Το σύνολο των μιγαδικών αριθμών $\mathbb{C}$

Μιγαδικός αριθμός $z = \underbrace{a}_{\text{πραγματικό μέρος}} + \underbrace{jb}_{\text{φανταστικό μέρος}}$

j : φανταστική μονάδα

$\boxed{j^2 = -1}$

$z_1 = a_1 + jb_1$

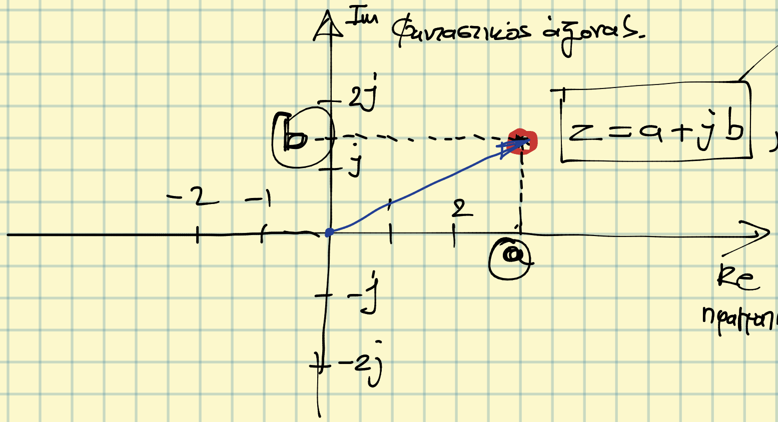
$z_2 = a_2 + jb_2$

(+) $z_1 + z_2 = a_1 + jb_1 + a_2 + jb_2 = (a_1 + a_2) + j(b_1 + b_2)$

(−) $z_1 - z_2 = (a_1 - a_2) + j(b_1 - b_2)$

(·) $z_1 \cdot z_2 = (a_1 + jb_1)(a_2 + jb_2) = a_1 a_2 + a_1 j b_2 + j b_1 a_2 + j^2 b_1 b_2 = a_1 \cdot a_2 - b_1 b_2 + j(a_1 b_2 + b_1 a_2)$

(/) $\dfrac{z_1}{z_2} = \dfrac{a_1 + jb_1}{a_2 + jb_2} = \dots\dots$



$\boxed{z = a + jb}$, μιγαδικό επίπεδο

Μέτρο μιγαδικού αριθμού $|z| = \sqrt{a^2 + b^2}$

SOS

$z \cdot \bar{z} = |z|^2$

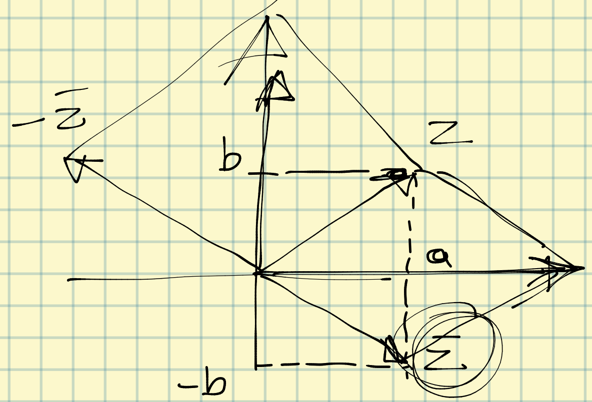
$|z_1 \cdot z_2| = |z_1| \cdot |z_2|$

$\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$

Βασικές Ιδιότητες

$z = a + jb$

συζυγής μιγαδικός αριθμός του z είναι ο $\bar{z}$ ή $z^* = a - jb$



Αν $z = a + jb$ η καρτεσιανή μορφή του μιγαδικού

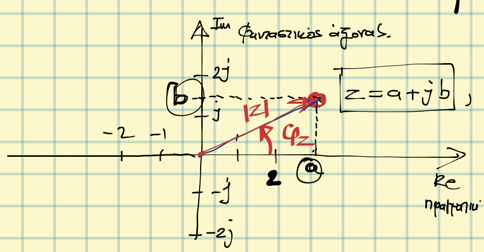
τότε με

$z = |z| \angle \varphi_z$

όπου $|z| = \sqrt{a^2 + b^2}$ (Μέτρο)

και $\varphi_z = \arctan \dfrac{b}{a} = \tan^{-1} \dfrac{b}{a}$ (φάση)

→ πολική μορφή του μιγαδικού αριθμού.



$\boxed{z + \bar{z} = 2\,\mathrm{Re}\{z\} = 2a}$

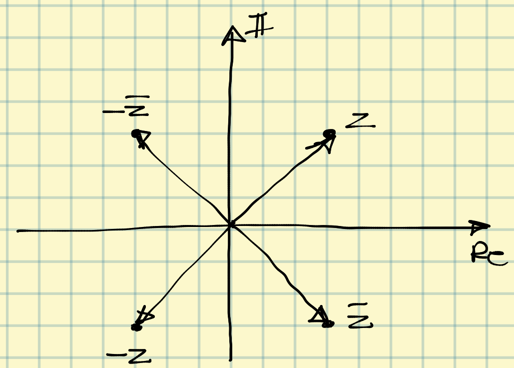
$\boxed{z - \bar{z} = 2\,\mathrm{Im}(z) = 2bj}$

$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

$\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$

$\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$

$\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\bar{z}_1}{\bar{z}_2}$



$z_1 = a_1 + jb_1 = \sqrt{a_1^2+b_1^2}\,\angle \operatorname{atan}\frac{b_1}{a_1} = |z_1|\angle\varphi_{z_1}$

