$$

The hedge ratio is calculated as $\rho(\sigma_s/\sigma_F)$, where ρ is the correlation coefficient between $\varepsilon_{S,t}$ and $\varepsilon_{F,t}$, and σ_S and σ_F are the standard deviations of $\varepsilon_{S,t}$ and $\varepsilon_{F,t}$ respectively.

Finally, Brooks *et al*. (2002) use a bivariate VECM, which is given by

$$\begin{aligned} &\Delta Y_t = \mu + \sum_{i=1}^{4} \Gamma_i \Delta Y_{t-i} + \Pi v_{t-1} + \varepsilon_t \\ &Y_t = \begin{bmatrix} S_t \\ F_t \end{bmatrix}; \mu = \begin{bmatrix} \mu_F \\ \mu_S \end{bmatrix}; \Gamma_i = \begin{bmatrix} \Gamma^F_{i,F} & \Gamma^F_{i,S} \\ \Gamma^S_{i,F} & \Gamma^S_{i,S} \end{bmatrix}; \Pi = \begin{bmatrix} \pi_F \\ \pi_S \end{bmatrix}; \\ &\varepsilon_t = \begin{bmatrix} \varepsilon_{F,t} \\ \varepsilon_{S,t} \end{bmatrix} \end{aligned} \tag{15}$$

However, a disadvantage of VECM is that it does not ensure the conditional variance–covariance matrix of spot and futures returns to be positive definite, see Lien *et al*. (2002).

IV. DATA

The data employed in this study comprise 525 daily observations on the FTSE/ASE-20 stock index and stock index futures contract (August 1999–August 2001) and 415 daily observations on the FTSE/ASE Mid 40 stock index and stock index futures contract (January 2000–August 2001). Closing prices for spot indices were obtained from Datastream, and closing futures prices were obtained from the official web page of the Athens Derivatives Exchange (www.adex.ase.gr).

The FTSE/ASE-20 comprises 20 Greek companies, quoted on the Athens Stock Exchange (ASE), with the largest market capitalization (blue chips), while the FTSE/ASE Mid 40 comprises 40 mid-capitalization Greek companies. Futures contracts are quoted on the Athens Derivatives Exchange (ADEX). The price of a futures contract is measured in index points multiplied by the contract multiplier, which is 5 Euros for the FTSE/ASE-20 contract and 10 Euros for the FTSE/ASE Mid 40 contract. There are four delivery months: March, June, September and December. Trading takes place in the three nearest delivery months, although volume in the far contract is very small. Both futures contracts are cash-settled and marked to market on the last trading day, which is the third Friday in the delivery (expiration) month at 14:30 Athens time.

V. EMPIRICAL RESULTS

First, unit root tests for log-stock prices and log-futures prices are applied for FTSE/ASE-20 and FTSE/ASE Mid 40. ADF and PP tests results for both series indicate presence of a unit root in each series. As a result, the null hypothesis of integration cannot be rejected. For the data in first differences, ADF and PP tests for both series are significant, indicating that the differenced series are not integrated. Therefore, all series are $I(1)$, and cointegration tests can be used to confirm whether there exists such a cointegrating structure between spot and futures markets. Johansen's approach suggests that spot and futures are cointegrated, with one cointegration relationship. Thus, there exists a linear combination of the Greek spot and futures price, which is not integrated.

A. The conventional approach

The optimal hedge ratio can be derived from the regression in Equation 1, where the returns to holding spot asset are regressed on the returns to holding the hedging instruments, see Ederington (1979). Table 1 presents these results for FTSE/ASE-20 index (Panel A) and FTSE/ASE Mid 40 index (Panel B). The hedge ratio for FTSE/ASE-20 is 0.916 086 and for FTSE/ASE Mid 40 is 0.703 317. In both cases, the estimated hedge ratio is significantly less than unity (i.e. $b < 1$).

