Digital Signal Processing Laboratory
Department of Electrical & Computer Engineering

Digital Signal Processing Laboratory

Laboratory 1

Introduction to signals

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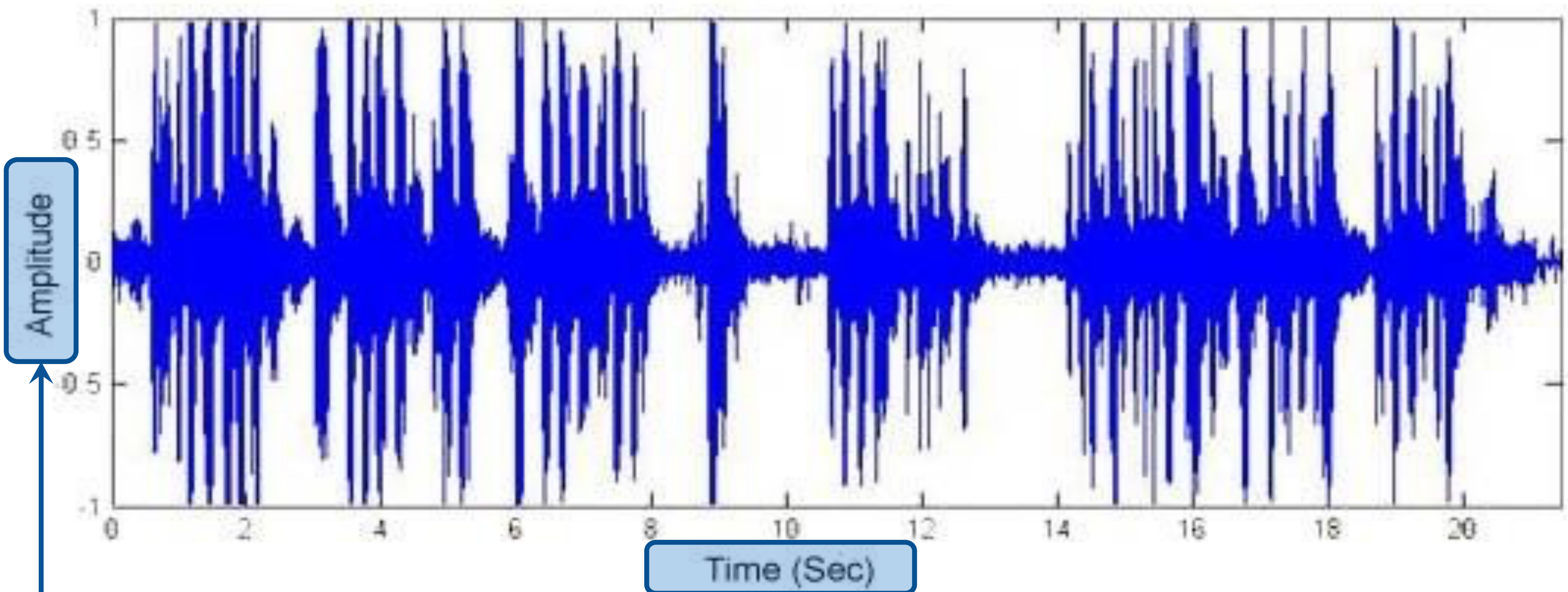
What is a signal?

- We define a signal as the set of values that a quantity receives (dependent variable), when it varies as a function of other quantities (independent variables).
- Mathematically this is expressed as a function or sequence of one or more independent variables.
- Signals contain information about the behavior or nature of a phenomenon.

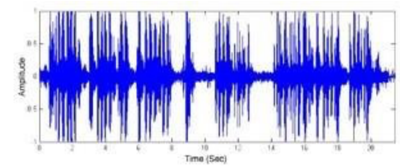
Example of a signal



- Dependent variable
- Independent Variable

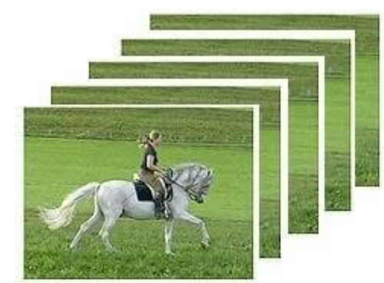
Signal dimensions

- Depending on the number of independent variables we have signals:
  - ➢ one variable or one dimension (one-dimensional, 1-D)
  - ➢ two variables or two dimensions (two-dimensional, 2-D)
  - ➢ multi-variable or multi-dimensional (multidimensional, n-D)



- Speech, music, maximum daily temperature are examples of one-dimensional signals, where the independent variable is the time.

- An image is a typical example of a two-dimensional signal. The dependent variable is the brightness of the image and the two independent variables are the two spatial coordinates.



- A three-dimensional signal can be a sequence of images (video), where the two independent variables are spatial coordinates and the third is time.

Types of signals

Signals are classified into two broad categories:

- continuous-time signals (analog).
- discrete-time signals.

We refer to them as discrete "time", but it is possible that the independent variable is some other physical quantity, such as distance or temperature.

