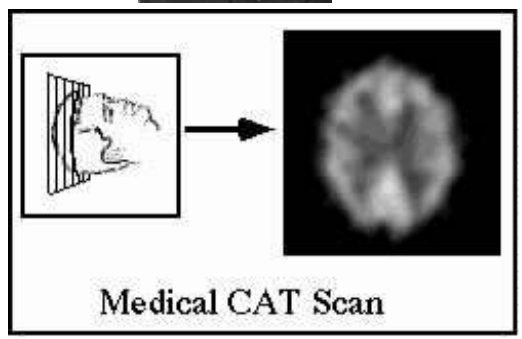
Advances in Image Processing and Computer Vision
Lecture 1

Many thanks to Ulas Bagci (Northwestern University) for sharing his experience and course material

Introduction

Basic

Historical data…



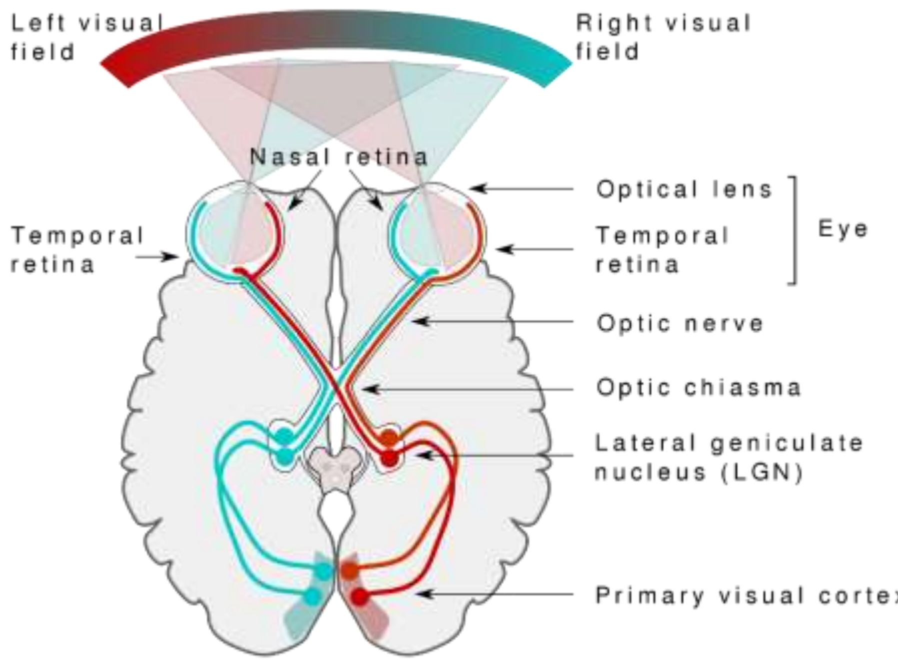
Nobel prizes

Roentgen (1901): Discovery of X-rays

Hounsfield & Cormack (1979): Computed tomography

Designing Imaging systems-Imaging Chain

The path of information in the human visual system: The visual stimulus from the outside world is captured in the sensing organ (eye), and then transmitted through the optic nerve to the brain, ending in the visual cortex where the information is processed and the sense of vision is realized.

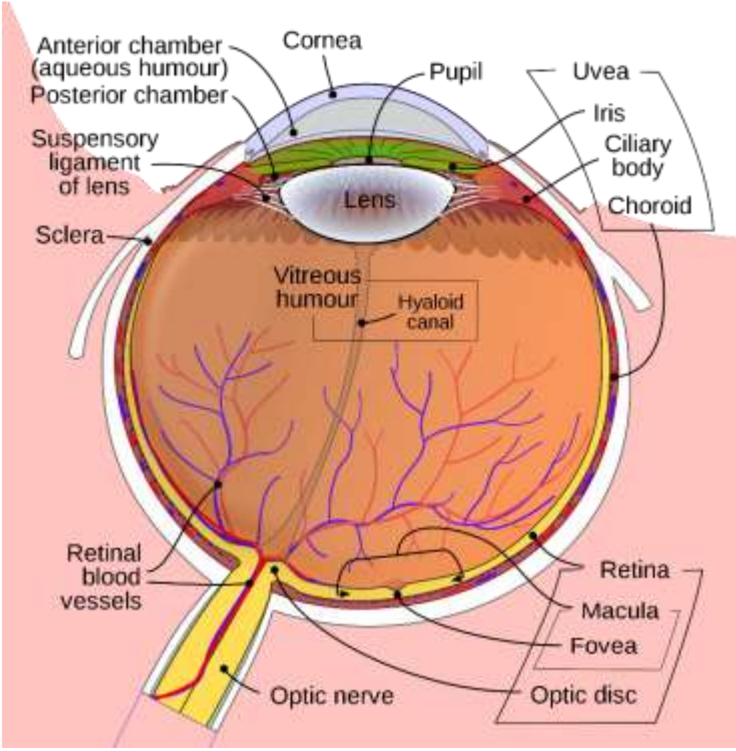


https://en.wikipedia.org/wiki/Visual_system#/media/File:Human_visual_pathway.svg

Designing Imaging systems-Imaging Chain

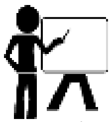
Light entering the eye is refracted as it passes through the cornea. It then passes through the pupil (controlled by the iris) and is further refracted by the lens. The cornea and lens act together as a compound lens to project an inverted image onto the retina.

The retina is the inner membrane of the eye. When the eye focuses (movement through muscles) the light from an external object is depicted in the retina through the receptor cells that transmit neuronal signals to the brain to be processed.