B. An error correction approach

Following Chou *et al*. (1996) and Lien (1996), existence of cointegration between spot and futures prices will lead to a downwardly biased hedge ratio if the error correction term is neglected. Engle and Granger (1987) show that if two or more series are cointegrated, then there exists an error correction model (ECM). The ECM incorporates both short- and long-run information of data. In the present case, S and F are cointegrated, and therefore, the optimal hedge ratio can be calculated from an Error Correction model, see Equation 12.

An ECM is applied to obtain alternative estimates for the hedge ratio, so they can compared with those obtained from the conventional method. Thus, last-period's equilibrium error is taken into account. The results are reported in Table 2 for FTSE/ASE-20 index and Table 3 for FTSE/ASE Mid 40 index.

The results show a hedge ratio of 0.912302 for FTSE/ASE-20 and a hedge ratio of 0.715086 for FTSE/ASE Mid 40. Both hedge ratio coefficients in the hedge equations (ECM) are significantly different from zero at 5% significance level. Ghosh and Clayton (1996) argue that the superiority of the optimal hedge ratios arises from the

Table 1. *Hedge ratio estimates (OLS)*

	Coefficient	*t*-statistic
Panel FTSE/ASE-20		
c	−0.0001	−0.5006
ΔF_t	0.9160	43.1613*
Panel B. FTSE/ASE Mid 40		
c	−0.0008	−1.3243
ΔF_t	0.7033	22.8321

Notes: * Indicates significance at the 5% level.
Model: $\Delta S_t = c + b\Delta F_t + u_t$.

Table 2. *FTSE/ASE-20: ECM (Equation 12)*

	Coefficient	*t*-statistic
c	−0.0004	−1.5802
ΔS_{t-1}	−0.2818	−4.8445*
ΔS_{t-2}	−0.2131	−3.9041*
ΔS_{t-3}	−0.0433	−0.6623
ΔS_{t-4}	−0.1230	−2.1007*
ΔS_{t-5}	−0.1366	−2.2981*
ΔS_{t-6}	−0.1936	−3.1683*
ΔS_{t-7}	−0.1296	−2.5641*
ΔS_{t-8}	−0.0963	−1.6360
ΔF_t	0.9123	49.1043*
ΔF_{t-1}	0.3370	5.9183*
ΔF_{t-2}	0.1836	3.2803*
ΔF_{t-3}	0.0564	0.8762
ΔF_{t-4}	0.1240	2.0376*
ΔF_{t-5}	0.1406	2.3332*
ΔF_{t-6}	0.1731	2.9063*
ΔF_{t-7}	0.0966	1.9358*
ΔF_{t-8}	0.0718	1.1355
ε_{t-1}	−0.1016	−3.2383*

Notes: *Indicates significance at the 5% level.
Model: $\Delta S_t = c + a\varepsilon_{t-1} + b\Delta F_t + \Sigma_{i=1}^{n}\theta_i\Delta F_{t-i} + \Sigma_{j=1}^{k}\phi_j\Delta S_{t-j} + u_t$ where $\Delta S_t = S_t - S_{t-1}$, $\Delta F_t = F_t - F_{t-1}$ and $\varepsilon_{t-1} = S_{t-1} - (a + bF_{t-1})$.

Table 3. *FTSE/ASE Mid 40: ECM (Equation 12)*

	Coefficient	t-Statistic
c	−0.0007	−1.4268
ΔS_{t-1}	−0.3784	−5.7405*
ΔS_{t-2}	−0.3602	−4.2029*
ΔS_{t-3}	−0.1302	−1.5842
ΔS_{t-4}	−0.0677	−1.0650
ΔS_{t-5}	−0.0079	−0.1314
ΔS_{t-6}	−0.0558	−0.9386
ΔS_{t-7}	−0.1548	−2.6793*
ΔS_{t-8}	−0.1303	−2.0068*
ΔF_t	0.7150	28.1097*
ΔF_{t-1}	0.4531	8.1799*
ΔF_{t-2}	0.3003	3.7098*
ΔF_{t-3}	0.1742	2.3552*
ΔF_{t-4}	0.1016	1.7073*
ΔF_{t-5}	0.0152	0.2860
ΔF_{t-6}	0.0606	1.1103
ΔF_{t-7}	0.0960	1.8438*
ΔF_{t-8}	0.0998	1.6234
ε_{t-1}	−0.0546	−1.1832