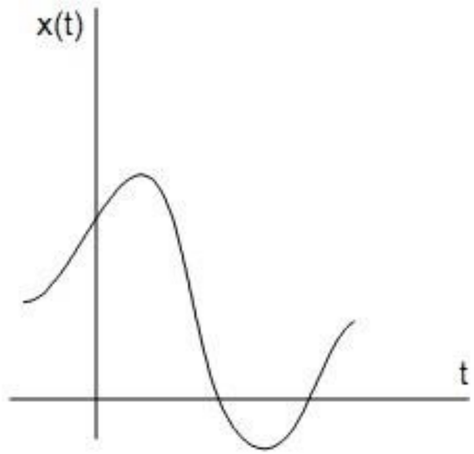
Continuous time signals

In continuous time signals, the independent variable is continuous. That is, these signals are defined in each time instant of the independent variable from the beginning to the end.

For example such signals are:

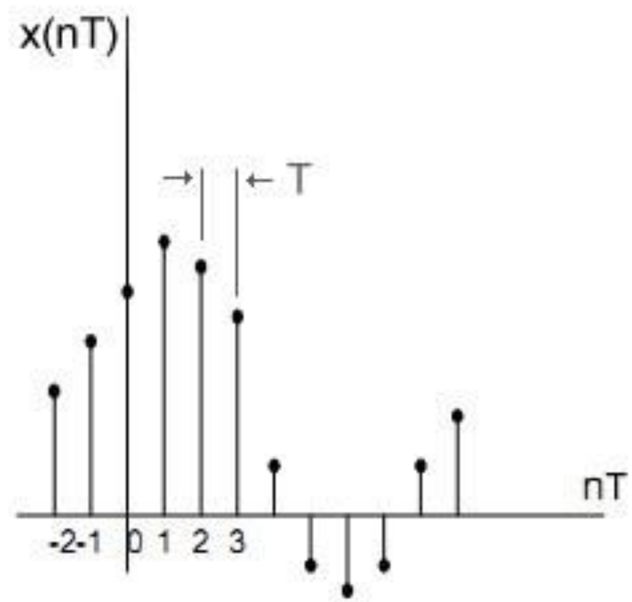
- *Speech, as a function of time*
- *Atmospheric pressure, as a function of altitude.*

An analog signal is described as a function x(t), where t is a real number.



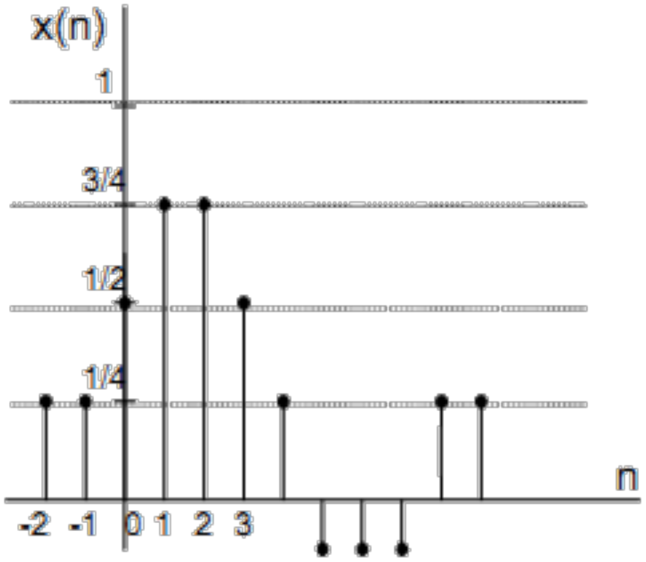
Discrete time signals

- In discrete time signals, the independent variable is discrete.
- That is, these signals are defined only for specific values (discrete set of values) of the independent variable.
- A discrete time signal can be described by a function x(nT), where T is the period between adjacent values and n is an integer ($-\infty<n<\infty$).

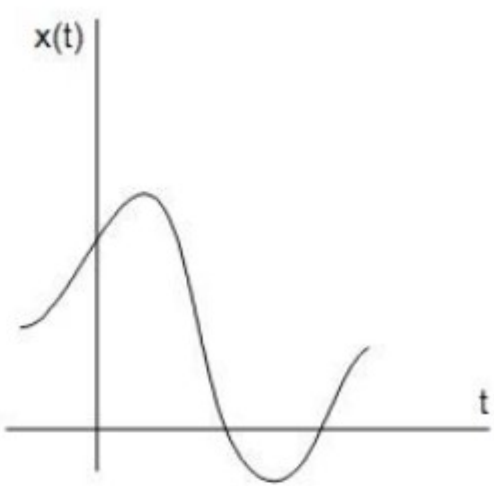


Digital signals

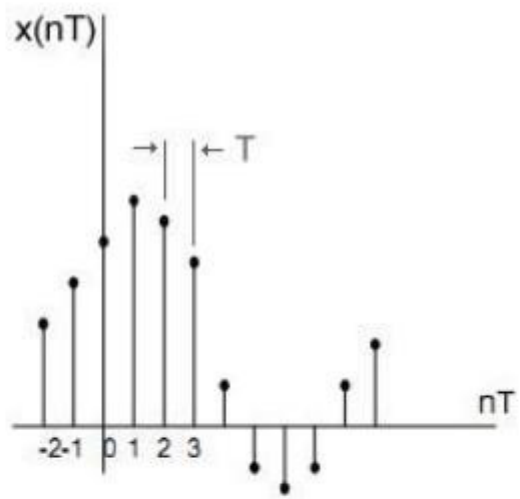
- When each sample of a discrete-time signal is quantized and then encoded, the final signal is referred to as a digital signal.
- A quantized signal can only receive discrete values usually (but not always) of equal distance. The output from a computer is an example of a digital signal.
- Of course, an analogue signal can be converted to digital by time sampling, quantization and encoding, so it can only be represented in bits.



