Στοιχειώδη σήματα και εφαρμογές

Κ.Σπανάκης

Τμήμα Ηλεκτρολόγων Μηχ/κών και Μηχ/κών Η/Υ

kspan@ics.forth.gr

1 Ασκήσεις

Άσκηση 1η

- Να υπολογιστούν η σταθερά συνιστώσα και οι συντελεστές της εκθετικής σειράς Fourier των κάτωθι περιοδικών σημάτων:

Άσκηση 1η

- Να υπολογιστούν η σταθερά συνιστώσα και οι συντελεστές της εκθετικής σειράς Fourier των κάτωθι περιοδικών σημάτων:

  1. $x(t) = \begin{cases} e^{-t}, & 0 < t \leq 1 \\ 0, & 1 < t \leq 2 \end{cases}$

Άσκηση 1η

- Να υπολογιστούν η σταθερά συνιστώσα και οι συντελεστές της εκθετικής σειράς Fourier των κάτωθι περιοδικών σημάτων:
  1. $x(t) = \begin{cases} e^{-t}, & 0 < t \leq 1 \\ 0, & 1 < t \leq 2 \end{cases}$
  2. $x(t) = t, \ -T < t \leq T$

Άσκηση 1η

- Να υπολογιστούν η σταθερά συνιστώσα και οι συντελεστές της εκθετικής σειράς Fourier των κάτωθι περιοδικών σημάτων:
  1. $x(t) = \begin{cases} e^{-t}, & 0 < t \leq 1 \\ 0, & 1 < t \leq 2 \end{cases}$
  2. $x(t) = t, \; -T < t \leq T$
  3. $x(t) = \begin{cases} 0, & 1 < |t| \leq 1.5 \\ t+1, & 0 \leq t \leq 1 \\ -t+1, & -1 \leq t < 0 \end{cases}$

Λύση 1ου σήματος: Σταθερά Συνιστώσα

- $x(t) = \begin{cases} e^{-t}, & 0 < t \leq 1 \\ 0, & 1 < t \leq 2 \end{cases} \Rightarrow T = 2 \Rightarrow \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \Rightarrow \alpha_0 =$
$\frac{1}{T}\int_0^2 x(t)dt = \frac{1}{2}\int_0^1 e^{-t}dt + \frac{1}{2}\int_1^2 0dt = \frac{1}{2}\int_0^1 e^{-t}dt = -\frac{1}{2}\int_0^1 -e^{-t}dt =$
$-\frac{1}{2}\int_0^1 (e^{-t})'dt = -\frac{1}{2}\left[e^{-t}\right]_0^1 = -\frac{e^{-1}-e^0}{2} = \frac{1-e^{-1}}{2} = \frac{1-\frac{1}{e}}{2} = \frac{\frac{e-1}{e}}{2} = \frac{e-1}{2e}$

Λύση 1ου σήματος: Συντελεστές Fourier

- $x(t) = \begin{cases} e^{-t}, & 0 < t \leq 1 \\ 0, & 1 < t \leq 2 \end{cases} \Rightarrow T = 2 \Rightarrow \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \Rightarrow \alpha_k =$
$\frac{1}{T}\int_0^2 x(t)e^{-jk\omega_0 t}dt = \frac{1}{2}\int_0^1 e^{-t}e^{-jk\omega_0 t}dt + \frac{1}{2}\int_1^2 0 \cdot e^{-jk\omega_0 t}dt = \frac{1}{2}\int_0^1 e^{-(jk\omega_0+1)t}dt$

Λύση 1ου σήματος: Συντελεστές Fourier

- $x(t) = \begin{cases} e^{-t}, & 0 < t \leq 1 \\ 0, & 1 < t \leq 2 \end{cases} \Rightarrow T = 2 \Rightarrow \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \Rightarrow \alpha_k =$
$\frac{1}{T}\int_0^2 x(t)e^{-jk\omega_0 t}dt = \frac{1}{2}\int_0^1 e^{-t}e^{-jk\omega_0 t}dt + \frac{1}{2}\int_1^2 0\cdot e^{-jk\omega_0 t}dt = \frac{1}{2}\int_0^1 e^{-(jk\omega_0+1)t}dt$

Λύση 1ου σήματος: Συντελεστές Fourier

- $x(t) = \begin{cases} e^{-t}, & 0 < t \leq 1 \\ 0, & 1 < t \leq 2 \end{cases} \Rightarrow T = 2 \Rightarrow \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \Rightarrow \alpha_k =$
$\frac{1}{T}\int_0^2 x(t)e^{-jk\omega_0 t}dt = \frac{1}{2}\int_0^1 e^{-t}e^{-jk\omega_0 t}dt + \frac{1}{2}\int_1^2 0 \cdot e^{-jk\omega_0 t}dt = \frac{1}{2}\int_0^1 e^{-(jk\omega_0+1)t}dt$
$\xrightarrow[du=-(jk\omega_0+1)dt]{u=-(jk\omega_0+1)t} \alpha_k = \frac{1}{2}\int_{-(jk\omega_0+1)\cdot 0}^{-(jk\omega_0+1)\cdot 1} e^u \left(-\frac{du}{jk\omega_0+1}\right) = -\frac{1}{2}\frac{1}{jk\pi+1}\int_0^{-(jk\pi+1)} e^u du$

