A880. This means that it modulates 220 Hz downward but 440 Hz upwardan asymmetrical modulation. The average center frequency of the carrier changes, which usually means that the perceived center pitch is detuned by a significant interval. This detuning is caused by the modulation index, which means that the bandwidth and the center frequency are linked. From a musical standpoint, this linkage is not ideal. We want to be able to increase the modulation index without shifting the center frequency. See Hutchins (1975) for an analysis of exponential FM.

In digital modulation the sidebands are spaced equally around the carrier; hence the term *linear FM*. As the modulation index increases, the center frequency remains the same. All digital FM is linear, and at least one manufacturer, Serge Modular, makes a linear FM analog oscillator module.

Analysis and FM

Since FM techniques can create many different families of spectra, it might be useful to have an analysis/resynthesis procedure linked to FM, similar to those used with additive and subtractive techniques. Such a procedure could take an existing sound and translate it into parameter values for an FM instrument. By plugging those values into the instrument, we could hear an approximation of that sound via FM synthesis. The general name for this type of procedure is *parameter estimation* (see chapter 13). Various attempts have been made to try to approximate a given steady-state spectrum automatically using FM (Justice 1979; Risberg 1982). The problem of estimating the FM parameters for complex evolving sounds is difficult (Kronland-Martinet and Grossmann 1991; Horner, Beauchamp, and Haken 1992).

As the power of digital hardware has increased, some of the motivation for estimating FM parameters has diminished. FM synthesis was originally proposed as a computationally efficient method, but now more powerful synthesis methods (such as additive synthesis) are no longer so difficult. Only a certain class of sounds are well modeled as modulations. Additive synthesis and physical models (see chapter 7) may be more appropriate models of traditional instruments.

Multiple-Carrier FM

By *multiple-carrier frequency modulation* (MC FM), we mean an FM instrument in which one oscillator simultaneously modulates two or more carrier oscillators. The output of the carriers sum to a composite waveform that

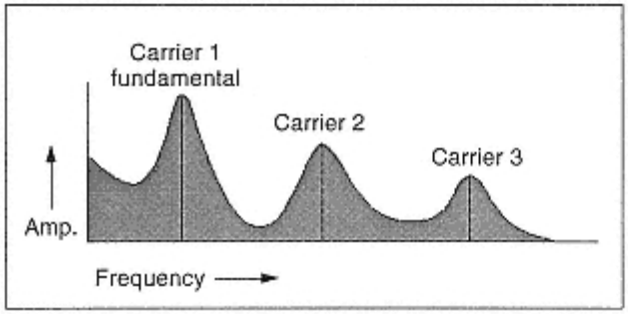

Figure 6.15
A spectrum with three formant regions created with a three-carrier FM instrument.

superposes the modulated spectra. Multiple carriers can create *formant regions* (peaks) in the spectrum, as shown in figure 6.15. The presence of formant regions is characteristic of the spectrum of the human voice and most traditional instruments. Another justification for separate carrier systems is to set different decay times for each formant region. This is useful in simulating brasslike tones where the upper partials decay more rapidly than the lower partials.

Figure 6.16 shows a triple-carrier FM instrument. In order to indicate clearly the multiple-carrier structure, the figure omits envelope controls and waveform tables. The amplitudes of the carriers are independent. When the *Carrier 2* and *Carrier 3* amplitudes are some fraction of *Carrier 1*, the instrument generates formant regions around the frequencies of the second and third carriers.

The equation for a multiple-carrier FM waveform at time t is simply the addition of n simple FM equations:

$$MCFM_t = A^{w1} \times \sin(C1_t + [I1 \times \sin(M)]) \ldots$$
$$+ A^{wn} \times \sin(Cn_t + [In \times \sin(M)])$$

where A is an amplitude constant, $0 < A$ 1.0,

$w1$ is the weighting of *Carrier 1*,

wn is the weighting of *Carrier n*,

$C1$ is the fundamental pitch = 2   × carrier frequency 1 (in Hz),

Cn is the formant frequency = 2   × carrier frequency n (in Hz), where Cn is an integer multiple of $C1$,

M is modulating frequency, usually set to be equal to $C1$ (Chowning 1989).

