Example

The following table presents the activities required for the implementation of a project and their duration in weeks.

Δραστηριότητα	**Διάρκεια (σε εβδομάδες)**
1	5
2	7
3	6
4	3
5	4
6	2
7	6
8	5

The dependencies of the activities are shown in the table below:

Activity Number		Activity Number
1	must be finished before	4, 7 can start
2	must be finished before	5
3	must be finished before	5,6
4	must be finished before	7
5	must be finished before	8
6	must be finished before	8

Questions

1) Calculate the minimum completion time of the project

2) Calculate the critical path

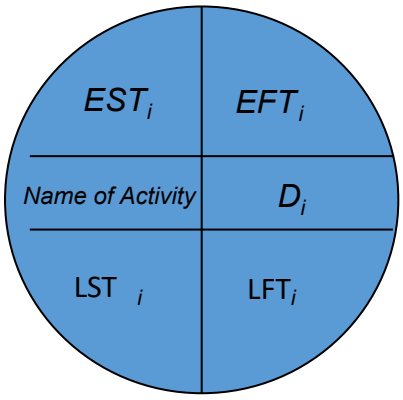
3) If activity 5 is delayed for 3 weeks, how will the total execution time of the project be affected and why? What happens to critical activities?

4) If activity 7 is delayed for 3 weeks, how will the total execution time of the project be affected and why? What happens to critical activities?

Note:

You should use the notation presented in the course:

- ESTi is the earliest starting time for activity i
- LSTi is the latest starting time for activity i
- EFTi is the earliest finish time for activity i
- LFTi is the latest finish time for activity i
- Di is the duration of activity i
- Fi is the float/slack of activity i



Solution

1) Calculate the total execution time of the project.

To calculate the total execution time, we first construct the project network and apply the forward pass.

Earliest start time:
ESTi = max(EFTj), where j are activities on which activity i depends.
If an activity has no predecessors, ESTi = 0.

Thus:
EST1 = EST2 = EST3 = 0
EST4 = max[EST1 + T1] = max[0+5] = max[5] = 5
EST5 = max[EST2 + T2, EST3 + T3] = max[0+7, 0+6] = max[7,6] = 7
EST6 = max[EST3 + T3] = max[0+6] = max[6] = 6
EST7 = max[EST1 + T1, EST4 + T4] = max[0+5, 5+3] = max[5, 8] = 8
EST8 = max[EST5 + T5, EST6 + T6] = max[7+4, 6+2] = max[11, 8] = 11
EST9 = max[EST7 + T7, EST8 + T8] = max[8+6, 11+5] = max[14, 16] = 16

Earliest finish time:
EFTi = ESTi + Ti

Thus:
EFT1 = EST1 + T1 = 0+5 = 5
EFT2 = EST2 + T2 = 0+7 = 7
EFT3 = EST3 + T3 = 0+6 = 6
EFT4 = EST4 + T4 = 5+3 = 8
EFT5 = EST5 + T5 = 7+4 = 11
EFT6 = EST6 + T6 = 6+2 = 8
EFT7 = EST7 + T7 = 8+6 = 14
EFT8 = EST8 + T8 = 11+5 = 16
EFT9 = EST9 + T9 = 16+0 = 16

The earliest finish time of the final activity is 16 weeks which is equal to the total execution time of the whole project.

2) Calculate the critical path.

To find the critical path, we apply the backward pass and then compute each activity's float.