Λύση 1ου σήματος: Συντελεστές Fourier

- $x(t) = \begin{cases} e^{-t}, & 0 < t \leq 1 \\ 0, & 1 < t \leq 2 \end{cases} \Rightarrow T = 2 \Rightarrow \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \Rightarrow \alpha_k =$
$\frac{1}{T}\int_0^2 x(t)e^{-jk\omega_0 t}dt = \frac{1}{2}\int_0^1 e^{-t}e^{-jk\omega_0 t}dt + \frac{1}{2}\int_1^2 0\cdot e^{-jk\omega_0 t}dt = \frac{1}{2}\int_0^1 e^{-(jk\omega_0+1)t}dt$
$\xrightarrow[du=-(jk\omega_0+1)dt]{u=-(jk\omega_0+1)t} \alpha_k = \frac{1}{2}\int_{-(jk\omega_0+1)\cdot 0}^{-(jk\omega_0+1)\cdot 1} e^u\left(-\frac{du}{jk\omega_0+1}\right) = -\frac{1}{2}\frac{1}{jk\pi+1}\int_0^{-(jk\pi+1)} e^u du$
$= \frac{1}{2(jk\pi+1)}\int_{-(jk\pi+1)}^0 e^u du = \frac{1}{2(jk\pi+1)}\left[e^u\right]_{-(jk\pi+1)}^0$
- $k \neq 0 \Rightarrow \alpha_k = \frac{1}{2(jk\pi+1)}\left(e^0 - e^{-1-jk\pi}\right) = \frac{1}{2(jk\pi+1)}\left(1 - \frac{e^{-jk\pi}}{e}\right) =$
$\frac{(-jk\pi+1)}{2(jk\pi+1)(-jk\pi+1)}\left(1 - \frac{\cos(k\pi) - j\sin(k\pi)}{e}\right) =$
$\begin{cases} \frac{(-jk\pi+1)}{2(1+k^2\pi^2)}\left(1-\frac{1}{e}\right), & k = 2n \\ \frac{(-jk\pi+1)}{2(1+k^2\pi^2)}\left(1+\frac{1}{e}\right), & k = 2n+1 \end{cases}$

Λύση 2ου σήματος: Σταθερά Συνιστώσα

- $x(t) = t, \; -T < t \leq T \Rightarrow T_0 = T - (-T) = 2T \Rightarrow \omega_0 = \dfrac{2\pi}{T_0} = \dfrac{2\pi}{2T} = \dfrac{\pi}{T} \Rightarrow \alpha_k = \dfrac{1}{T_0}\displaystyle\int_{-T}^{T} x(t)e^{-jk\omega_0 t}dt = \dfrac{1}{2T}\displaystyle\int_{-T}^{T} te^{-jk\omega_0 t}dt \xrightarrow{k=0} \alpha_0 = \dfrac{1}{2T}\displaystyle\int_{-T}^{T} tdt = 0$, διότι η συνάρτηση $x(t) = t$ είναι περιττή.

Λύση 2ου σήματος: Συντελεστές Fourier

- $x(t) = t, \ -T < t \leq T \Rightarrow T_0 = T - (-T) = 2T \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T} = \frac{\pi}{T} \Rightarrow \alpha_k =$
$\frac{1}{T_0}\int_{-T}^{T} x(t)e^{-jk\omega_0 t}dt = \frac{1}{2T}\int_{-T}^{T} te^{-jk\omega_0 t}dt \xrightarrow[du=-jk\omega_0 dt]{u=-jk\omega_0 t} \alpha_k =$
$\frac{1}{2T}\int_{-T(-jk\omega_0)}^{T(-jk\omega_0)} \left(-\frac{u}{jk\omega_0}\right) e^u \left(-\frac{du}{jk\omega_0}\right) = \frac{1}{2T}\int_{jkT\omega_0}^{-jkT\omega_0} \left(-\frac{u}{jk\omega_0}\right) e^u \left(-\frac{du}{jk\omega_0}\right)$

Λύση 2ου σήματος: Συντελεστές Fourier

- $x(t) = t, \ -T < t \leq T \Rightarrow T_0 = T - (-T) = 2T \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T} = \frac{\pi}{T} \Rightarrow \alpha_k = \frac{1}{T_0}\int_{-T}^{T} x(t)e^{-jk\omega_0 t}dt = \frac{1}{2T}\int_{-T}^{T} te^{-jk\omega_0 t}dt \xrightarrow[du=-jk\omega_0 dt]{u=-jk\omega_0 t} \alpha_k = \frac{1}{2T}\int_{-T(-jk\omega_0)}^{T(-jk\omega_0)} \left(-\frac{u}{jk\omega_0}\right) e^u \left(-\frac{du}{jk\omega_0}\right) = \frac{1}{2T}\int_{jkT\omega_0}^{-jkT\omega_0} \left(-\frac{u}{jk\omega_0}\right) e^u \left(-\frac{du}{jk\omega_0}\right)$

Λύση 2ου σήματος: Συντελεστές Fourier

- $x(t) = t,\ -T < t \leq T \Rightarrow T_0 = T - (-T) = 2T \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T} = \frac{\pi}{T} \Rightarrow \alpha_k =$
$\frac{1}{T_0}\int_{-T}^{T} x(t)e^{-jk\omega_0 t}dt = \frac{1}{2T}\int_{-T}^{T} te^{-jk\omega_0 t}dt \xrightarrow[du=-jk\omega_0 dt]{u=-jk\omega_0 t} \alpha_k =$
$\frac{1}{2T}\int_{-T(-jk\omega_0)}^{T(-jk\omega_0)} \left(-\frac{u}{jk\omega_0}\right) e^u \left(-\frac{du}{jk\omega_0}\right) = \frac{1}{2T}\int_{jkT\omega_0}^{-jkT\omega_0} \left(-\frac{u}{jk\omega_0}\right) e^u \left(-\frac{du}{jk\omega_0}\right)$
$= \frac{1}{2T}\left(-\frac{1}{jk\omega_0}\right)^2 \int_{jk\pi}^{-jk\pi} ue^u du = \frac{1}{2T}\frac{1}{j^2k^2\omega_0^2}\int_{jk\pi}^{-jk\pi} ue^u du =$