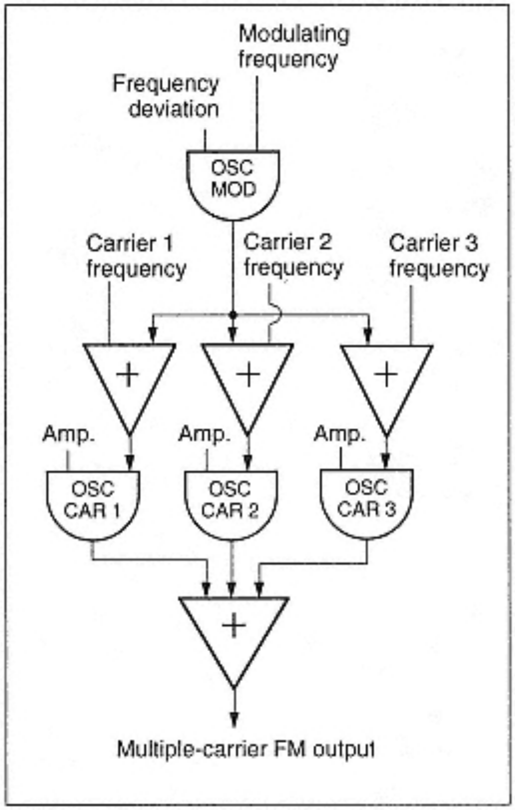

Figure 6.16
Triple-carrier FM instrument driven by a single modulating oscillator (OSC MOD).

I1 is the modulation index of *C1*

In is the modulation index of *Cn*

The exponents *w1* and *wn* determine how the relative contribution of the carriers vary with the overall amplitude *A*.

Musical Applications of MC FM

Documented applications of MC FM strive to simulate the sounds of traditional instrument tones. With MC FMor any synthesis technique, for that matterthe secret of realistic simulation is attention to detail in all aspects of the soundamplitude, frequency, spectral envelopes, vibrato, and musical context.

A straightforward application of MC FM is in the synthesis of trumpet-like tones. Risset and Mathews's (1969) analysis of trumpet-like tones showed a nearly harmonic spectrum, a 20 25 ms rise time of the amplitude envelope (with high partials building up more slowly), a small quasi-random frequency fluctuation, and a formant peak in the region of 1500 Hz. Morrill (1977) developed both single-carrier and double-carrier FM instruments for brass tone synthesis based on these data. A double-carrier instrument sounds more realistic, since each carrier produces frequencies for different parts of the spectrum. In particular, *C1* generates the fundamental and the first five to seven partials, while *C2* is set at 1500 Hz, the main formant region of the trumpet. Each carrier has its own amplitude envelope for adjusting the balance between the two carrier systems in the composite spectrum. For example, in loud trumpet tones, the upper partials standout.

Chowning (1980, 1989) applied the MC FM technique to the synthesis of vowel sounds sung by a soprano and by a low bass voice. He determined that a combination of periodic and random vibrato must be applied to all frequency parameters for realistic simulation of the vocal tones. "Without vibrato the synthesized tones are unnatural sounding" (Chowning 1989, p. 62). A quasi-periodic vibrato makes the frequencies "fuse" into a vocal-like tone. In Chowning's simulations, the *vibrato percent deviation V* is defined by the relation

$$V = 0.2 \times \log(\text{pitch}).$$

Hence for a pitch of 440 Hz, V is about 1.2 percent or 5.3 Hz in depth. The frequency of the vibrato ranges from 5.0 to 6.5 Hz according to the fundamental frequency range of the pitches F3 to F6.

Multiple-Modulator FM

In multiple-modulator frequency modulation (MM FM) more than one oscillator modulates a single carrier oscillator. Two basic configurations are possible: *parallel* and *series* (figure 6.17). MM FM is easiest to understand when the number of modulators is limited to two and their waveforms are sinusoidal.

Parallel MM FM

In parallel MM FM, two sine waves simultaneously modulate a single carrier sine wave. The modulation generates sidebands at frequencies of the form:

$$C \pm (i \times M1) \pm (k \times M2)$$

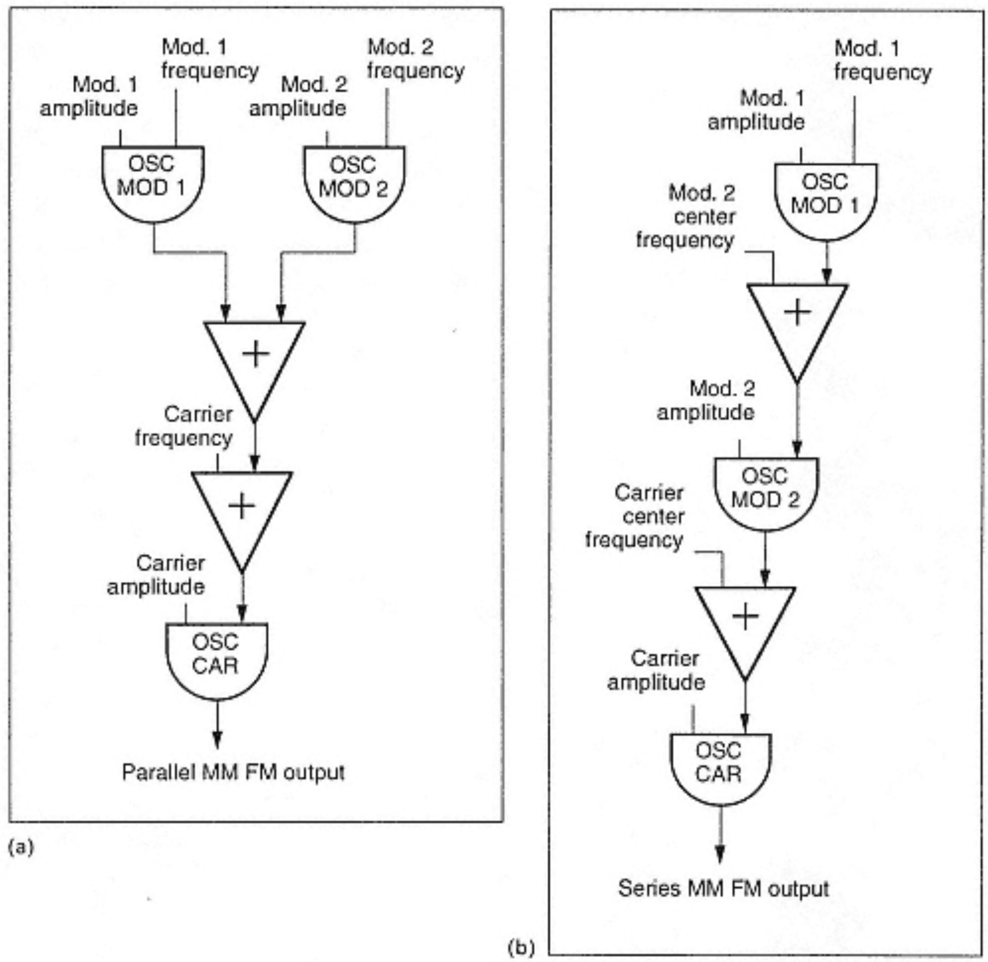

Figure 6.17
MM FM instruments. (*a*) Parallel MM FM. (*b*) Series MM FM.

where *i* and *k* are integers and *M1* and *M2* are the modulating frequencies. In parallel MM FM, it is as though each of the sidebands produced by one of the modulators is modulated as a carrier by the other modulator. The explosion in the number of partials is clear in figure 6.18, which lists both the primary and secondary modulation products.

The wave equation of the parallel double-modulator FM signal at time *t* is as follows:

$$PMMFMt = A \times \sin\{Ct + [I1 \times \sin(M1t)] + [I2 \times \sin(M2t)]\}.$$

For mathematical descriptions of the spectra produced by this class of techniques, see Schottstaedt (1977) and LeBrun (1977).