Simplified diagram of a section of the eye

https://en.wikipedia.org/wiki/Visual_system#/media/File:Schematic_diagram_of_the_human_eye_en.svg



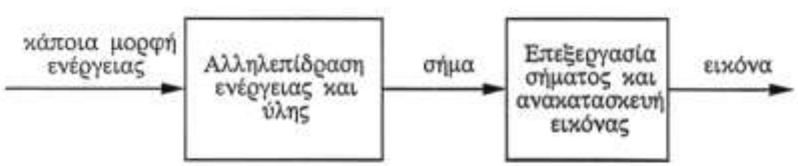
Basic

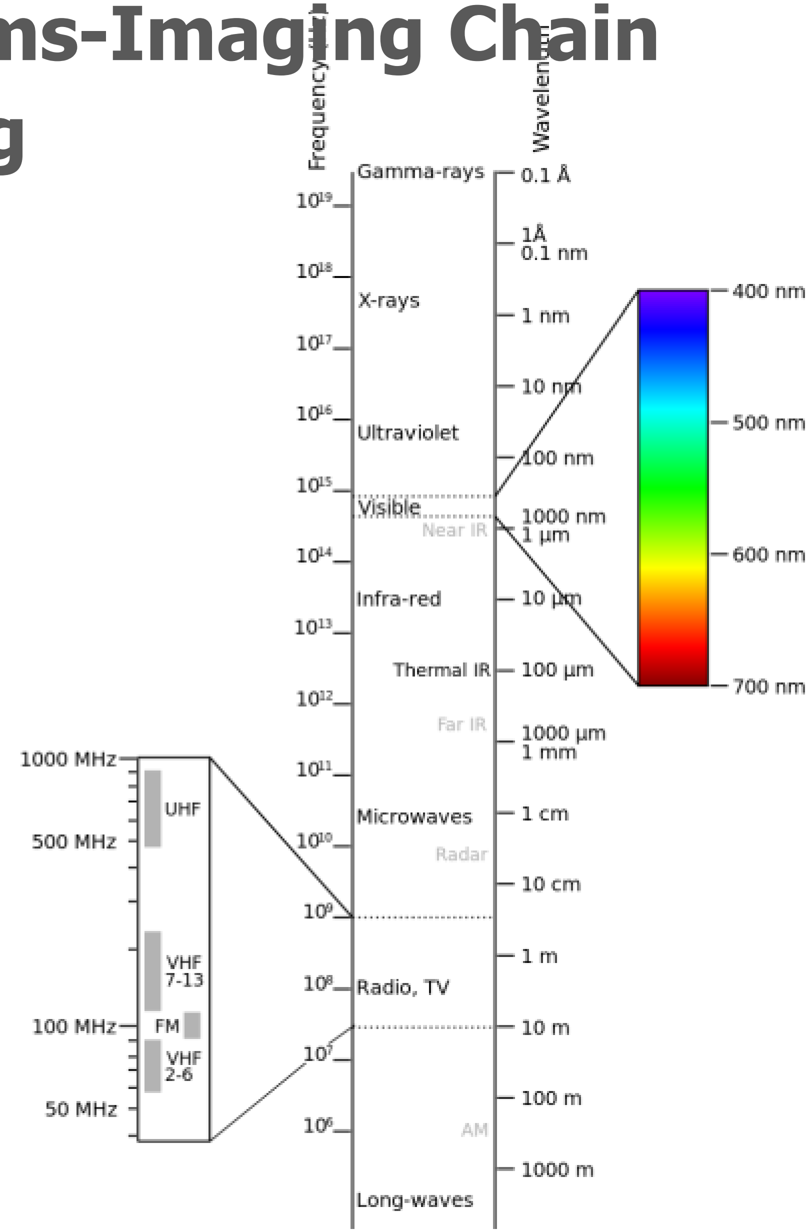
Designing Imaging systems-Imaging Chain

EM radiation and Imaging

EM radiation types

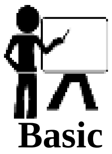
- Gamma radiation
- X-ray radiation
- Ultraviolet radiation
- Visible radiation
- Infrared radiation
- Terahertz radiation
- Microwave radiation
- Radio waves