Λύση 2ου σήματος: Συντελεστές Fourier

- $x(t) = t, \ -T < t \leq T \Rightarrow T_0 = T - (-T) = 2T \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T} = \frac{\pi}{T} \Rightarrow \alpha_k =$
$\frac{1}{T_0}\int_{-T}^{T} x(t)e^{-jk\omega_0 t}dt = \frac{1}{2T}\int_{-T}^{T} te^{-jk\omega_0 t}dt \xrightarrow[du=-jk\omega_0 dt]{u=-jk\omega_0 t} \alpha_k =$
$\frac{1}{2T}\int_{-T(-jk\omega_0)}^{T(-jk\omega_0)} \left(-\frac{u}{jk\omega_0}\right) e^{u} \left(-\frac{du}{jk\omega_0}\right) = \frac{1}{2T}\int_{jkT\omega_0}^{-jkT\omega_0} \left(-\frac{u}{jk\omega_0}\right) e^{u} \left(-\frac{du}{jk\omega_0}\right)$
$= \frac{1}{2T}\left(-\frac{1}{jk\omega_0}\right)^2 \int_{jk\pi}^{-jk\pi} ue^{u}du = \frac{1}{2T}\frac{1}{j^2k^2\omega_0^2}\int_{jk\pi}^{-jk\pi} ue^{u}du =$
$\frac{1}{2T}\left(-\frac{1}{k^2\omega_0^2}\right)\int_{jk\pi}^{-jk\pi} ue^{u}du = \frac{1}{2Tk^2\omega_0^2}\int_{-jk\pi}^{jk\pi} ue^{u}du = \frac{1}{2Tk^2\omega_0^2}\int_{-jk\pi}^{jk\pi} u\left(e^{u}\right)' du =$

Λύση 2ου σήματος: Συντελεστές Fourier

- $x(t) = t, \ -T < t \leq T \Rightarrow T_0 = T - (-T) = 2T \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T} = \frac{\pi}{T} \Rightarrow \alpha_k =$
  $\frac{1}{T_0}\int_{-T}^{T} x(t)e^{-jk\omega_0 t}dt = \frac{1}{2T}\int_{-T}^{T} te^{-jk\omega_0 t}dt \xrightarrow[du=-jk\omega_0 dt]{u=-jk\omega_0 t} \alpha_k =$
  $\frac{1}{2T}\int_{-T(-jk\omega_0)}^{T(-jk\omega_0)} \left(-\frac{u}{jk\omega_0}\right) e^u \left(-\frac{du}{jk\omega_0}\right) = \frac{1}{2T}\int_{jkT\omega_0}^{-jkT\omega_0} \left(-\frac{u}{jk\omega_0}\right) e^u \left(-\frac{du}{jk\omega_0}\right)$
  $= \frac{1}{2T}\left(-\frac{1}{jk\omega_0}\right)^2 \int_{jk\pi}^{-jk\pi} ue^u du = \frac{1}{2T}\frac{1}{j^2k^2\omega_0^2}\int_{jk\pi}^{-jk\pi} ue^u du =$
  $\frac{1}{2T}\left(-\frac{1}{k^2\omega_0^2}\right)\int_{jk\pi}^{-jk\pi} ue^u du = \frac{1}{2Tk^2\omega_0^2}\int_{-jk\pi}^{jk\pi} ue^u du = \frac{1}{2Tk^2\omega_0^2}\int_{-jk\pi}^{jk\pi} u\left(e^u\right)' du =$
  $\frac{1}{2Tk^2\omega_0^2}\left(\left[ue^u\right]_{-jk\pi}^{jk\pi} - \int_{-jk\pi}^{jk\pi} u' e^u du\right) = \frac{1}{2Tk^2\omega_0^2}\left(\left[ue^u\right]_{-jk\pi}^{jk\pi} - \int_{-jk\pi}^{jk\pi} e^u du\right) =$

Λύση 2ου σήματος: Συντελεστές Fourier

- $x(t) = t, \ -T < t \leq T \Rightarrow T_0 = T - (-T) = 2T \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T} = \frac{\pi}{T} \Rightarrow \alpha_k =$
$\frac{1}{T_0}\int_{-T}^{T} x(t)e^{-jk\omega_0 t}dt = \frac{1}{2T}\int_{-T}^{T} te^{-jk\omega_0 t}dt \xrightarrow[du=-jk\omega_0 dt]{u=-jk\omega_0 t} \alpha_k =$
$\frac{1}{2T}\int_{-T(-jk\omega_0)}^{T(-jk\omega_0)}\left(-\frac{u}{jk\omega_0}\right)e^u\left(-\frac{du}{jk\omega_0}\right) = \frac{1}{2T}\int_{jkT\omega_0}^{-jkT\omega_0}\left(-\frac{u}{jk\omega_0}\right)e^u\left(-\frac{du}{jk\omega_0}\right)$
$= \frac{1}{2T}\left(-\frac{1}{jk\omega_0}\right)^2\int_{jk\pi}^{-jk\pi} ue^u du = \frac{1}{2T}\frac{1}{j^2k^2\omega_0^2}\int_{jk\pi}^{-jk\pi} ue^u du =$
$\frac{1}{2T}\left(-\frac{1}{k^2\omega_0^2}\right)\int_{jk\pi}^{-jk\pi} ue^u du = \frac{1}{2Tk^2\omega_0^2}\int_{-jk\pi}^{jk\pi} ue^u du = \frac{1}{2Tk^2\omega_0^2}\int_{-jk\pi}^{jk\pi} u\left(e^u\right)' du =$
$\frac{1}{2Tk^2\omega_0^2}\left(\left[ue^u\right]_{-jk\pi}^{jk\pi} - \int_{-jk\pi}^{jk\pi} u' e^u du\right) = \frac{1}{2Tk^2\omega_0^2}\left(\left[ue^u\right]_{-jk\pi}^{jk\pi} - \int_{-jk\pi}^{jk\pi} e^u du\right) =$
$\frac{1}{2Tk^2\omega_0^2}\left(\left[ue^u\right]_{-jk\pi}^{jk\pi} - \int_{-jk\pi}^{jk\pi} u' e^u du\right) = \frac{1}{2Tk^2\omega_0^2}\left(\left[ue^u\right]_{-jk\pi}^{jk\pi} - \left[e^u\right]_{-jk\pi}^{jk\pi}\right) =$