Notes: *Indicates significance at the 5% level.
Model:
$\Delta S_t = c + a\varepsilon_{t-1} + b\Delta F_t + \sum_{i=1}^{n} \theta_i \Delta F_{t-i} + \sum_{j=1}^{k} \phi_j \Delta S_{t-j} + u_t$
where $\Delta S_t = S_t - S_{t-1}$, $\Delta F_t = F_t - F_{t-1}$ and $\varepsilon_{t-1} = S_{t-1} - (a + bF_{t-1})$.

likelihood ratio test. In addition, comparing estimated hedge ratio for FTSE/ASE-20, it is concluded that the hedge ratio estimated by Equation 12 is less than the one estimated by Equation 1. This implies that the conventional model overestimates the number of futures contracts needed to hedge the spot portfolio, Ghosh and Clayton (1996). Since the hedge ratio, estimated by ECM, is lower than the one estimated by OLS, then it could be said that the hedge ratio estimated by the ECM is more efficient in reducing the risk of spot change. Also, this implies that Greek investors need fewer contracts to hedge their spot risk. In other words, the lower ratio helps investors to improve their hedging performance at a lower cost. Also, the hedge ratio, estimated by ECM, significantly improves the performance of the hedging activity, while ECM provides better hedge ratio. On the other hand, for FTSE/ASE Mid 40, it is found that the traditional model underestimates the number of futures contracts, needed to hedge the spot portfolio. In this case, there is the observation of Ghosh and Clayton (1996) that portfolio managers can incur significant loss by using the traditional hedge ratio.

C. A Vector Error Correction approach

Furthermore, Ghosh (1993) and Lien (1996) suggest that, if spot and futures prices are cointegrated, non-inclusion of the error correction term in VAR model used to estimate the hedge ratio will lead to misspecification problems and under-estimation of the true optimal hedge ratio. In this section, a Vector Error Correction Model (VECM) is used to estimate hedge ratios. In particular, the unconditional variances of the spot prices (σ_{SS}), futures prices (σ_{FF}) and the covariance (σ_{SF}) of the two series is obtained from the residual covariance matrix of the VECM. The hedge ratio from VECM is thus calculated as $h = \sigma_{SF}/\sigma_{FF}$. The results are presented in Table 4. As expected, the hedge ratios, estimated from VECM, are greater than the ones obtained from other models. This is consistent with Ghosh (1993).

Table 4. *Hedge ratio estimates (VECM)*

Variances	FTSE/ASE-20	FTSE/ASE Mid 40
σ_{SS}	0.000 362	0.000 501
σ_{FF}	0.000 379	0.000 762
σ_{SF}	0.000 346	0.000 549
h	0.912 928	0.720 472

Notes: Model: $h = \sigma_{SF}/\sigma_{FF}$.

D. The multivariate GARCH model

The final approach for estimating time-varying hedge ratios is by employing a multivariate GARCH (M-GARCH) model. In this paper, a restricted version of the bivariate BEKK of Engle and Kroner (1995) is employed. The Bivariate cointegration model, with GARCH error structure, (BGARCH), incorporates a time varying conditional correlation coefficient between spot and futures prices and generates time-varying hedge ratios. Several BGARCH models (not reported in detail) are applied to the data, so the variance of each series can be modelled and hedge ratios estimated. In particular, to account for cointegration, the mean equations (first moment) is modelled with a bivariate error correction model, see Engle and Granger (1987), and in addition, time-varying variances and covariances are taken into account, by modelling the second moment with a bivariate GARCH(1,1) model as proposed in Engle and Kroner (1995) For estimation, a BHHH algorithm with the Marquardt correction is used. Akaike's information criterion (AIC) is used to select the best model (representation). Accordingly, the lower AIC value selects the model with the better fit to the data.