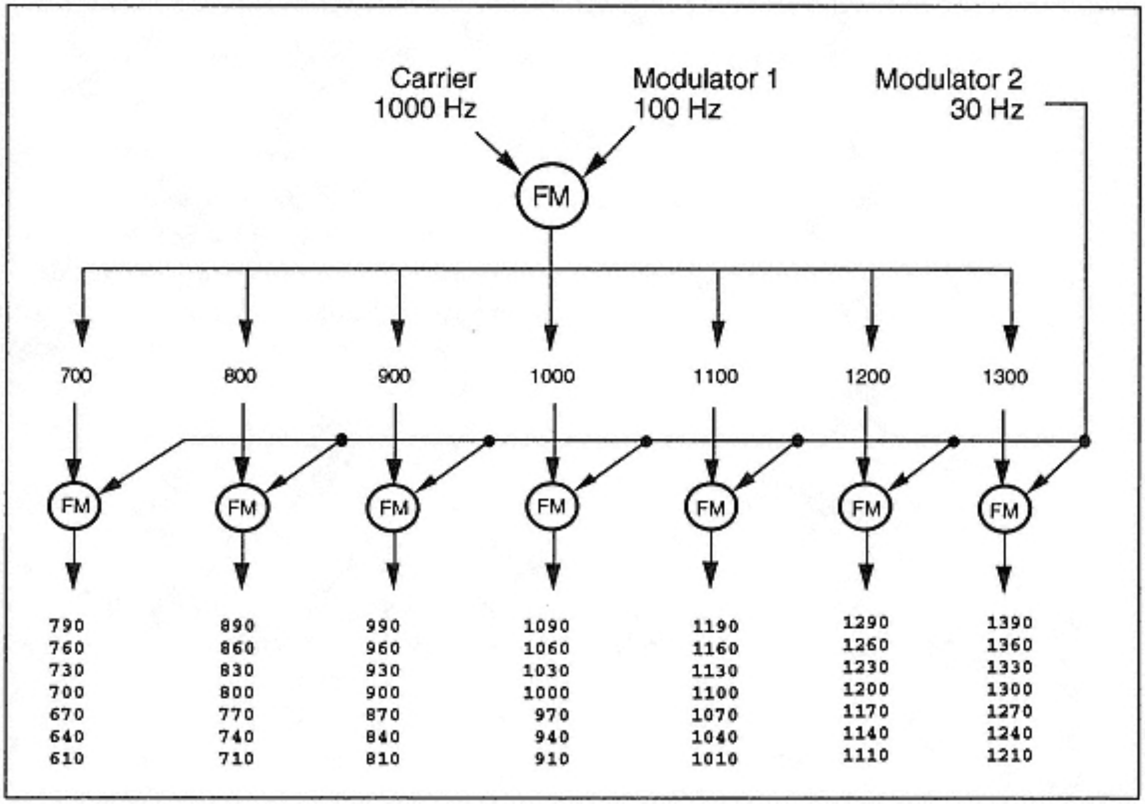

Figure 6.18
This diagram depicts the explosion in the number of partials produced by parallel MM FM. Each of the components emitted by the modulation of the *Carrier* by *Modulator 1* is then modulated by *Modulator 2*, producing the list of spectral components shown at the bottom.

Series MM FM

In series MM FM the modulating sine wave *M1* is itself modulated by *M2*. This creates a complicated modulating wave with a potentially immense number of sinusoidal sideband components, depending on the index of modulation. The instantaneous amplitude of series double-modulator FM is given in the following equation, adapted from Schottstaedt (1977):

$$SMMFMt = A \times \sin\{Ct + [I1 \times \sin(M1t + [I2 \times \sin(M2t)])]\}.$$

The differences between the parallel and serial equations reflects the configuration of the oscillators. In practice, *I2* determines the number of significant sidebands in the modulating signal and *I1* determines the number of sidebands in the output signal. Even small values of *I1* and *I2* create complex waveforms. The ratio *M1*:*C* determines the placement of the carrier's sidebands, each of which has sidebands of its own at intervals determined by *M2:M1*. Hence, each sideband is modulated and is also a modulator.

Musical Applications of MM FM

Schottstaedt (1977) used double-modulator FM to simulate certain characteristics of piano tones. He set the first modulator to approximately the carrier frequency, and the second modulator to approximately four times the carrier frequency. According to Schottstaedt, if the carrier and the first modulator are exactly equal, the purely harmonic result sounds artificial, like the sound of an electric (amplified tuning bar) piano. This need for inharmonicity in piano tones agrees with the findings of acousticians (Blackham 1965; Backus 1977).

Schottstaedt made the amplitudes of the modulating indexes frequency-dependent. That is, as the carrier frequency increases, the modulation index decreases. The result is a spectrum that is rich in the lower register but becomes steadily simpler as the pitch rises. Since the length of decay of a piano tone also varies with pitch (low tones decay longer), he used a pitch-dependent decay time.

Chowning and Schottstaedt also worked on the simulation of stringlike tones using triple-modulator FM, where the *C:M1:M2* ratio was 1:3:4, and the modulation indexes were frequency dependent (Schottstaedt 1977). Chowning also developed a deep bass voice using a combination MC FM and MM FM instrument. See Chowning (1980, 1989) for more details on this instrument.

Feedback FM

Feedback FM is a widely used synthesis technique, due to Yamaha's patented application of the method in its digital synthesizers (Tomisawa 1981). In this section we describe three types of feedback FM: *one-oscillator feedback, two-oscillator feedback*, and *three-oscillator indirect feedback*.

Feedback FM solves certain problems associated with simple (nonfeedback) FM methods. When the modulation index increases in simple FM, the amplitude of the partials vary unevenly, moving up and down according to the Bessel functions (figure 6.19). This undulation in the amplitude of the partials lends an unnatural "electronic sound" characteristic to the simple FM spectrum; it makes simulations of traditional instruments more difficult.