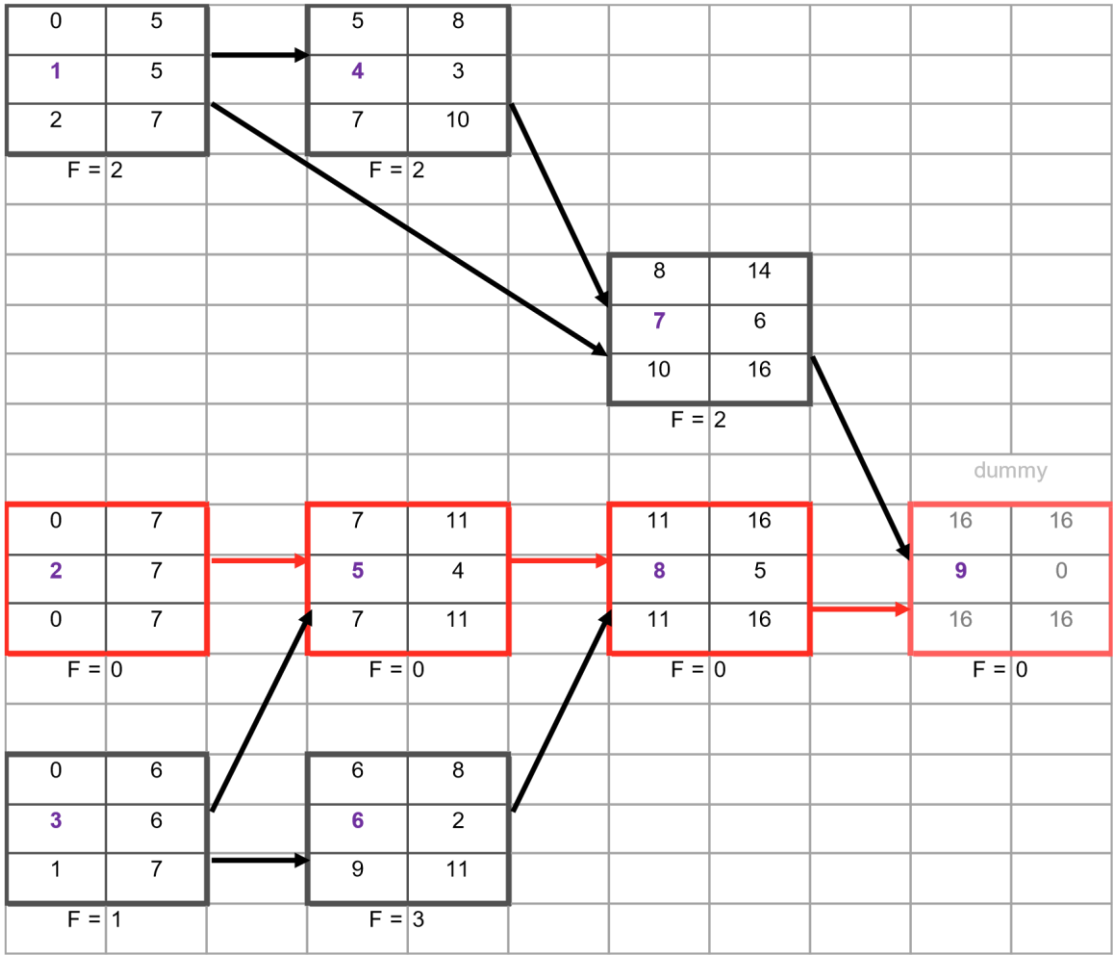
Backward pass (latest start time):
LST9 = LFT9 – T9 = 16-0 = 16
LST8 = LFT8 – T8 = 11
LST7 = LFT7 – T7 = 16-6 =10
LST6 = LFT6 – T6 = 11-2 = 9
LST5 = LFT5 – T5 = 11-4 = 7
LST4 = LFT4 – T4 = 10-3 = 7
LST3 = LFT3 – T3 = 7-6 = 1
LST2 = LFT2 – T2 = 7-7 = 0
LST1 = LFT1 – T1 = 7-5 = 2

Latest finish times:
LFT9 = EFT9 = 16
LFT8 = min[LST9] = min[16] = 16
LFT7 = min[LST9] = min[16] = 16
LFT6 = min[LST8] = min[11] = 11
LFT5 = min[LST8] = min[11] = 11
LFT4 = min[LST7] = min[10] = 10
LFT3 = min[LST5, LST6] = min[7,9] = 7
LFT2 = min[LST5] = min[7] = 7

Float:
F1 = LST1 – EST1 = 2 – 0 = 2
F2 = LST2 – EST2 = 0 – 0 = 0
F3 = LST3 – EST3 = 1 – 0 = 1
F4 = LST4 – EST4 = 7 – 5 = 2
F5 = LST5 – EST5 = 7 – 7 = 0
F6 = LST6 – EST6 = 9 – 6 = 3
F7 = LST7 – EST7 = 10 – 8 = 2
F8 = LST8 – EST8 = 11 – 11 = 0
F9 = LST9 – EST9 = 16 – 16 = 0