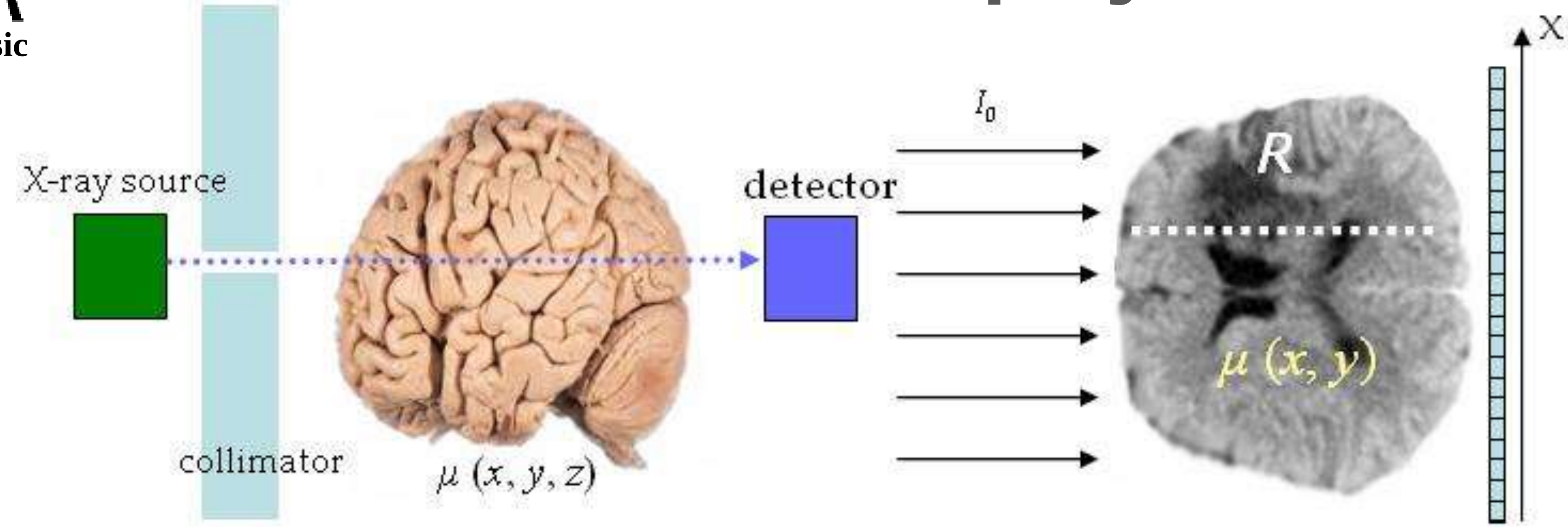




https://en.wikipedia.org/wiki/Electromagnetic_spectrum

Reconstruction from projections





The projected object from an angle *θ*:

$$g_\theta(x) = -\ln \frac{I(x)}{I_o} = \int_{source}^{\det ector} \mu(\mathrm{x}, y)\,\mathrm{d}\,y$$

The problem of reconstruction can be set as the attempt to calculate the function *μ*(x, y) from projections

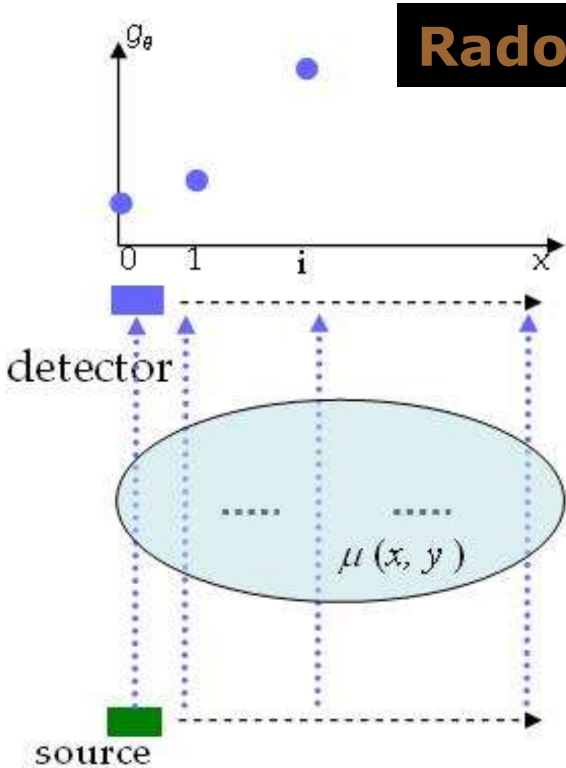
Reconstruction from projections

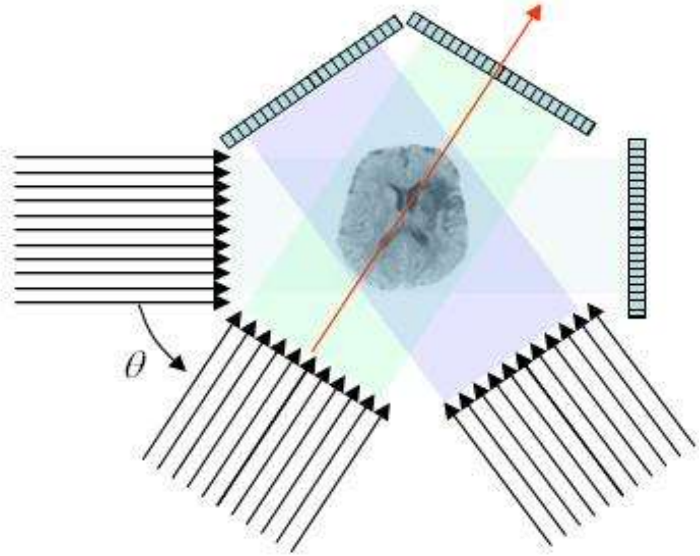
Basic

Continuous rotation of the source-detector system for acquisition of projections from different angles.

Radon transform of $\mu(x,y)$.