Specifically, mean hedge ratios are calculated from Equation 9. Table 5 reports the results from BGARCH (1,1) models, with 1, 2, 3, 4, 5, and 6 lags in the mean equation, for FTSE/ASE-20 (Panel A) and FTSE/ASE Mid 40 (Panel B). Although most empirical applications have restricted attention to BGARCH (1,1) model, with one lag for ΔS and one lag for ΔF, it is found that, for the present data, the BGARCH (1,1) model with two lags for ΔS and two lags for ΔF has the lowest AIC value. Therefore, this model is selected. The mean hedge ratio is 0.923 596 for FTSE/ASE-20 index, and 0.754 221 for FTSE/ASE Mid 40 index. Remarkably, hedge ratios estimated by BGARCH models are greater than the ones

Table 5. *Mean of the hedge ratio (BGARCH (1,1) model)*

Lags	AIC	Hedge ratio
Panel A. FTSE/ASE-20		
1	−12.321 02	0.925 789
2*	−12.329 52	0.923 596
3	−12.328 50	0.924 630
4	−12.320 40	0.925 031
5	−12.311 15	0.925 859
6	−12.319 67	0.928 269
Panel B. FTSE/ASE Mid 40		
1	−10.858 79	0.746 485
2*	−10.920 41	0.754 221
3	−10.919 90	0.755 136
4	−10.909 70	0.753 801
5	−10.901 47	0.754 189
6	−10.885 26	0.754 372

Notes: *Selected model.
The lag length is determined using the AIC.
Model: Bivariate cointegration GARCH(1,1) of spot and futures:

$$S_t = a_0 + a_1(S_{t-1} - \gamma F_{t-1}) + \varepsilon_{st}$$
$$F_t = \beta_0 + \beta_1(S_{t-1} - \gamma F_{t-1}) + \varepsilon_{ft}$$
$$\begin{bmatrix} \varepsilon_{st} \\ \varepsilon_{ft} \end{bmatrix} | \Psi_{t-1} \sim N(0, H_t)$$
$$h_{st}^2 = c_s + a_s \varepsilon_{s,t-1}^2 + b_s h_{s,t-1}^2$$
$$h_{ft}^2 = c_f + a_f \varepsilon_{f,t-1}^2 + b_f h_{f,t-1}^2$$
$$\text{Hedge ratio: } b_t = \frac{h_{sf,t}}{h_{ff,t}} = \left(\frac{\text{Cov}(\varepsilon_{st}, \varepsilon_{ft})}{\text{Var}(\varepsilon_{ft})}\right).$$

obtained from all other models. Hence, hedge ratios estimated by multivariate GARCH models should be more efficient in reducing risk of spot prices. Furthermore, this implies that all other models underestimate the number of futures contracts needed to hedge spot prices. In summary, the results show that the hedge ratio obtained from the BGARCH (1,1) model is superior to hedge ratios obtained from other models, and that this hedge ratio should provide better hedging.

VI. CONCLUSIONS

When using stock index futures for hedging (a technique to minimize risk mainly), estimates of the so-called hedge ratio are required. Various approaches for risk minimization lead to different estimation approaches for the hedge ratio. First, the naive or one-to-one hedge assumes that the correlation between the spot and futures is perfect (hedge ratio = 1). This hedge ratio fails to recognize that the correlation between spot and futures prices is less than perfect and ignores the stochastic nature of futures and spot prices, as well as time variation of hedge ratios. A second approach estimates the hedge ratio as the OLS coefficient of a regression of spot returns on futures return, see Ederington (1979), and Anderson and Danthine (1981).