Λύση 2ου σήματος: Συντελεστές Fourier

- $x(t) = t, \ -T < t \leq T \Rightarrow T_0 = T - (-T) = 2T \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T} = \frac{\pi}{T} \Rightarrow \alpha_k =$
$\frac{1}{T_0}\int_{-T}^{T} x(t)e^{-jk\omega_0 t}dt = \frac{1}{2T}\int_{-T}^{T} te^{-jk\omega_0 t}dt \xrightarrow[du=-jk\omega_0 dt]{u=-jk\omega_0 t} \alpha_k =$
$\frac{1}{2T}\int_{-T(-jk\omega_0)}^{T(-jk\omega_0)} \left(-\frac{u}{jk\omega_0}\right) e^u \left(-\frac{du}{jk\omega_0}\right) = \frac{1}{2T}\int_{jkT\omega_0}^{-jkT\omega_0} \left(-\frac{u}{jk\omega_0}\right) e^u \left(-\frac{du}{jk\omega_0}\right)$
$= \frac{1}{2T}\left(-\frac{1}{jk\omega_0}\right)^2 \int_{jk\pi}^{-jk\pi} ue^u du = \frac{1}{2T}\frac{1}{j^2k^2\omega_0^2}\int_{jk\pi}^{-jk\pi} ue^u du =$
$\frac{1}{2T}\left(-\frac{1}{k^2\omega_0^2}\right)\int_{jk\pi}^{-jk\pi} ue^u du = \frac{1}{2Tk^2\omega_0^2}\int_{-jk\pi}^{jk\pi} ue^u du = \frac{1}{2Tk^2\omega_0^2}\int_{-jk\pi}^{jk\pi} u\left(e^u\right)' du =$
$\frac{1}{2Tk^2\omega_0^2}\left(\left[ue^u\right]_{-jk\pi}^{jk\pi} - \int_{-jk\pi}^{jk\pi} u' e^u du\right) = \frac{1}{2Tk^2\omega_0^2}\left(\left[ue^u\right]_{-jk\pi}^{jk\pi} - \int_{-jk\pi}^{jk\pi} e^u du\right) =$
$\frac{1}{2Tk^2\omega_0^2}\left(\left[ue^u\right]_{-jk\pi}^{jk\pi} - \int_{-jk\pi}^{jk\pi} u' e^u du\right) = \frac{1}{2Tk^2\omega_0^2}\left(\left[ue^u\right]_{-jk\pi}^{jk\pi} - \left[e^u\right]_{-jk\pi}^{jk\pi}\right) =$
$\frac{1}{2Tk^2\omega_0^2}\left[(u-1)e^u\right]_{-jk\pi}^{jk\pi} = \frac{1}{2Tk^2\omega_0^2}\left[(jk\pi - 1)e^{jk\pi} - (-jk\pi - 1)e^{-jk\pi}\right] =$

Λύση 2ου σήματος: Συντελεστές Fourier

- $x(t) = t, \ -T < t \leq T \Rightarrow T_0 = T - (-T) = 2T \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T} = \frac{\pi}{T} \Rightarrow \alpha_k =$
$\frac{1}{T_0}\int_{-T}^{T} x(t)e^{-jk\omega_0 t}dt = \frac{1}{2T}\int_{-T}^{T} te^{-jk\omega_0 t}dt \xrightarrow[du=-jk\omega_0 dt]{u=-jk\omega_0 t} \alpha_k =$
$\frac{1}{2T}\int_{-T(-jk\omega_0)}^{T(-jk\omega_0)}\left(-\frac{u}{jk\omega_0}\right)e^u\left(-\frac{du}{jk\omega_0}\right) = \frac{1}{2T}\int_{jkT\omega_0}^{-jkT\omega_0}\left(-\frac{u}{jk\omega_0}\right)e^u\left(-\frac{du}{jk\omega_0}\right)$
$= \frac{1}{2T}\left(-\frac{1}{jk\omega_0}\right)^2\int_{jk\pi}^{-jk\pi} ue^u du = \frac{1}{2T}\frac{1}{j^2k^2\omega_0^2}\int_{jk\pi}^{-jk\pi} ue^u du =$
$\frac{1}{2T}\left(-\frac{1}{k^2\omega_0^2}\right)\int_{jk\pi}^{-jk\pi} ue^u du = \frac{1}{2Tk^2\omega_0^2}\int_{-jk\pi}^{jk\pi} ue^u du = \frac{1}{2Tk^2\omega_0^2}\int_{-jk\pi}^{jk\pi} u\left(e^u\right)' du =$
$\frac{1}{2Tk^2\omega_0^2}\left(\left[ue^u\right]_{-jk\pi}^{jk\pi} - \int_{-jk\pi}^{jk\pi} u' e^u du\right) = \frac{1}{2Tk^2\omega_0^2}\left(\left[ue^u\right]_{-jk\pi}^{jk\pi} - \int_{-jk\pi}^{jk\pi} e^u du\right) =$
$\frac{1}{2Tk^2\omega_0^2}\left(\left[ue^u\right]_{-jk\pi}^{jk\pi} - \int_{-jk\pi}^{jk\pi} u' e^u du\right) = \frac{1}{2Tk^2\omega_0^2}\left(\left[ue^u\right]_{-jk\pi}^{jk\pi} - \left[e^u\right]_{-jk\pi}^{jk\pi}\right) =$
$\frac{1}{2Tk^2\omega_0^2}\left[(u-1)e^u\right]_{-jk\pi}^{jk\pi} = \frac{1}{2Tk^2\omega_0^2}\left[(jk\pi - 1)e^{jk\pi} - (-jk\pi - 1)e^{-jk\pi}\right] =$
$\frac{1}{2Tk^2\omega_0^2}\left[(jk\pi - 1)(\cos k\pi + j\sin k\pi) - (-jk\pi - 1)(\cos k\pi - j\sin k\pi)\right] =$
$\frac{1}{2Tk^2\omega_0^2}\cdot 2jk\pi\cos k\pi = \frac{2jk\pi\cos k\pi}{2T\omega_0 k^2\omega_0} = \frac{j\cos k\pi}{k\omega_0} = (-1)^k\frac{j}{k\omega_0}, \ \ k \neq 0$