$$R = x\cos\theta + y\sin\theta$$

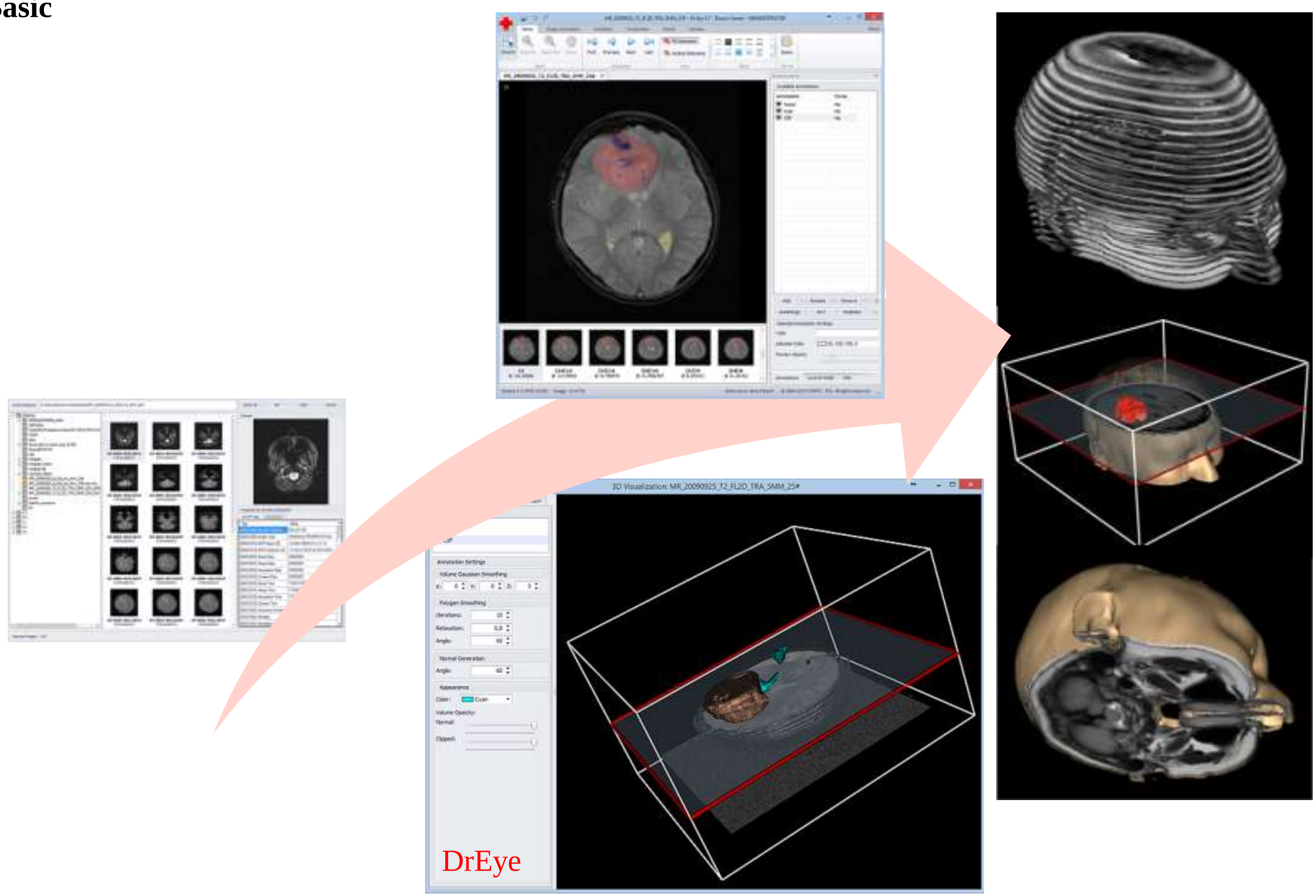




$$g_\theta(R) = \int_x \int_y \mu(\mathrm{x}, y)\, \delta(x\cos\theta + y\sin\theta - R)\, dy\, dx$$