$z_2 = |z_2|\angle\varphi_{z_2}$

$$z_1\cdot z_2 = \left(|z_1|\angle\varphi_{z_1}\right)\cdot\left(|z_2|\angle\varphi_{z_2}\right)$$

$$= |z_1|\cdot|z_2|\angle\left(\varphi_{z_1}+\varphi_{z_2}\right)$$

$$\frac{z_1}{z_2} = \frac{|z_1|\angle\varphi_{z_1}}{|z_2|\angle\varphi_{z_2}} = \frac{|z_1|}{|z_2|}\angle\left(\varphi_{z_1}-\varphi_{z_2}\right)$$

$$z\cdot\bar{z} = |z|^2$$

$z_1 = a_1 + jb_1$

$z_2 = a_2 + jb_2$

$$\frac{z_1}{z_2} = \frac{(a_1+jb_1)}{(a_2+jb_2)} = \frac{z_1\cdot\bar{z}_2}{z_2\cdot\bar{z}_2} = \frac{z_1\cdot\bar{z}_2}{|z_2|^2} = \frac{(a_1+jb_1)(a_2-jb_2)}{a_2^2+b_2^2}$$

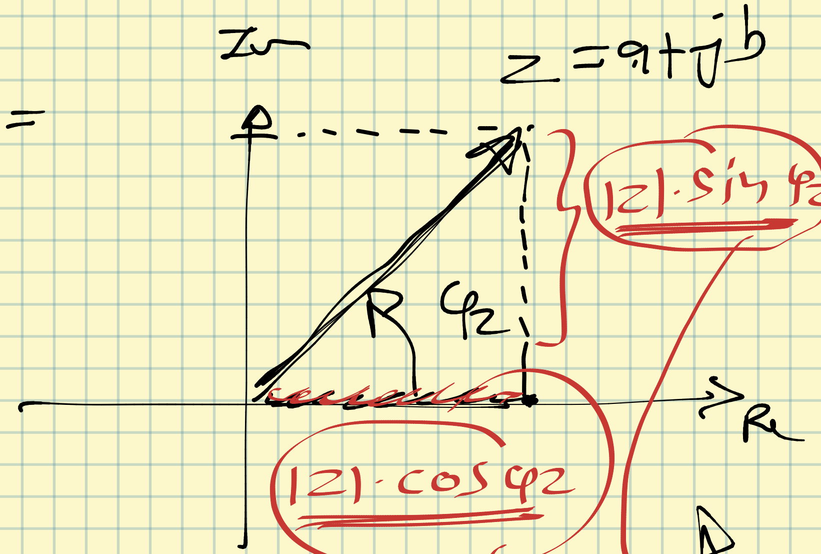
$$= \frac{a_1a_2 - a_1b_2 j + jb_1a_2 - j^2 b_1b_2}{a_2^2+b_2^2} =$$

$$= \frac{a_1a_2+b_1b_2+j(b_1a_2-a_1b_2)}{a_2^2+b_2^2}$$

$$= \underbrace{\frac{a_1a_2+b_1b_2}{a_2^2+b_2^2}}_{a'} + j\underbrace{\frac{(b_1a_2-a_1b_2)}{a_2^2+b_2^2}}_{b'}$$

$$= a' + jb'$$

$$\frac{\sqrt{a_1^2+b_1^2}}{\sqrt{a_2^2+b_2^2}}\angle\left(\tan^{-1}\frac{b_1}{a_1} - \tan^{-1}\frac{b_2}{a_2}\right)$$



$$z_1\cdot z_2 = |z_1|\cdot e^{j\varphi_1}\cdot|z_2|e^{j\varphi_2}$$

$$= |z_1|\cdot|z_2|\, e^{j\varphi_1}\cdot e^{j\varphi_2}$$

$$= |z_1|\cdot|z_2|\, e^{j(\varphi_1+\varphi_2)}$$

$$\frac{z_1}{z_2} = \frac{|z_1|\,e^{j\varphi_1}}{|z_2|\,e^{j\varphi_2}} = \frac{|z_1|}{|z_2|}\,e^{j\varphi_1}\cdot e^{-j\varphi_2} = \frac{|z_1|}{|z_2|}\cdot e^{j(\varphi_1-\varphi_2)}$$

Εκθετική μορφή μιγάδικου $z = a+jb = |z|\angle\varphi_z$

$$z = |z|\cdot e^{j\varphi_z} \to z_1 = |z_1|\cdot e^{j\varphi_1}$$

$$z_2 = |z_2|\, e^{j\varphi_2}$$

τριγωνομετρική μορφή μιγαδικού αριθμ.

$$z = a+jb = |z|\left(\cos\varphi_z + j\sin\varphi_z\right)$$

$a = |z|\cos\varphi_z$, $b = |z|\sin(\varphi_z)$, $\varphi_z = \operatorname{atan}\left(\frac{b}{a}\right)$!

$$z = a + jb = \underbrace{|z|\angle\varphi_z}_{\text{πολική}} = \underbrace{|z|\cdot e^{j\varphi_z}}_{\text{εκθετική μορφή}} = \underbrace{|z|(\cos\varphi_z + j\sin\varphi_z)}_{\text{τριγωνομετρική}}$$

$$|z| = \sqrt{a^2+b^2}$$

$$\varphi_z = \operatorname{atan}\frac{b}{a}$$

$$a = |z|\cos\varphi_z$$

$$b = |z|\sin\varphi_z$$