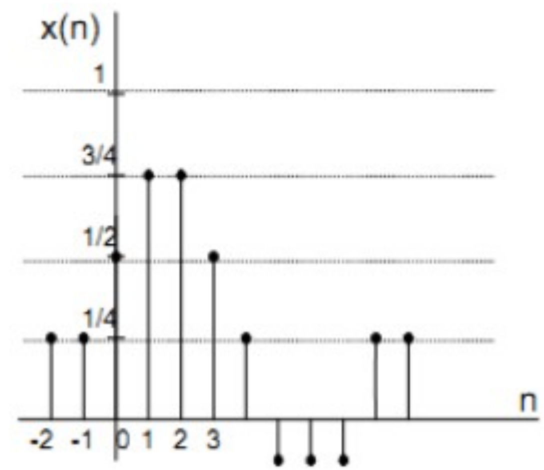
Signal types



Continuous time signal



Discrete time signal



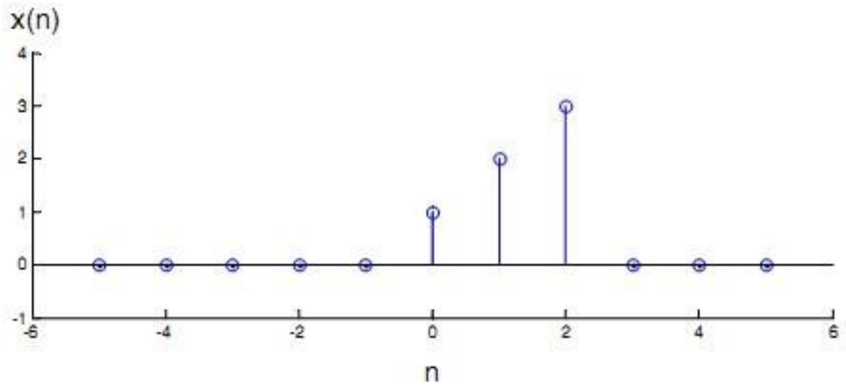
Digital signal

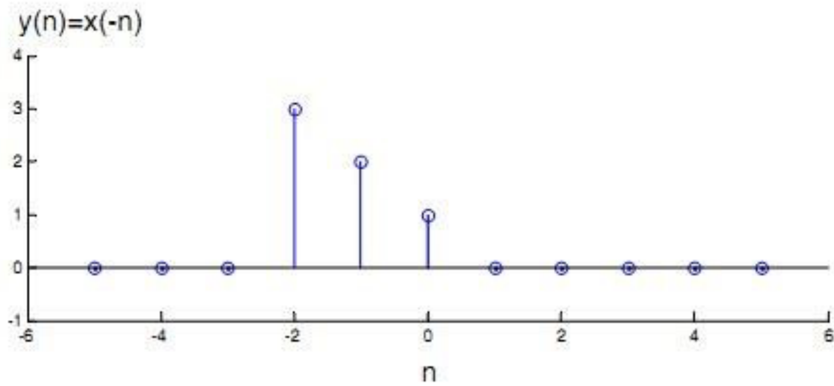
Signal transformations

Time inversion

In general, if a signal x(n) has a transformation applied to the independent variable n, for example f(n), then the signal x(f(n)) is obtained.

- If **f(n) = -n** then we say that the signal has undergone an inversion and changes to the signal y(n)=x(-n)
- The graph of x(-n) is symmetric with respect to the vertical axis with that of x(n).



