$$\int \cos^2(2x+3)\,dx \overset{u=2x+3}{=}$$

$$du = d(2x+3) = 2\,dx$$

$$\left[\cos 2x = 2\cos^2 x - 1 \Rightarrow \cos^2 x = \frac{1+\cos 2x}{2}\right]$$

$$\int \cos^2 u\,\frac{du}{2} = \frac{1}{2}\int \cos^2 u\,du = \frac{1}{2}\int\left(\frac{1+\cos 2u}{2}\right)du$$

$$= \frac{1}{4}\left[\int 1\,du + \int \cos 2u\,du\right] = \frac{1}{4}\left[u + \int \cos 2u\,du\right] =$$

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$$= \frac{1}{4}\left[u + \frac{1}{2}\sin 2u + c\right] = \frac{u}{4} + \frac{\sin 2u}{8} + \frac{c}{4} = \qquad \frac{c}{4} = C$$

$$= \frac{u}{4} + \frac{\sin 2u}{8} + C = \frac{2x+3}{4} + \frac{\sin(4x+6)}{8} + C$$

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$$A = \int_0^1 (\ln x)^2\,dx = \int_0^1 (x)' \ln^2 x\,dx = \left[x \ln^2 x\right]_0^1 - \int_0^1 x\,(\ln^2 x)'\,dx.$$

$$(\ln^2 x)' = (\ln x \cdot \ln x)' = (\ln x)' \cdot \ln x + \ln x \cdot (\ln x)' =$$

$$= \frac{1}{x}\ln x + \ln x \cdot \frac{1}{x} = \frac{2\ln x}{x}$$

Ape $\quad A = \left[x \ln^2 x\right]_0^1 - \int_0^1 \cancel{x} \cdot \frac{2\ln x}{\cancel{x}}\,dx =$

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$$= \left[1 \cdot \overset{0}{\cancel{\ln 1}} - \lim_{x \to 0} x \ln x\right] - 2\int_0^1 \ln x \, dx =$$

$$= [0 - 0] - 2\int_0^1 (x)' \ln x \, dx =$$

$$= -2\left[[x \ln x]_0^1 - \int_0^1 x (\ln x)' \, dx\right] = -2\left[\left[1 \cdot \overset{0}{\cancel{\ln 1}} - \overset{0}{\cancel{\lim_{x \to 0} x \ln x}}\right] -\right.$$

$$\left. - \int_0^1 \cancel{x} \cdot \frac{1}{\cancel{x}} \, dx\right] = -2\left[-[x]_0^1\right] = 2[1 - 0] =$$

$$= 2$$

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$$A = \int x^2 \cos x \, dx = \int x^2 (\sin x)' \, dx = x^2 \sin x - \int (x^2)' \sin x \, dx$$

$$= x^2 \sin x - \int 2x \sin x \, dx = x^2 \sin x + 2 \int -x \sin x \, dx$$

$$= x^2 \sin x + 2 \int x (\cos x)' \, dx = x^2 \sin x + 2 \Big[x \cos x -$$

$$- \int x' \cos x \, dx \Big] = x^2 \sin x + 2x \cos x - 2 \int \cos x \, dx =$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x$$

$$A = \int_0^1 \frac{1}{\sqrt{1-x}}\,dx$$

$$u = \sqrt{1-x} \Rightarrow du = d\left(\sqrt{1-x}\right) =$$

$$= \frac{1}{2\sqrt{1-x}} \cdot (1-x)'\,dx = -\frac{1}{2u}\,dx \Rightarrow$$

$$\xrightarrow{\quad x \qquad\qquad\qquad u \quad}$$

$$\rightarrow \quad dx = -2u\,du$$

$$0 \rightarrow \sqrt{1-0} = \sqrt{1} = 1$$

$$1 \rightarrow \sqrt{1-1} = \sqrt{0} = 0$$

$$f(x) = \sqrt{x}$$

$$g(x) = 1-x$$

$$A = \int_1^0 \frac{1}{u}(-2u\,du) = -\int_1^0 \frac{2\cancel{u}}{\cancel{u}}\,du = \int_0^1 2\,du =$$

$$= [2u]_0^1 = 2 \cdot 1 - 2 \cdot 0 = 2$$

$$\left(f(g(x))\right)' = f'(g(x)) \cdot g'(x) = \frac{1}{2\sqrt{1-x}}(-1)$$

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e^x, $\ln x$, $\cos x$, $\sin x$, x^p,

$x^3 + x^2$

$\sqrt{x} = x^{1/2}$

$\ln e^u = u$

$\sqrt{\tan x + 1}$

$\sin(\ln x)$

$$x = e^u$$

$$\Updownarrow$$

$$A = \int_{e^{\pi/4}}^{e^{3\pi/4}} \sin(\ln x)\,dx$$

Θετω $\overbrace{\ln x = u} \Rightarrow du = d(\ln x)$

$= \dfrac{dx}{x} \Rightarrow dx = x\,du = e^u\,du$

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x	$u = \ln x$
$e^{\pi/4}$	$\pi/4$
$e^{3\pi/4}$	$3\pi/4$

Άρα $A = \int_{\pi/4}^{3\pi/4} \sin u \cdot e^u \, du =$

$$= \int_{\pi/4}^{3\pi/4} \sin u \, (e^u)' \, du = \left[\sin u \, e^u\right]_{\frac{\pi}{4}}^{3\pi/4} - \int_{\frac{\pi}{4}}^{3\pi/4} (\sin u)' \cdot e^u \, du =$$

$$= \underbrace{\left[\sin\frac{3\pi}{4} \, e^{3\pi/4} - \sin\frac{\pi}{4} \, e^{\pi/4}\right)}_{B} - \int_{\pi/4}^{3\pi/4} \cos u \cdot e^u \, du = B - \int_{\frac{\pi}{4}}^{3\pi/4} \cos u \, (e^u)' \, du$$

$$= B - \left[\left[e^u \cos u\right]_{\frac{\pi}{4}}^{3\pi/4} - \int_{\pi/4}^{3\pi} (\cos u)' e^u \, du\right] = B - \left[\Gamma - \int_{\pi/4}^{3\pi/4} -\sin u \, e^u \, dx\right] =$$

$$= B - \Gamma - \underbrace{\int_{\pi/4}^{3\pi/4} \sin u \, e^{u} \, du}_{A}$$

$$A = B - \Gamma - A \Rightarrow 2A = B - \Gamma \Rightarrow A = \frac{B - \Gamma}{2}$$

$$\int h(x)\,dx = \int f'(x)\,g(x)\,dx = f \cdot g - \int f(x)\,g'(x)\,dx$$

$$\int (f \cdot g)' = \int (f' \cdot g + f \cdot g') \Rightarrow fg = \int f' g \, dx + \int f g' \, dx \Rightarrow$$

$$\int \underbrace{f' g \, dx}_{h} = f g - \int f g' \, dx$$

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