Λύση 3ου σήματος: Σταθερά Συνιστώσα

- $x(t) = t, \ -1.5 < t \leq 1.5 \Rightarrow T_0 = 1.5 - (-1.5) = 3 \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{3} \Rightarrow \alpha_k =$ $\frac{1}{T_0}\int_{-1.5}^{1.5} x(t)dt = \frac{1}{T_0}\int_{-1.5}^{-1} 0dt + \frac{1}{T_0}\int_{-1}^{0}(-t+1)dt + \frac{1}{T_0}\int_{0}^{1}(t+1)dt + \frac{1}{T_0}\int_{1}^{1.5} 0dt =$ $\frac{1}{T_0}\left[-\frac{t^2}{2}+t\right]_{-1}^{0} + \frac{1}{T_0}\left[\frac{t^2}{2}+t\right]_{0}^{1} = \frac{1}{3}\left[-\frac{0^2}{2}+0-\left(-\frac{(-1)^2}{2}+(-1)\right)\right] +$ $\frac{1}{3}\left[\frac{1^2}{2}+1-\left(\frac{0^2}{2}+0\right)\right] = \frac{1}{3}\cdot\left(-\frac{3}{2}\right)+\frac{1}{3}\cdot\frac{3}{2} = -\frac{1}{2}+\frac{1}{2} = 0$

Λύση 3ου σήματος: Συντελεστές Fourier

- $x(t) = t,\ -1.5 < t \leq 1.5 \Rightarrow T_0 = 1.5 - (-1.5) = 3 \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{3} \Rightarrow \alpha_k = \frac{1}{T_0}\int_{-1.5}^{1.5} x(t)e^{-jk\omega_0 t}dt = \frac{1}{T_0}\int_{-1.5}^{-1} 0e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{-1}^{0}(-t+1)e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{0}^{1}(t+1)e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{1}^{1.5} 0e^{-jk\omega_0 t}dt =$

Λύση 3ου σήματος: Συντελεστές Fourier

- $x(t) = t, \ -1.5 < t \leq 1.5 \Rightarrow T_0 = 1.5 - (-1.5) = 3 \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{3} \Rightarrow \alpha_k = \frac{1}{T_0}\int_{-1.5}^{1.5} x(t)e^{-jk\omega_0 t}dt = \frac{1}{T_0}\int_{-1.5}^{-1} 0e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{-1}^{0}(-t+1)e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{0}^{1}(t+1)e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{1}^{1.5} 0e^{-jk\omega_0 t}dt =$

Λύση 3ου σήματος: Συντελεστές Fourier

- $x(t) = t,\ -1.5 < t \leq 1.5 \Rightarrow T_0 = 1.5 - (-1.5) = 3 \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{3} \Rightarrow \alpha_k =$
$\frac{1}{T_0}\int_{-1.5}^{1.5} x(t)e^{-jk\omega_0 t}dt = \frac{1}{T_0}\int_{-1.5}^{-1} 0e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{-1}^{0}(-t+1)e^{-jk\omega_0 t}dt +$
$\frac{1}{T_0}\int_{0}^{1}(t+1)e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{1}^{1.5} 0e^{-jk\omega_0 t}dt =$
$\frac{1}{T_0}\int_{-1}^{0}(-t+1)e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{0}^{1}(t+1)e^{-jk\omega_0 t}dt =$

Λύση 3ου σήματος: Συντελεστές Fourier

- $x(t) = t,\ -1.5 < t \leq 1.5 \Rightarrow T_0 = 1.5 - (-1.5) = 3 \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{3} \Rightarrow \alpha_k =$
$\frac{1}{T_0}\int_{-1.5}^{1.5} x(t)e^{-jk\omega_0 t}dt = \frac{1}{T_0}\int_{-1.5}^{-1} 0e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{-1}^{0}(-t+1)e^{-jk\omega_0 t}dt +$
$\frac{1}{T_0}\int_{0}^{1}(t+1)e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{1}^{1.5} 0e^{-jk\omega_0 t}dt =$
$\frac{1}{T_0}\int_{-1}^{0}(-t+1)e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{0}^{1}(t+1)e^{-jk\omega_0 t}dt =$
$\frac{1}{T_0}\int_{-1}^{0} -te^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{-1}^{0} 1\cdot e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{0}^{t} e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{0}^{1} 1\cdot$
$e^{-jk\omega_0 t}dt \xRightarrow[du=-jk\omega_0 dt]{u=-jk\omega_0 t}$

Λύση 3ου σήματος: Συντελεστές Fourier

- $x(t) = t,\ -1.5 < t \le 1.5 \Rightarrow T_0 = 1.5 - (-1.5) = 3 \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{3} \Rightarrow \alpha_k =$
$\frac{1}{T_0}\int_{-1.5}^{1.5} x(t)e^{-jk\omega_0 t}dt = \frac{1}{T_0}\int_{-1.5}^{-1} 0e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{-1}^{0}(-t+1)e^{-jk\omega_0 t}dt +$
$\frac{1}{T_0}\int_{0}^{1}(t+1)e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{1}^{1.5} 0e^{-jk\omega_0 t}dt =$
$\frac{1}{T_0}\int_{-1}^{0}(-t+1)e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{0}^{1}(t+1)e^{-jk\omega_0 t}dt =$
$\frac{1}{T_0}\int_{-1}^{0} -te^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{-1}^{0} 1\cdot e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{0}^{t} e^{-jk\omega_0 t}dt + \frac{1}{T_0}\int_{0}^{1} 1\cdot$
$e^{-jk\omega_0 t}dt \xrightarrow[du=-jk\omega_0 dt]{u=-jk\omega_0 t}$
$\alpha_k = \frac{1}{T_0}\int_{-1\cdot(-jk\omega_0)}^{0\cdot(-jk\omega_0)} -\left(-\frac{u}{jk\omega_0}\right)e^u\left(-\frac{du}{jk\omega_0}\right) + \frac{1}{T_0}\int_{-1\cdot(-jk\omega_0)}^{0\cdot(-jk\omega_0)} 1\cdot e^u\left(-\frac{du}{jk\omega_0}\right) +$
$\frac{1}{T_0}\int_{0\cdot(-jk\omega_0)}^{1\cdot(-jk\omega_0)}\left(-\frac{u}{jk\omega_0}\right)e^u\left(-\frac{du}{jk\omega_0}\right) + \frac{1}{T_0}\int_{0\cdot(-jk\omega_0)}^{1\cdot(-jk\omega_0)} 1\cdot e^u\left(-\frac{du}{jk\omega_0}\right) \Rightarrow$