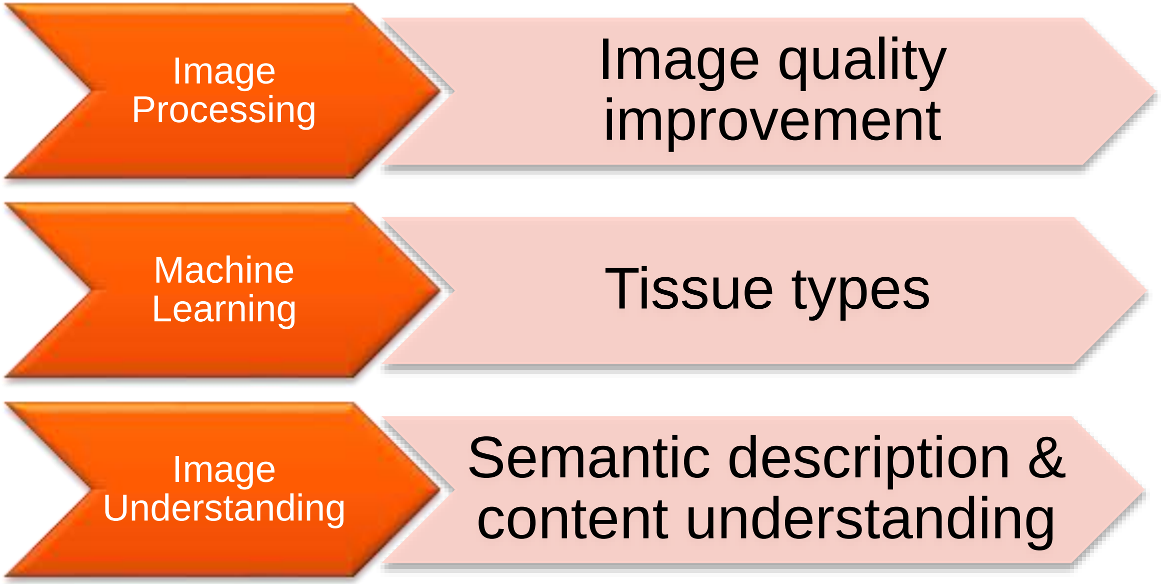
Basic

Visualization and processing of medical images



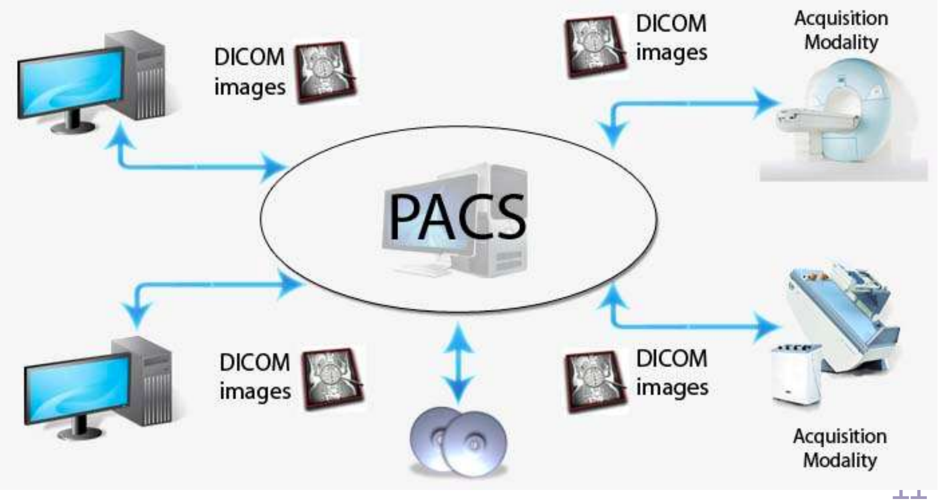
Medical Imaging

- We can see inside the human body in ways that are less invasive or (completely non-invasive)
- We can even see metabolic/functional/molecular activities which are not visible to naked eye

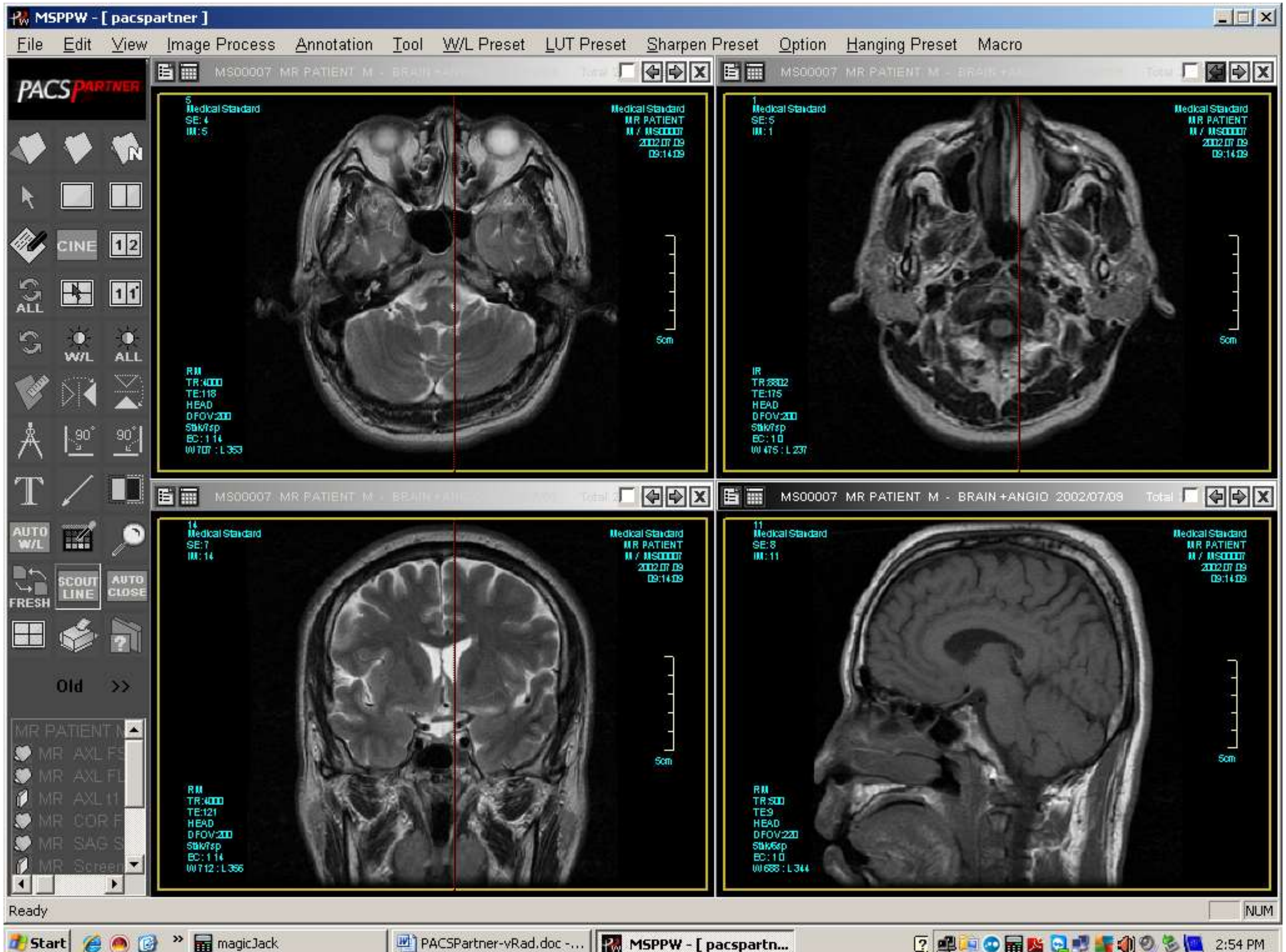


where do radiologists interpret scans?