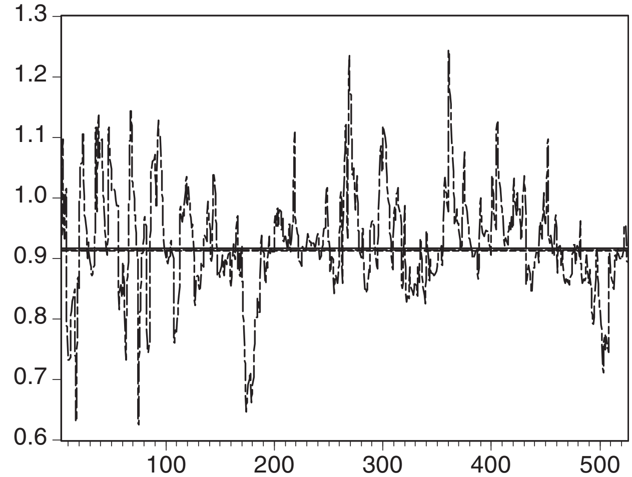

Fig. 1. *Hedge ratios for FTSE/ASE-20. (——) HR_OLS, (– – – –), HR_VECM (- - - - -) HR_ECM, (—·—) HR_BGARCH*

This approach imposes a constant hedge ratio, as well as constant conditional second moments over time. Since financial returns exhibit time-varying conditional heteroscedasticity, the hedge ratio may be treated as varying and estimated using GARCH models. The papers by Cecchetti *et al.* (1988), Myers and Thompson (1989) and Baillie and Myers (1991) highlight the issue of time-varying covariance matrices (and thus hedging ratios) in futures prices models. More specifically, Baillie and Myers (1991), and Kroner and Sultan (1993) highlight the use of M-GARCH models to capture time-variation and estimate variable hedge ratio by employing bivariate conditional heteroscedasticity models to measure optimal hedge ratio.

In this paper, the focus is on model specification and empirical comparison of several models for (optimal) hedge ratio estimation using data from Greek futures markets. In particular, we examine the behaviour of futures prices from FTSE/ASE-20 and FTSE/ASE Mid 40 indices is examined by employing various econometric methods, which include: the traditional regression model (OLS), ECM, VECM and the multivariate GARCH (M-GARCH) models; implying four alternative hedging strategies. First, empirical results show that error correction models (ECM) are superior to conventional (OLS) models. For FTSE/ASE-20, it is found that hedge ratio, estimated by ECM and VECM, is lower than the hedge ratios estimated by the traditional (OLS) method. Using a Bivariate cointegration model with GARCH (BGARCH), it is shown that the hedge ratio for FTSE/ASE-20 index is greater than hedge ratios obtained from OLS, ECM and VECM, respectively. For FTSE/ASE Mid 40, the conventional method underestimates the number of futures contracts needed to hedge the spot portfolio. However, the optimal hedge ratio, estimated from a BGARCH (1,1) model, is higher than hedge ratios, estimated by OLS, ECM and VECM, respectively. Hence, the

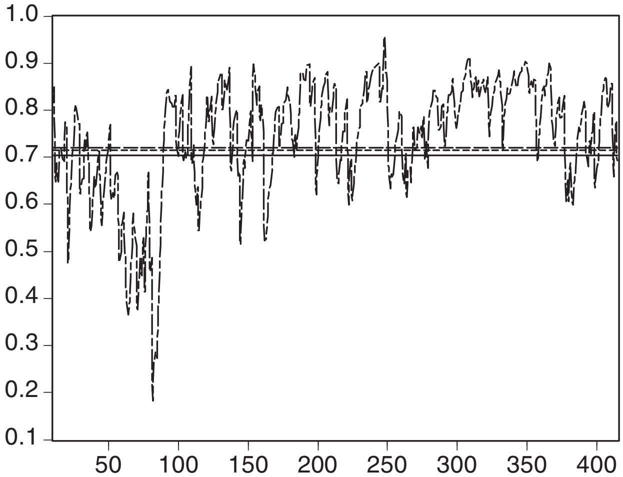

Fig. 2. *Hedge ratios for FTSE/ASE MID 40. (——) HR_OLS, (----), HR_VECM (-----) HR_ECM, (—-—) HR_BGARCH*

BGARCH (1,1) model is superior to the other models, in this respect. Finally, from Fig. 1 (for FTSE/ASE-20) and Fig. 2 (for FTSE/ASE Mid 40), it is clear that hedge ratios obtained from restricted versions of Bivariate BEKK (BGARCH (1,1) models), are time-varying, as new information arrives in the Greek market. Future research should examine the stability of the hedge ratios over time in Greek futures markets.

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