Λύση 3ου σήματος: Συντελεστές Fourier(Συνέχεια)

- $\alpha_k = \frac{1}{T_0}\int_{jk\omega_0}^{0} -\left(-\frac{u}{jk\omega_0}\right)e^{u}\left(-\frac{du}{jk\omega_0}\right) + \frac{1}{T_0}\int_{jk\omega_0}^{0} 1\cdot e^{u}\left(-\frac{du}{jk\omega_0}\right) + \frac{1}{T_0}\int_{0}^{-jk\omega_0}\left(-\frac{u}{jk\omega_0}\right)e^{u}\left(-\frac{du}{jk\omega_0}\right) + \frac{1}{T_0}\int_{0}^{-jk\omega_0} 1\cdot e^{u}\left(-\frac{du}{jk\omega_0}\right) \Rightarrow$

Λύση 3ου σήματος: Συντελεστές Fourier(Συνέχεια)

- $\alpha_k = \frac{1}{T_0}\int_{jk\omega_0}^{0} -\left(-\frac{u}{jk\omega_0}\right)e^{u}\left(-\frac{du}{jk\omega_0}\right) + \frac{1}{T_0}\int_{jk\omega_0}^{0} 1\cdot e^{u}\left(-\frac{du}{jk\omega_0}\right) + \frac{1}{T_0}\int_{0}^{-jk\omega_0}\left(-\frac{u}{jk\omega_0}\right)e^{u}\left(-\frac{du}{jk\omega_0}\right) + \frac{1}{T_0}\int_{0}^{-jk\omega_0} 1\cdot e^{u}\left(-\frac{du}{jk\omega_0}\right) \Rightarrow$

Λύση 3ου σήματος: Συντελεστές Fourier(Συνέχεια)

- $\alpha_k = \frac{1}{T_0}\int_{jk\omega_0}^{0} -\left(-\frac{u}{jk\omega_0}\right)e^u\left(-\frac{du}{jk\omega_0}\right) + \frac{1}{T_0}\int_{jk\omega_0}^{0} 1\cdot e^u\left(-\frac{du}{jk\omega_0}\right) +$
$\frac{1}{T_0}\int_{0}^{-jk\omega_0}\left(-\frac{u}{jk\omega_0}\right)e^u\left(-\frac{du}{jk\omega_0}\right) + \frac{1}{T_0}\int_{0}^{-jk\omega_0} 1\cdot e^u\left(-\frac{du}{jk\omega_0}\right) \Rightarrow$
$\alpha_k = -\frac{1}{T_0(jk\omega_0)^2}\int_{jk\omega_0}^{0} ue^u du - \frac{1}{T_0(jk\omega_0)}\int_{jk\omega_0}^{0} e^u du + \frac{1}{T_0(jk\omega_0)^2}\int_{0}^{-jk\omega_0} ue^u du -$
$\frac{1}{T_0(jk\omega_0)}\int_{0}^{-jk\omega_0} e^u du \Rightarrow$

Λύση 3ου σήματος: Συντελεστές Fourier(Συνέχεια)

- $$\alpha_k = \frac{1}{T_0}\int_{jk\omega_0}^{0} -\left(-\frac{u}{jk\omega_0}\right)e^u\left(-\frac{du}{jk\omega_0}\right) + \frac{1}{T_0}\int_{jk\omega_0}^{0} 1\cdot e^u\left(-\frac{du}{jk\omega_0}\right) + \frac{1}{T_0}\int_{0}^{-jk\omega_0}\left(-\frac{u}{jk\omega_0}\right)e^u\left(-\frac{du}{jk\omega_0}\right) + \frac{1}{T_0}\int_{0}^{-jk\omega_0} 1\cdot e^u\left(-\frac{du}{jk\omega_0}\right) \Rightarrow$$

  $$\alpha_k = -\frac{1}{T_0(jk\omega_0)^2}\int_{jk\omega_0}^{0} ue^u du - \frac{1}{T_0(jk\omega_0)}\int_{jk\omega_0}^{0} e^u du + \frac{1}{T_0(jk\omega_0)^2}\int_{0}^{-jk\omega_0} ue^u du - \frac{1}{T_0(jk\omega_0)}\int_{0}^{-jk\omega_0} e^u du \Rightarrow$$

  $$\alpha_k = -\frac{1}{T_0 j^2 k^2\omega_0^2}\left[u(e^u-1)\right]_{jk\omega_0}^{0} + \frac{1}{T_0\omega_0 jk}\int_{0}^{jk\omega_0} e^u du + \frac{1}{T_0 j^2 k^2\omega_0^2}\left[u(e^u-1)\right]_{0}^{-jk\omega_0} + \frac{1}{T_0\omega_0 jk}\int_{-jk\omega_0}^{0} e^u du \Rightarrow$$

Λύση 3ου σήματος: Συντελεστές Fourier(Συνέχεια επί της Συνέχειας)

- $\alpha_k = -\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u - 1)\right]_{jk\omega_0}^{0} + \frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u - 1)\right]_{0}^{-jk\omega_0} + \frac{1}{T_0 \omega_0 jk}\int_{-jk\omega_0}^{jk\omega_0} e^u du$