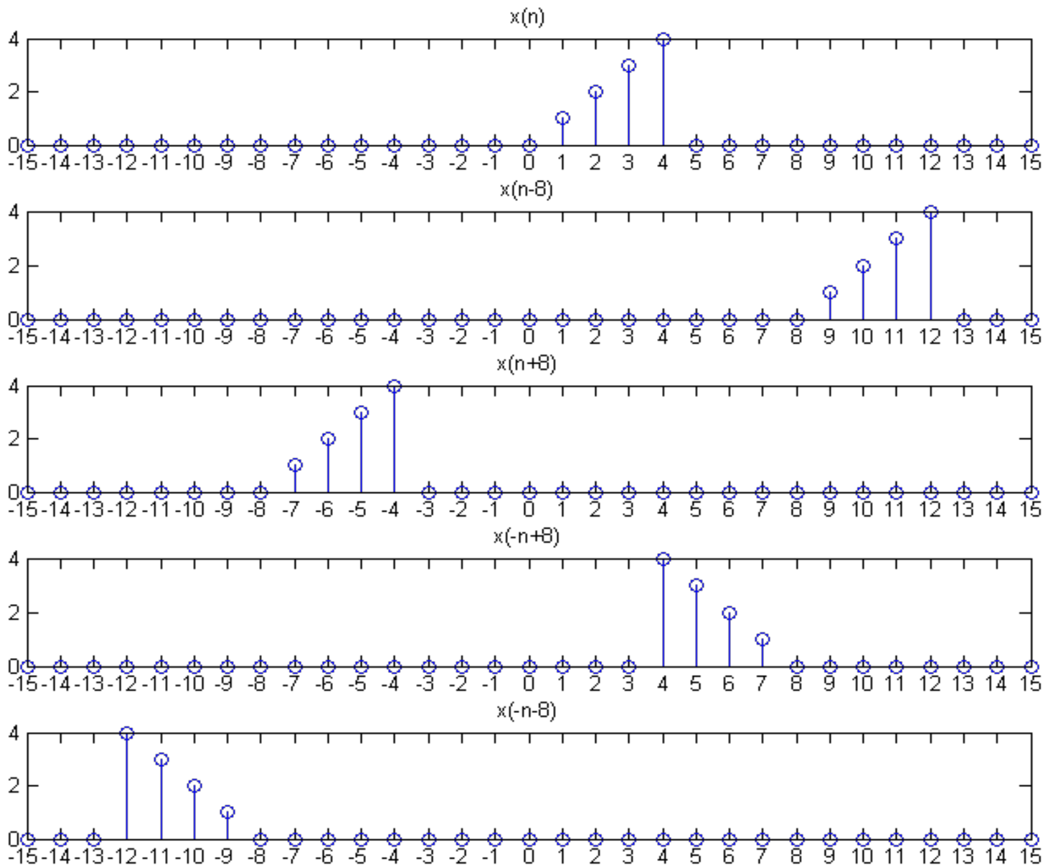


Signal transformations

Time shift

- If $f(n)=n-n_0$, then we say that the signal has undergone **a shift by n_0** and is transformed into the signal $y(n)= x(n-n_0)$
- In the graph of $x(n-n_0)$ there is a shift of the graph of $x(n)$ by n_0, on the horizontal axis.
- If $n_0 > 0$, then we observe a shift to the right **(we have a delay)**.
- If $n_0 < 0$, then we observe a shift to the left **(we have an advance)** and the signal can be written as $y(n)= x(n+n_0)$.

Signal transformations



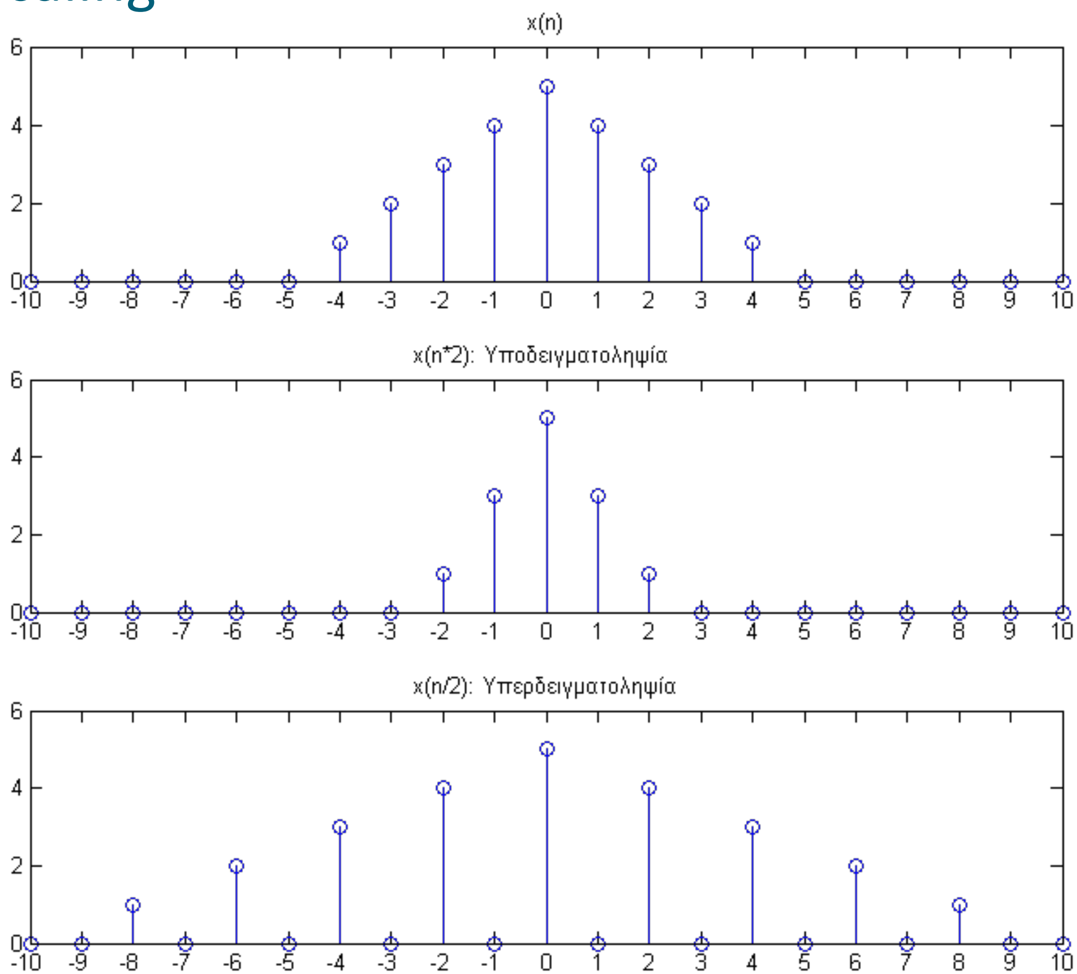
- x(n)
- x(n-8)
- x(n+8)
- x(-n+8)
- x(-n-8)