- Dedicated light source
- Darkened environment
- Limited distraction



PACS (example)

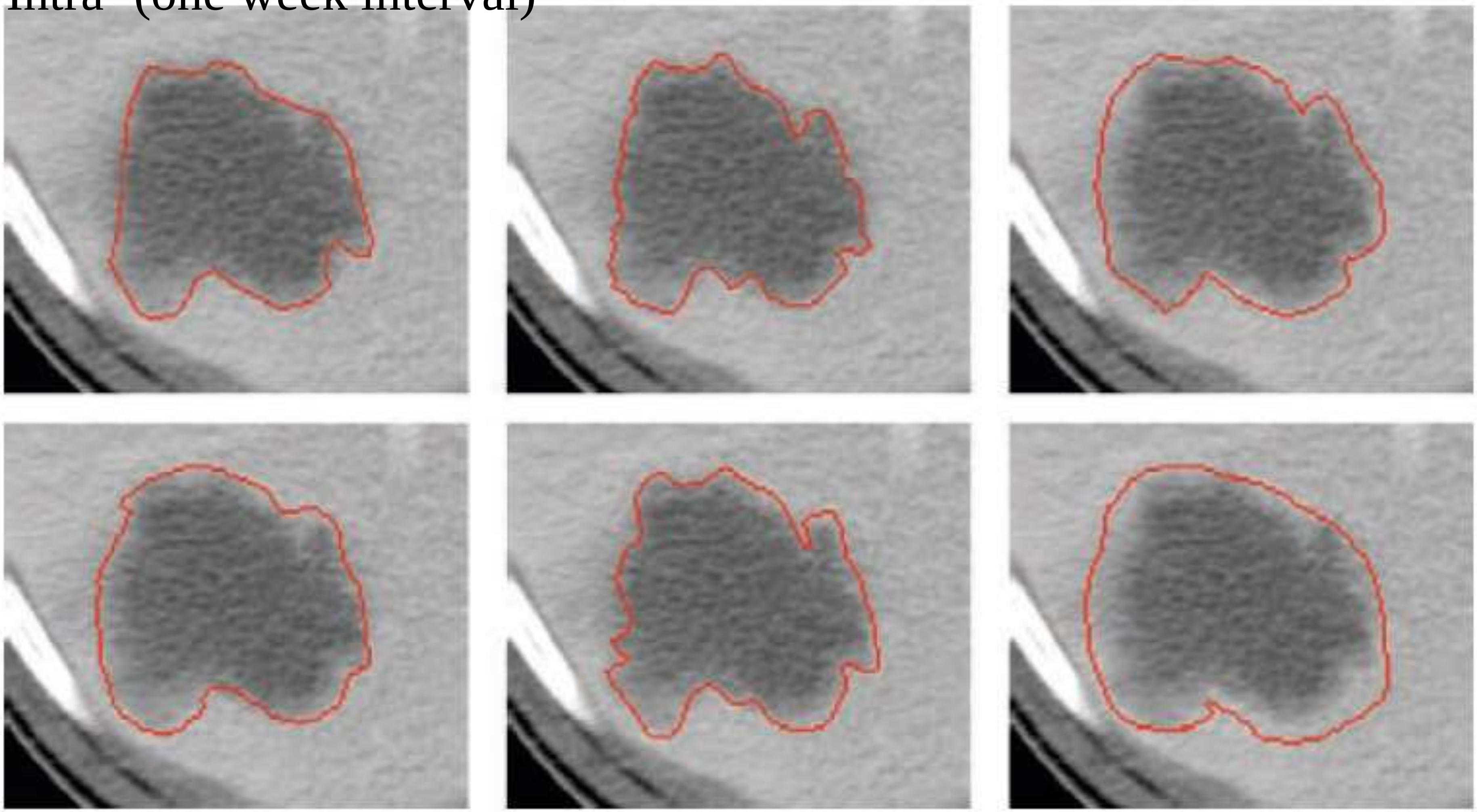


Medical Image Analysis-Need for objective and quantitative measurements

- Manual analysis is often accepted as surrogate of the truth (if biopsy or real ground truth is not available)
- However, manual analysis is highly subjective because it relies on the observer's perception.
  - Intra and inter-observer agreements/variabilities
- It is highly tedious
- Medical Image Analysis offers the technology to add precision and objectivity in diagnostic taks.

Observer Variability – Example: Liver lesion

Intra- (one week interval)

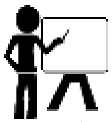


Inter-

Medical Image Analysis-Automated

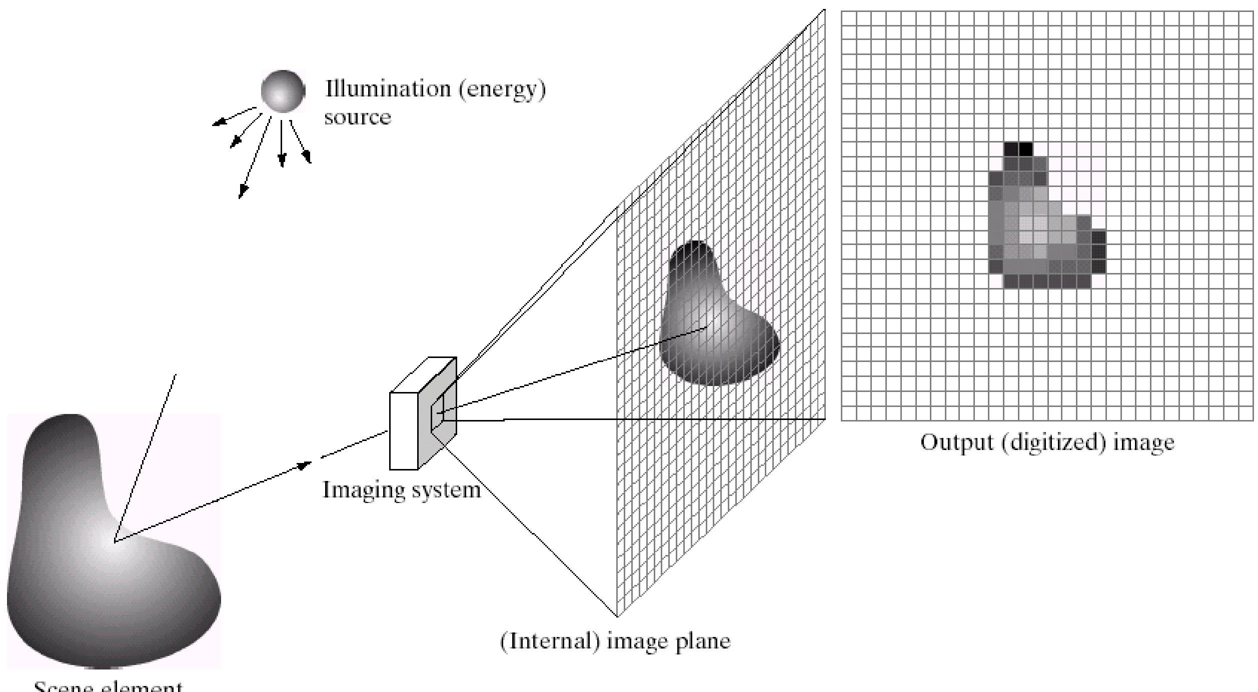
- Different strategies for image analysis exist. However, few of them are suited for medical applications.
- Medical Images are typically quite complex and require domain specific knowledge in order to design and implement efficient and clinically acceptable processing workflows.