Λύση 3ου σήματος: Συντελεστές Fourier(Συνέχεια επί της Συνέχειας)

- $\alpha_k = -\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u - 1)\right]_{jk\omega_0}^{0} + \frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u - 1)\right]_{0}^{-jk\omega_0} + \frac{1}{T_0 \omega_0 jk}\int_{-jk\omega_0}^{jk\omega_0} e^u du$

Λύση 3ου σήματος: Συντελεστές Fourier(Συνέχεια επί της Συνέχειας)

- $\alpha_k = -\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u - 1)\right]_{jk\omega_0}^{0} + \frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u - 1)\right]_{0}^{-jk\omega_0} + \frac{1}{T_0 \omega_0 jk}\int_{-jk\omega_0}^{jk\omega_0} e^u du$
  $\Rightarrow \alpha_k = -\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[0(e^0 - 1) - jk\omega_0(e^{jk\omega_0} - 1)\right] +$
  $\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[-jk\omega_0(e^{-jk\omega_0} - 1) - 0(e^u - 1)\right] + \frac{1}{T_0 \omega_0 jk}\left[e^u\right]_{-jk\omega_0}^{jk\omega_0} \xRightarrow{j^2=-1}$

Λύση 3ου σήματος: Συντελεστές Fourier(Συνέχεια επί της Συνέχειας)

- $\alpha_k = -\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u - 1)\right]_{jk\omega_0}^{0} + \frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u - 1)\right]_{0}^{-jk\omega_0} + \frac{1}{T_0 \omega_0 jk}\int_{-jk\omega_0}^{jk\omega_0} e^u du$

$$\Rightarrow \alpha_k = -\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[0(e^0 - 1) - jk\omega_0(e^{jk\omega_0} - 1)\right] +$$

$$\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[-jk\omega_0(e^{-jk\omega_0} - 1) - 0(e^u - 1)\right] + \frac{1}{T_0 \omega_0 jk}\left[e^u\right]_{-jk\omega_0}^{jk\omega_0} \xRightarrow{j^2=-1}$$

$$\alpha_k = -\frac{jk\omega_0(e^{jk\omega_0} - 1)}{T_0 k^2 \omega_0^2} + \frac{jk\omega_0(e^{-jk\omega_0} - 1)}{T_0 k^2 \omega_0^2} + \frac{1}{T_0 \omega_0 jk}\left[e^{jk\omega_0} - e^{-jk\omega_0}\right] \xRightarrow{\sin x = \frac{e^{jx} - e^{-jx}}{2j}}$$

Λύση 3ου σήματος: Συντελεστές Fourier(Συνέχεια επί της Συνέχειας)

- $\alpha_k = -\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u - 1)\right]_{jk\omega_0}^{0} + \frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u - 1)\right]_{0}^{-jk\omega_0} + \frac{1}{T_0 \omega_0 jk}\int_{-jk\omega_0}^{jk\omega_0} e^u du$

$$\Rightarrow \alpha_k = -\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[0(e^0 - 1) - jk\omega_0(e^{jk\omega_0} - 1)\right] +$$

$$\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[-jk\omega_0(e^{-jk\omega_0} - 1) - 0(e^u - 1)\right] + \frac{1}{T_0 \omega_0 jk}\left[e^u\right]_{-jk\omega_0}^{jk\omega_0} \xRightarrow{j^2=-1}$$

$$\alpha_k = -\frac{jk\omega_0(e^{jk\omega_0} - 1)}{T_0 k^2 \omega_0^2} + \frac{jk\omega_0(e^{-jk\omega_0} - 1)}{T_0 k^2 \omega_0^2} + \frac{1}{T_0 \omega_0 jk}\left[e^{jk\omega_0} - e^{-jk\omega_0}\right] \xRightarrow{\sin x = \frac{e^{jx} - e^{-jx}}{2j}}$$

$$\alpha_k = -\frac{j(e^{jk\omega_0} - 1)}{T_0 \omega_0 k} + \frac{j(e^{-jk\omega_0} - 1)}{T_0 \omega_0 k} + \frac{2j \sin k\omega_0}{T_0 \omega_0 jk} \xRightarrow{T_0 \omega_0 = 2\pi}$$

Λύση 3ου σήματος: Συντελεστές Fourier(Συνέχεια επί της Συνέχειας)

- $$\alpha_k = -\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u - 1)\right]_{jk\omega_0}^{0} + \frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u - 1)\right]_{0}^{-jk\omega_0} + \frac{1}{T_0 \omega_0 jk}\int_{-jk\omega_0}^{jk\omega_0} e^u du$$
  $$\Rightarrow \alpha_k = -\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[0(e^0 - 1) - jk\omega_0(e^{jk\omega_0} - 1)\right] +$$
  $$\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[-jk\omega_0(e^{-jk\omega_0} - 1) - 0(e^u - 1)\right] + \frac{1}{T_0 \omega_0 jk}\left[e^u\right]_{-jk\omega_0}^{jk\omega_0} \xrightarrow{j^2=-1}$$
  $$\alpha_k = -\frac{jk\omega_0(e^{jk\omega_0} - 1)}{T_0 k^2 \omega_0^2} + \frac{jk\omega_0(e^{-jk\omega_0} - 1)}{T_0 k^2 \omega_0^2} + \frac{1}{T_0 \omega_0 jk}\left[e^{jk\omega_0} - e^{-jk\omega_0}\right] \xrightarrow{\sin x = \frac{e^{jx} - e^{-jx}}{2j}}$$
  $$\alpha_k = -\frac{j(e^{jk\omega_0} - 1)}{T_0 \omega_0 k} + \frac{j(e^{-jk\omega_0} - 1)}{T_0 \omega_0 k} + \frac{2j \sin k\omega_0}{T_0 \omega_0 jk} \xrightarrow{T_0 \omega_0 = 2\pi}$$
  $$\alpha_k = -\frac{j(e^{jk\omega_0} - 1)}{2\pi k} + \frac{j(e^{-jk\omega_0} - 1)}{2\pi k} + \frac{2 \sin k\omega_0}{2\pi k} =$$
  $$\frac{-j(\cos k\omega_0 + j \sin k\omega_0 - 1) + j(\cos k\omega_0 - j \sin k\omega_0 - 1) + 2 \sin k\omega_0}{2\pi k} \Rightarrow$$