Signal transformations

Time scaling

- A signal undergoes scaling if **f(n)=Mn or f(n)=n/M**, where M is an integer.
- In the first case the discrete signal is said to have undergone subsampling and in the second case it is said to have undergone oversampling.
- In subsampling we obtain the signal y(n)= x(Mn) and in oversampling we obtain the signal y(n)= x(n/M), where n/M is an integer.
- The signal x(n/M) is not defined for non-integer values of the quotient n/M.

Time scaling



- x(n)

- Subsampling
  x(n*2)

- Oversampling
  x(n/2)

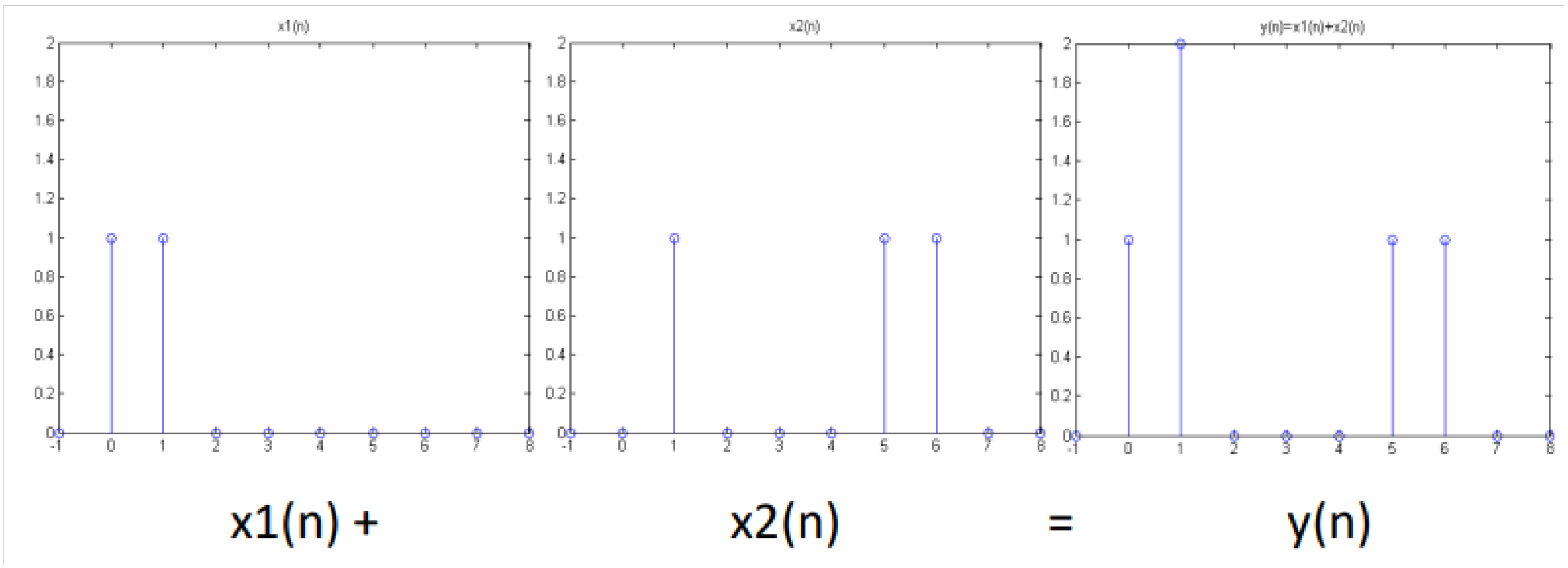
Discrete signal operations

Addition	x(n) + y(n)
Subtraction	x(n) – y(n)
Multiplication	x(n) * y(n)
Division	x(n) / y(n) with y(n)≠0

The operations are performed per element and for the same value of the independent variable.

Example of operations

Suppose we want to add the following two signals x1(n) and x2(n):