Digital Images and Processing


Basic

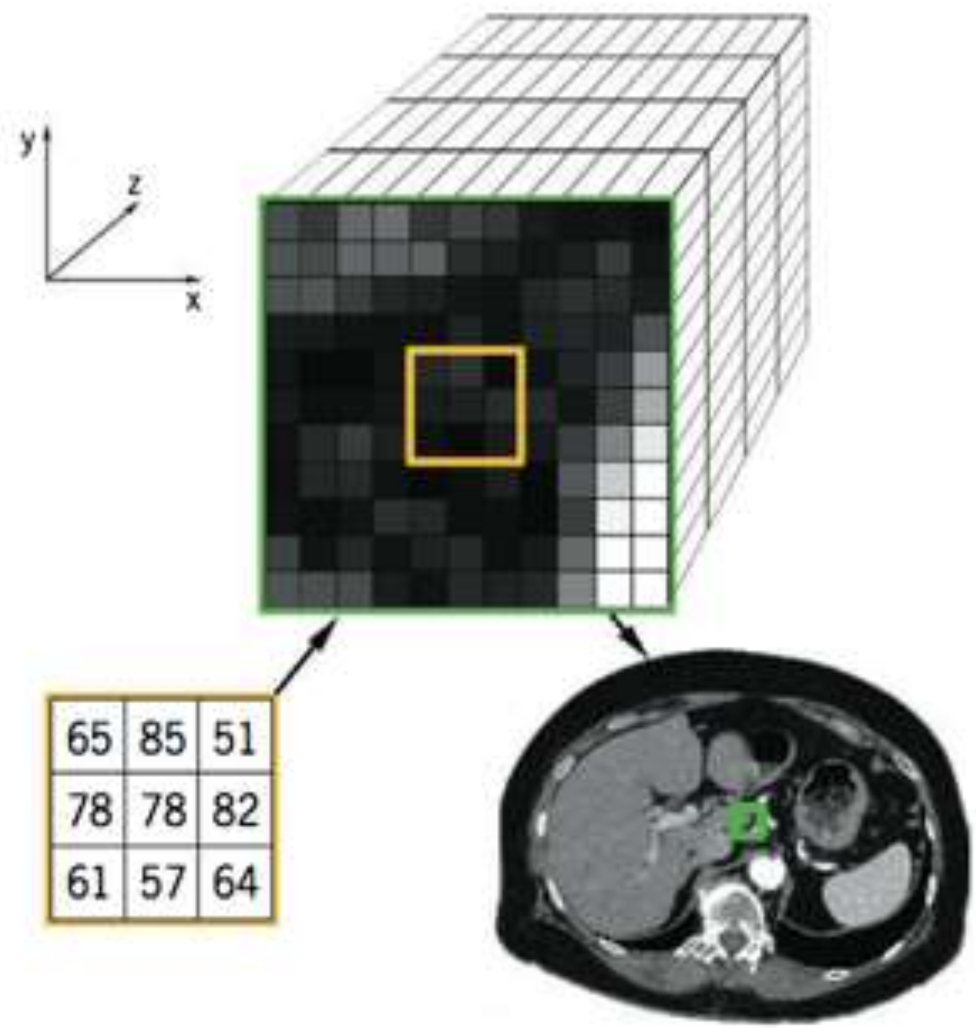
The digital Image

A digital image is a 2D discrete set of pixels f(x,y)



"Digital Image Processing", Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002