Λύση 3ου σήματος: Συντελεστές Fourier(Συνέχεια επί της Συνέχειας)

- $$\alpha_k = -\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u - 1)\right]_{jk\omega_0}^{0} + \frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u - 1)\right]_{0}^{-jk\omega_0} + \frac{1}{T_0\omega_0 jk}\int_{-jk\omega_0}^{jk\omega_0} e^u du$$
  $$\Rightarrow \alpha_k = -\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[0(e^0 - 1) - jk\omega_0(e^{jk\omega_0} - 1)\right] +$$
  $$\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[-jk\omega_0(e^{-jk\omega_0} - 1) - 0(e^u - 1)\right] + \frac{1}{T_0\omega_0 jk}\left[e^u\right]_{-jk\omega_0}^{jk\omega_0} \xRightarrow{j^2=-1}$$
  $$\alpha_k = -\frac{jk\omega_0(e^{jk\omega_0} - 1)}{T_0 k^2\omega_0^2} + \frac{jk\omega_0(e^{-jk\omega_0} - 1)}{T_0 k^2\omega_0^2} + \frac{1}{T_0\omega_0 jk}\left[e^{jk\omega_0} - e^{-jk\omega_0}\right] \xRightarrow{\sin x = \frac{e^{jx} - e^{-jx}}{2j}}$$
  $$\alpha_k = -\frac{j(e^{jk\omega_0} - 1)}{T_0\omega_0 k} + \frac{j(e^{-jk\omega_0} - 1)}{T_0\omega_0 k} + \frac{2j\sin k\omega_0}{T_0\omega_0 jk} \xRightarrow{T_0\omega_0 = 2\pi}$$
  $$\alpha_k = -\frac{j(e^{jk\omega_0} - 1)}{2\pi k} + \frac{j(e^{-jk\omega_0} - 1)}{2\pi k} + \frac{2\sin k\omega_0}{2\pi k} =$$
  $$\frac{-j(\cos k\omega_0 + j\sin k\omega_0 - 1) + j(\cos k\omega_0 - j\sin k\omega_0 - 1) + 2\sin k\omega_0}{2\pi k} \Rightarrow$$
  $$\alpha_k = \frac{-j\cos k\omega_0 - j^2\sin k\omega_0 + j + j\cos k\omega_0 - j^2\sin k\omega_0 - j + 2\sin k\omega_0}{2\pi k} =$$

Λύση 3ου σήματος: Συντελεστές Fourier(Συνέχεια επί της Συνέχειας)

- $\alpha_k = -\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u-1)\right]_{jk\omega_0}^{0} + \frac{1}{T_0 j^2 k^2 \omega_0^2}\left[u(e^u-1)\right]_{0}^{-jk\omega_0} + \frac{1}{T_0\omega_0 jk}\int_{-jk\omega_0}^{jk\omega_0} e^u du$

$$\Rightarrow \alpha_k = -\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[0(e^0-1) - jk\omega_0(e^{jk\omega_0}-1)\right] +$$

$$\frac{1}{T_0 j^2 k^2 \omega_0^2}\left[-jk\omega_0(e^{-jk\omega_0}-1) - 0(e^u-1)\right] + \frac{1}{T_0\omega_0 jk}\left[e^u\right]_{-jk\omega_0}^{jk\omega_0} \xRightarrow{j^2=-1}$$

$$\alpha_k = -\frac{jk\omega_0(e^{jk\omega_0}-1)}{T_0 k^2\omega_0^2} + \frac{jk\omega_0(e^{-jk\omega_0}-1)}{T_0 k^2\omega_0^2} + \frac{1}{T_0\omega_0 jk}\left[e^{jk\omega_0} - e^{-jk\omega_0}\right] \xRightarrow{\sin x = \frac{e^{jx}-e^{-jx}}{2j}}$$

$$\alpha_k = -\frac{j(e^{jk\omega_0}-1)}{T_0\omega_0 k} + \frac{j(e^{-jk\omega_0}-1)}{T_0\omega_0 k} + \frac{2j\sin k\omega_0}{T_0\omega_0 jk} \xRightarrow{T_0\omega_0=2\pi}$$

$$\alpha_k = -\frac{j(e^{jk\omega_0}-1)}{2\pi k} + \frac{j(e^{-jk\omega_0}-1)}{2\pi k} + \frac{2\sin k\omega_0}{2\pi k} =$$

$$\frac{-j(\cos k\omega_0 + j\sin k\omega_0 - 1) + j(\cos k\omega_0 - j\sin k\omega_0 - 1) + 2\sin k\omega_0}{2\pi k} \Rightarrow$$

$$\alpha_k = \frac{-j\cos k\omega_0 - j^2\sin k\omega_0 + j + j\cos k\omega_0 - j^2\sin k\omega_0 - j + 2\sin k\omega_0}{2\pi k} =$$

$$\frac{-(-1)\sin k\omega_0 - (-1)\sin k\omega_0 + 2\sin k\omega_0}{2\pi k} = \frac{4\sin k\omega_0}{2\pi k} = \frac{2\sin\left(\frac{2\pi k}{3}\right)}{\pi k} =$$

$$\frac{1}{\pi k}\cdot\begin{cases} 2\frac{\sqrt{3}}{2}, & k \mod 3 = 1 \\ -2\frac{\sqrt{3}}{2}, & k \mod 3 = 2 \\ 0, & k \mod 3 = 0 \end{cases} = \begin{cases} \frac{\sqrt{3}}{k\pi}, & k \mod 3 = 1 \\ -\frac{\sqrt{3}}{k\pi}, & k \mod 3 = 2 \\ 0, & k \mod 3 = 0 \end{cases}$$