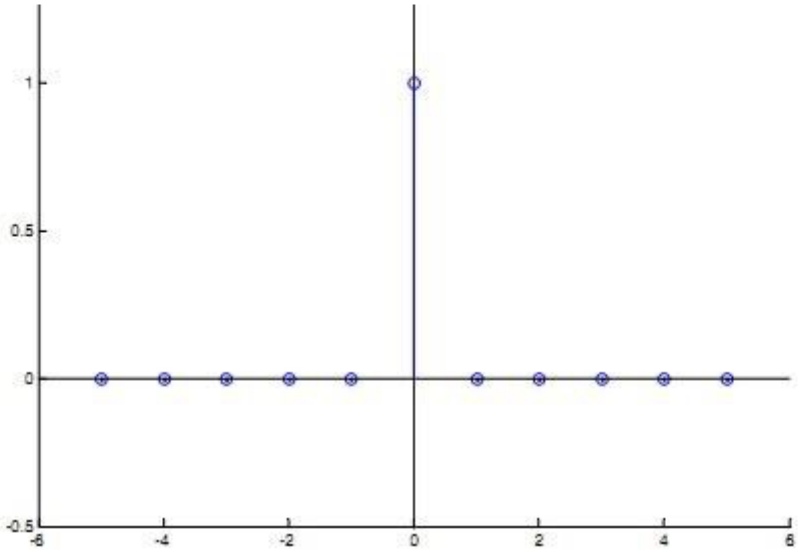


Fundamental sequences

Discrete delta sequence

The discrete sequence δ(n) or unit impulse sequence is defined δ(n) and originates from the relation:

$$\delta(n)=\begin{cases}1 & n=0\\ 0 & n\neq 0\end{cases}$$

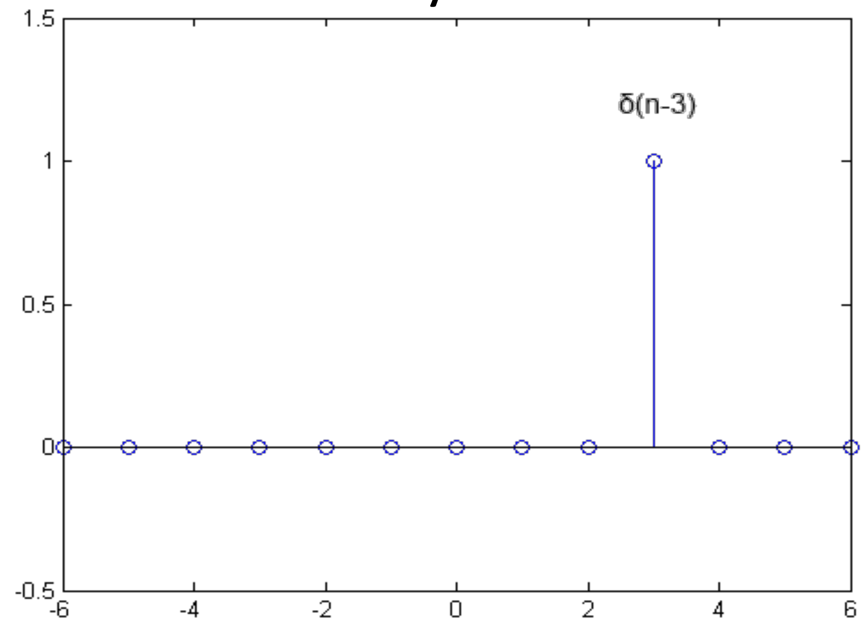


Fundamental sequences

Discrete deceleration sequence

- The discrete deceleration sequence is the sequence δ(n) where it has undergone a shift by k and is defined by:

$$\delta(n-k) = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$$



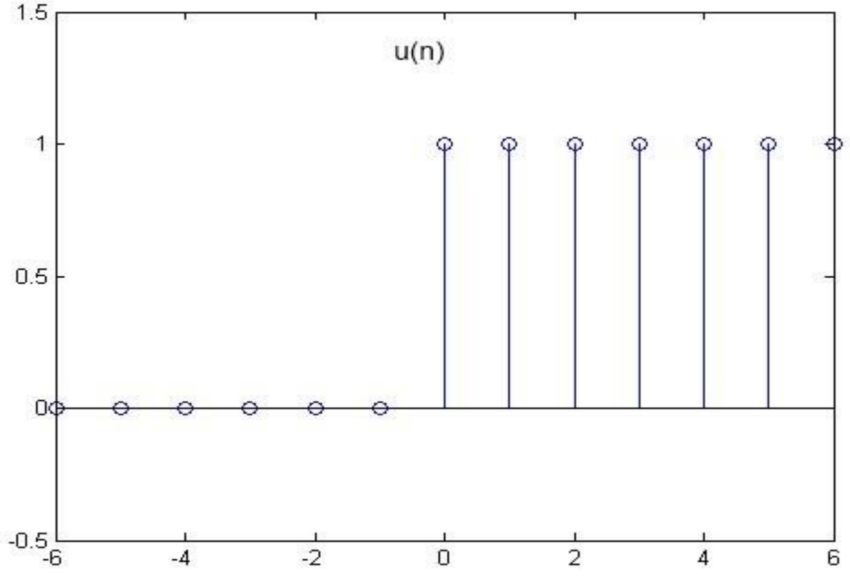
δ[n-3] equals 1 when n=3, and 0 when n ≠3

Fundamental sequences

Unit step sequence (unit step)

- The unit step sequence is denoted by u(n) and defined by:

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

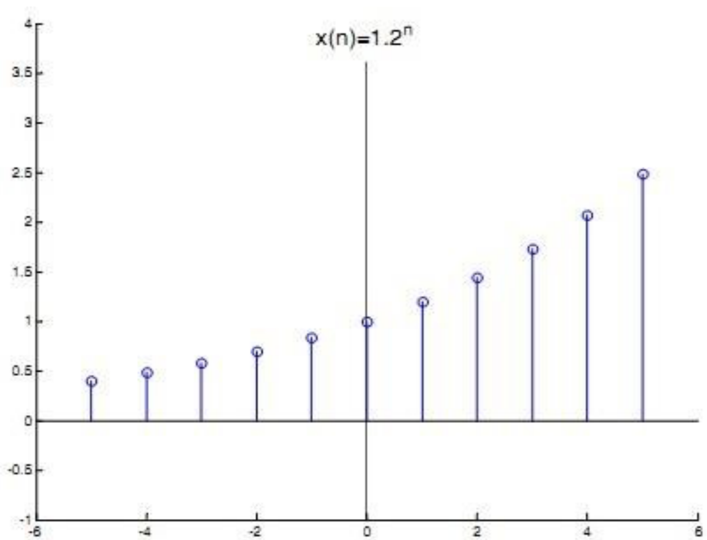


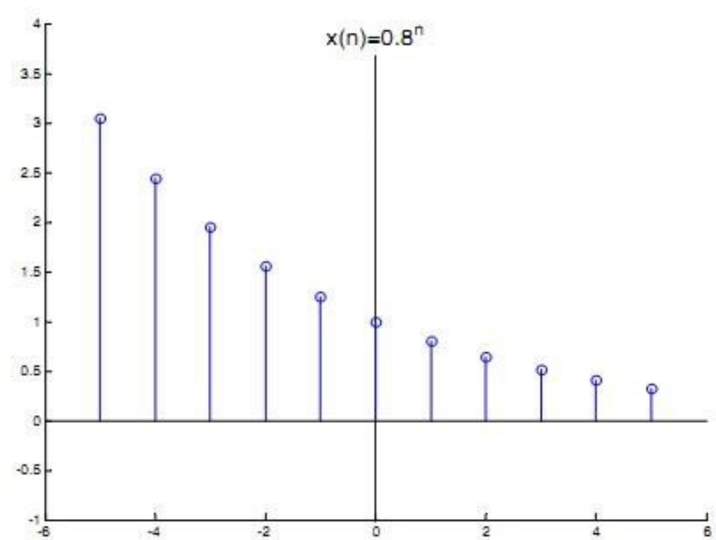
Note that u[n-3] equals 1 when n>=3, and 0 when n<3

Fundamental sequences

Exponential sequence

The exponential sequence is decreasing when|a|<1, increases when|a|>1 and it is constant when a=1, i.e. x(n)=u(n) and is defined by the relation: $x(n) = \alpha^{n} \forall n, a \in N$



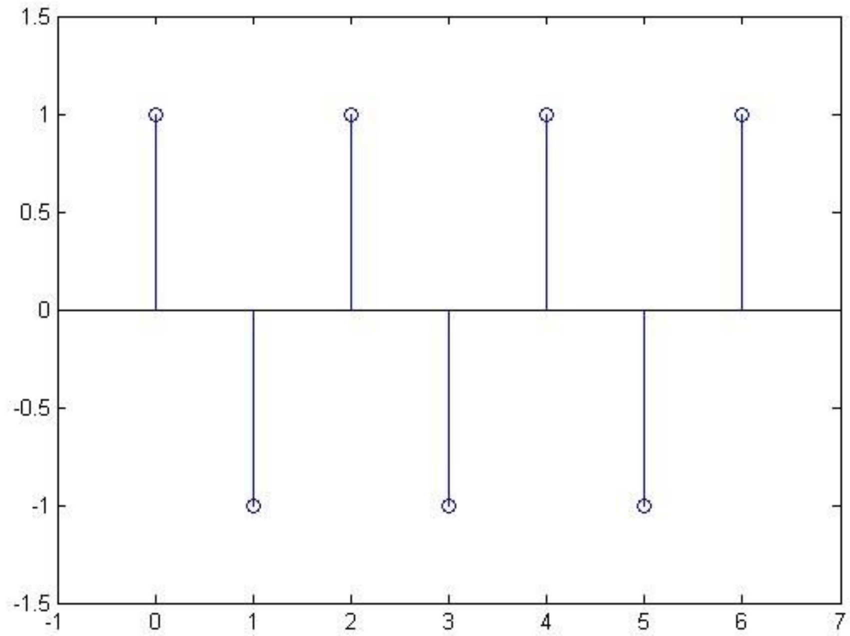


Fundamental sequences

Alternating sequence

The unitary alternating sequence is defined by:

$$x(n) = (-1)^n, n \geq 0$$



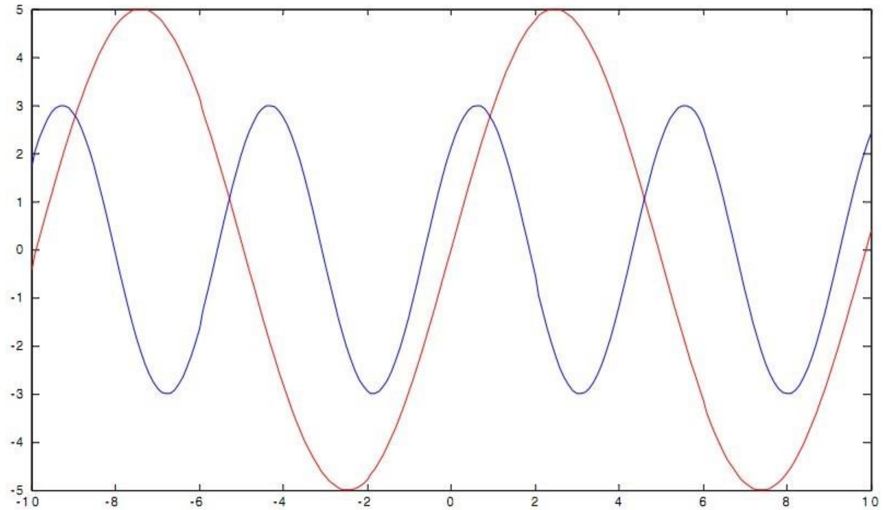
Fundamental sequences

Sine sequence

Sinusoidal sequences are defined by the relations, where A is the amplitude, ω is the frequency and ϕ is the phase:

$$x(n)=A\cos(\omega_o n+\varphi), \forall n \in N$$

$$x(n)=A\sin(\omega_o n+\varphi), \forall n \in N$$



With red color: $5\sin(\omega_1 t)$

With blue: $3\sin(\omega_2 t+\pi/4)$, $\omega_2=2\omega_1$

Implementation of δ(n-k)

Build a Python function that creates a discrete deceleration function.

- The function will take 3 arguments: k, and the limits of n.
- It will return a vector with the sequence δ and a vector with the values of n.

$$\delta(n-k) = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$$

Solution

```
import matplotlib.pyplot as plt

import numpy as np


def dk(k,min_n,max_n):

        n=range(min_n,max_n,1);

        d=np.zeros(len(n));

        for i in range(0, len(n)):

                if n[i]==k:

                        d[i]=1;

                        break;

    return n,d
```

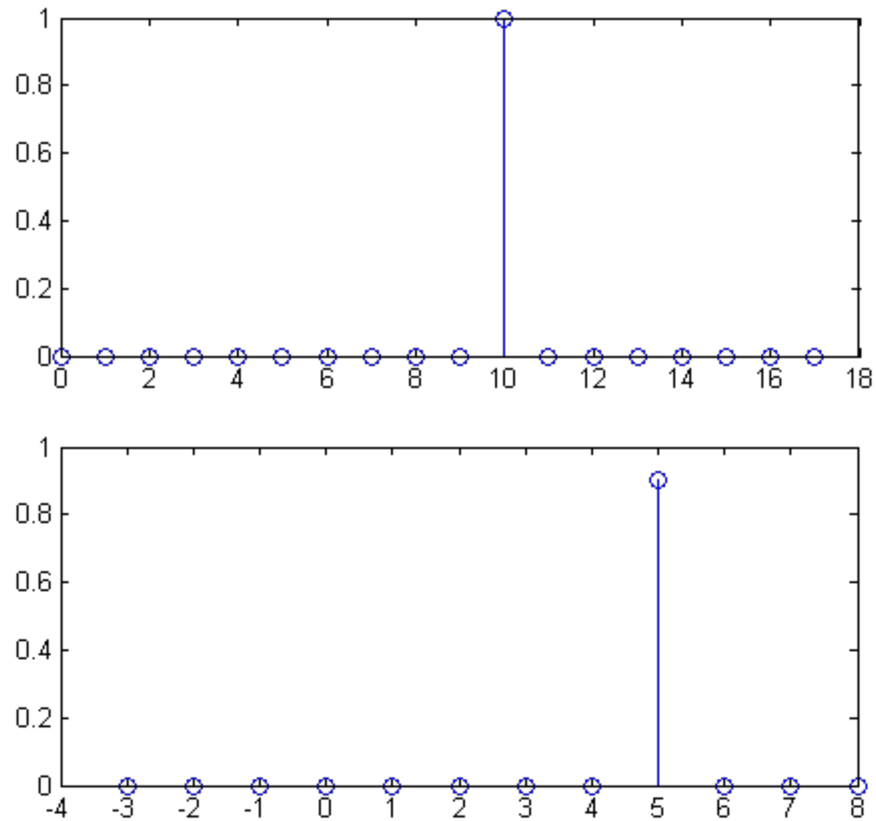
$$\delta(n-k)=\begin{cases}1 & n=k\\ 0 & n\neq k\end{cases}$$

Exercise

By using the dk function you created, graph the following discrete signals using the stem command.

- x1(n)=δ(n-10)   0<=n<=17
- x2(n)=0.9δ(n-5)   -3<=n<=8

```
[n1,x1]=dk(10,0,17);

[n2,x2]=dk(5,-3,8);

x2=x2*0.9;

plt.stem(n1, x1)

plt.show()

plt.stem(n2, x2)

plt.show()
```



Exercise

Graph the following discrete signals in the interval -6 ≤ n ≤ 8 on the same figure, using the stem command.

- δ(n+3)
- δ(n)
- δ(n-6)

```
[n1,d1]=dk(-3,-6,8);
[n2,d2]=dk(0,-6,8);
[n3,d3]=dk(6,-6,8);
plt.stem(n1,d1,'r')
plt.stem(n2,d2,'g')
plt.stem(n3,d3,'b')
```