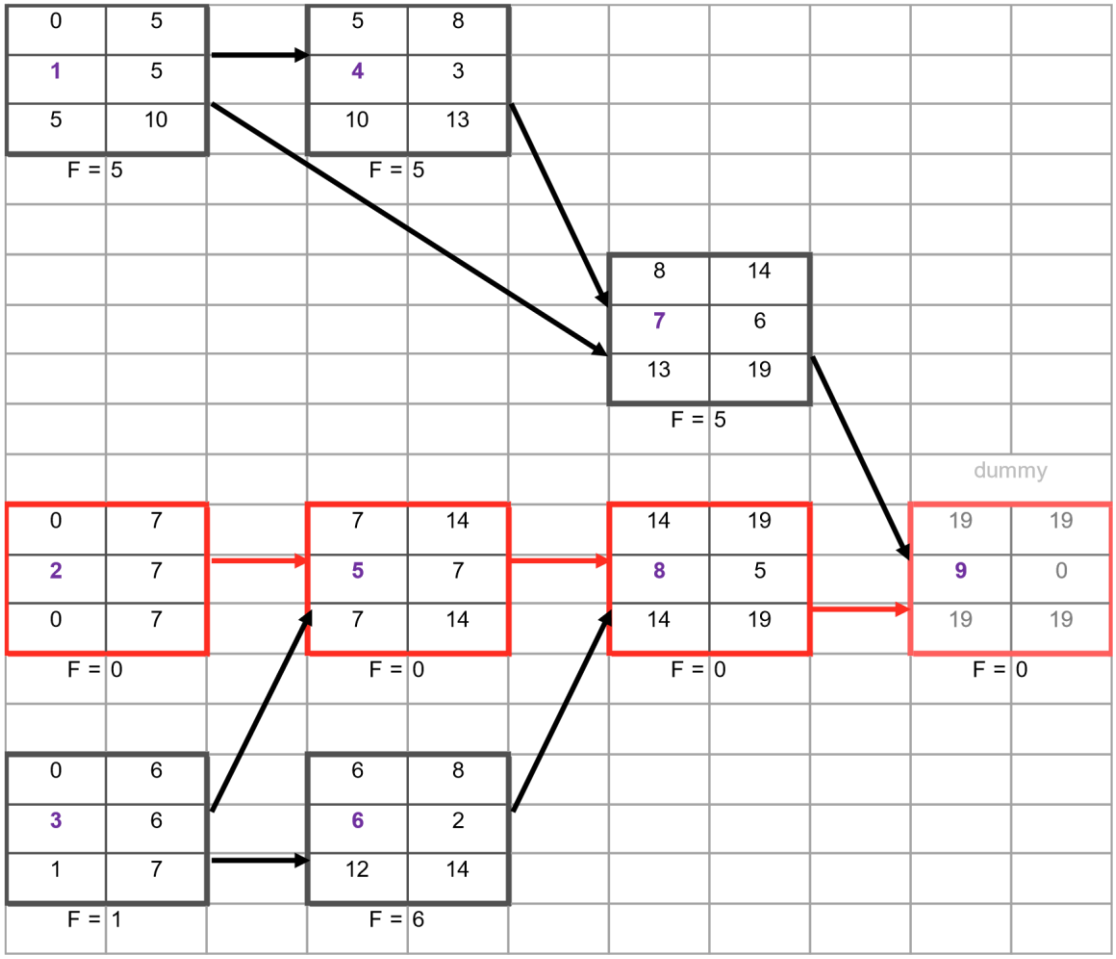
The **critical path** is the set of activities with **zero float (F=0)**: **2,5,8,9**

The network diagram is shown below.



3) If activity 5 is delayed for 3 weeks, how will the total execution time of the project be affected and why? What happens to critical activities?

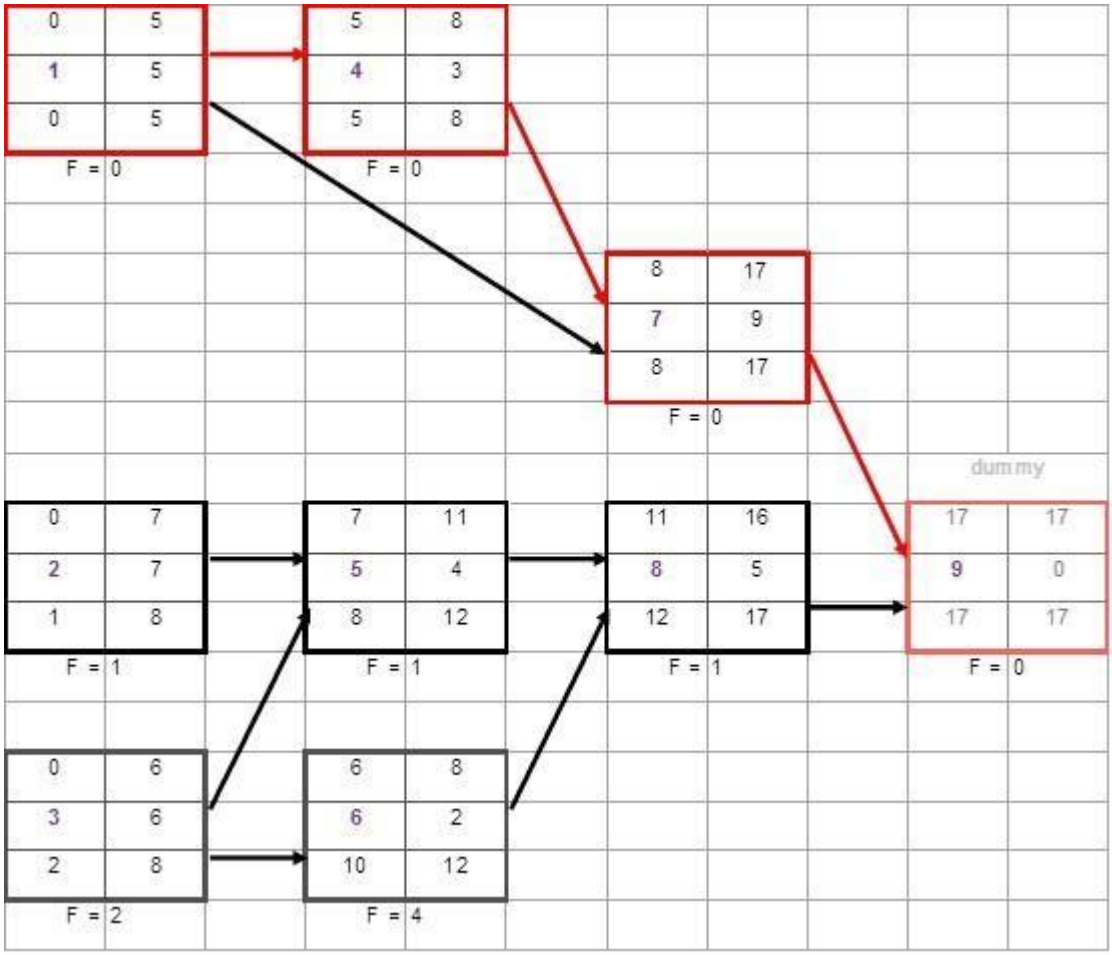
Since activity 5 is critical, a delay changes the total execution time. The project is delayed by 3 weeks, becoming 19 weeks. The critical path remains the same. The new network diagram is shown below. We calculate the network diagram and this time the duration of activity 5 is 7 (4 previous duration + 3 weeks of delay)



EST8 = max [EST5 + T5, EST6 + T6] = max [10 + 4, 6 + 2], max [14, 8] = 14
EST9 = max [EST7 + T7, EST8 + T8] = max [8 + 6, 14 + 5] = max [14, 19] = 19

4) If activity 7 is delayed for 3 weeks, how will the total execution time of the project be affected and why? What happens to critical activities?

Activity 7 is not on the critical path, but its float is F7 = 2 < 3.
Thus, the network must be recalculated.
The project is delayed by 1 week, reaching 17 weeks.
The new critical path becomes 1, 4, 7, 9.


0 5
1 5
0 5
F = 0
5 8
4 3
5 8
F = 0
8 17
7 9
8 17
F = 0
dummy
0 7
2 7
1 8
F = 1
7 11
5 4
8 12
F = 1
11 16
8 5
12 17
F = 1
17 17
9 0
17 17
F = 0
0 6
3 6
2 8
F = 2
6 8
6 2
10 12
F = 4