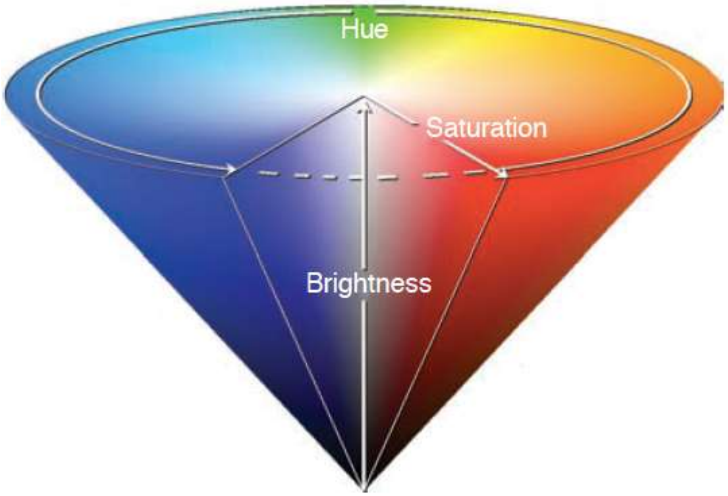
Digital Images



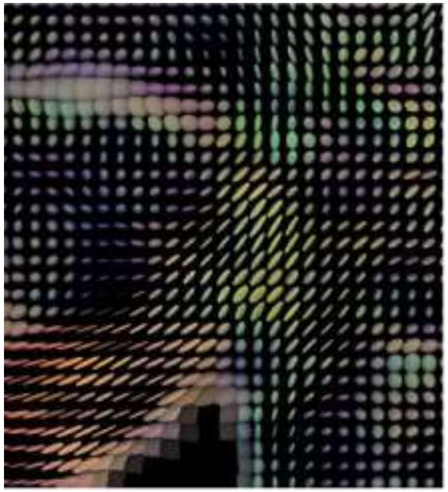
0	3	2	5	4	7	6	9	8
3	0	1	2	3	4	5	6	7
2	1	0	3	2	5	4	7	6
5	2	3	0	1	2	3	4	5
4	3	2	1	0	3	2	5	4
7	4	5	2	3	0	1	2	3
6	5	4	3	2	1	0	3	2
9	6	7	4	5	2	3	0	1
8	7	6	5	4	3	2	1	0

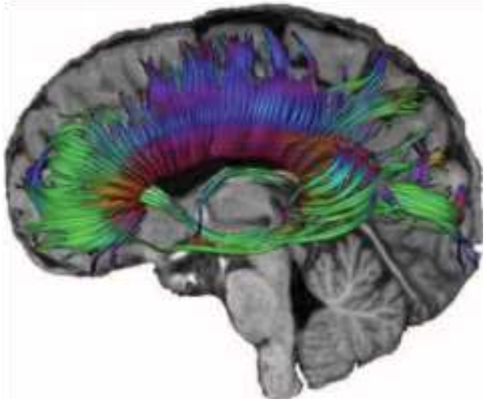
What computer sees!

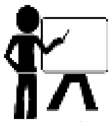
Image Types-Color



- Image has three channels (bands), each channel spans a-bit values.
- RGB, Hue-Saturation-Brightness






Basic

Digital Image

- It can be considered as a function f, from R^2 -> R :
  - $f(x, y)$ is the intensity at (x, y)
  - Images are rectangular matrices of non-infinite size
- A color image is a a vector function of three images R, G, B

$$f(x,y) = \begin{bmatrix} r(x,y) \\ g(x,y) \\ b(x,y) \end{bmatrix}$$

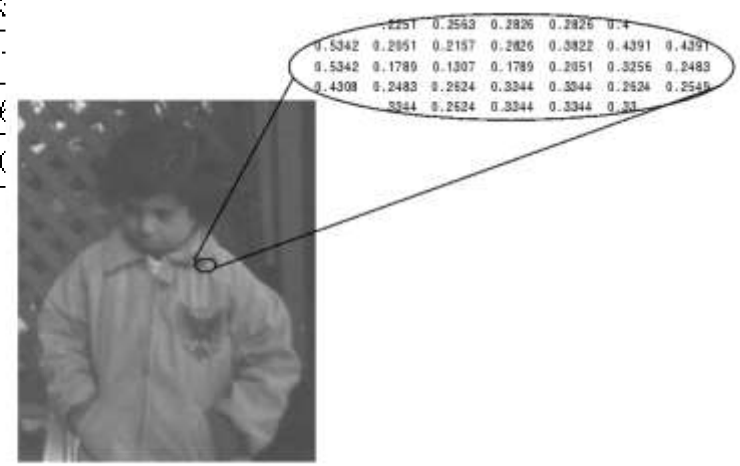
The digital Image

- We use discrete values in images with sampling
- The image can therefore be represented as a matrix with discrete, integer values.

4 j

3 i

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	[illegible]	[illegible]	[illegible]
0	89	144	147	187	[illegible]		
255	252	0	166	123	[illegible]		
166	63	127	17	1	[illegible]		



- The intensity at i=3, j=4 is $f(3, 4) = 46$

Image Types

- A scalar image has integer values

$$u \in \{0, 1, ..., 2^a - 1\}$$

a: level (bit)

Ex. If 8 bit (a=8), image spans from 0 to 255

0 black

255 white

Ex. If 1 bit (a=1), it is binary image, 0 and 1 only.

Types of digital Images

- **<u>Binary</u>**: Each pixel has two possible values only, 0 (black) και 1 (white).
- We need1 bit/pixel therefore they are very economical in storage.
- This representation is adequate for fingerprints, text documents and architectural designs.

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1
0	0	1	1	0	1	1	1	1	1
0	0	0	0	0	1	1	1	0	1
0	0	0	0	0	1	0	1	1	1
0	0	0	1	0	1	1	1	0	1
0	0	1	1	0	1	1	1	1	1

https://en.wikipedia.org/wiki/Binary_image

Types of digital Images

- **<u>Gray Scale</u>**: Each pixel value is a shade of gray, typically for the 8-bit case from 0 (black)- to 255 (white).
- In this case of 8-bit images the range is 255-0+1=256 shades of gray (Gray Levels or GL).
- 256 shades of gray are enough for the recognition of most physical objects.
- There are certain applications however that 12bit or 16bit images are used (e.g. medical imaging).

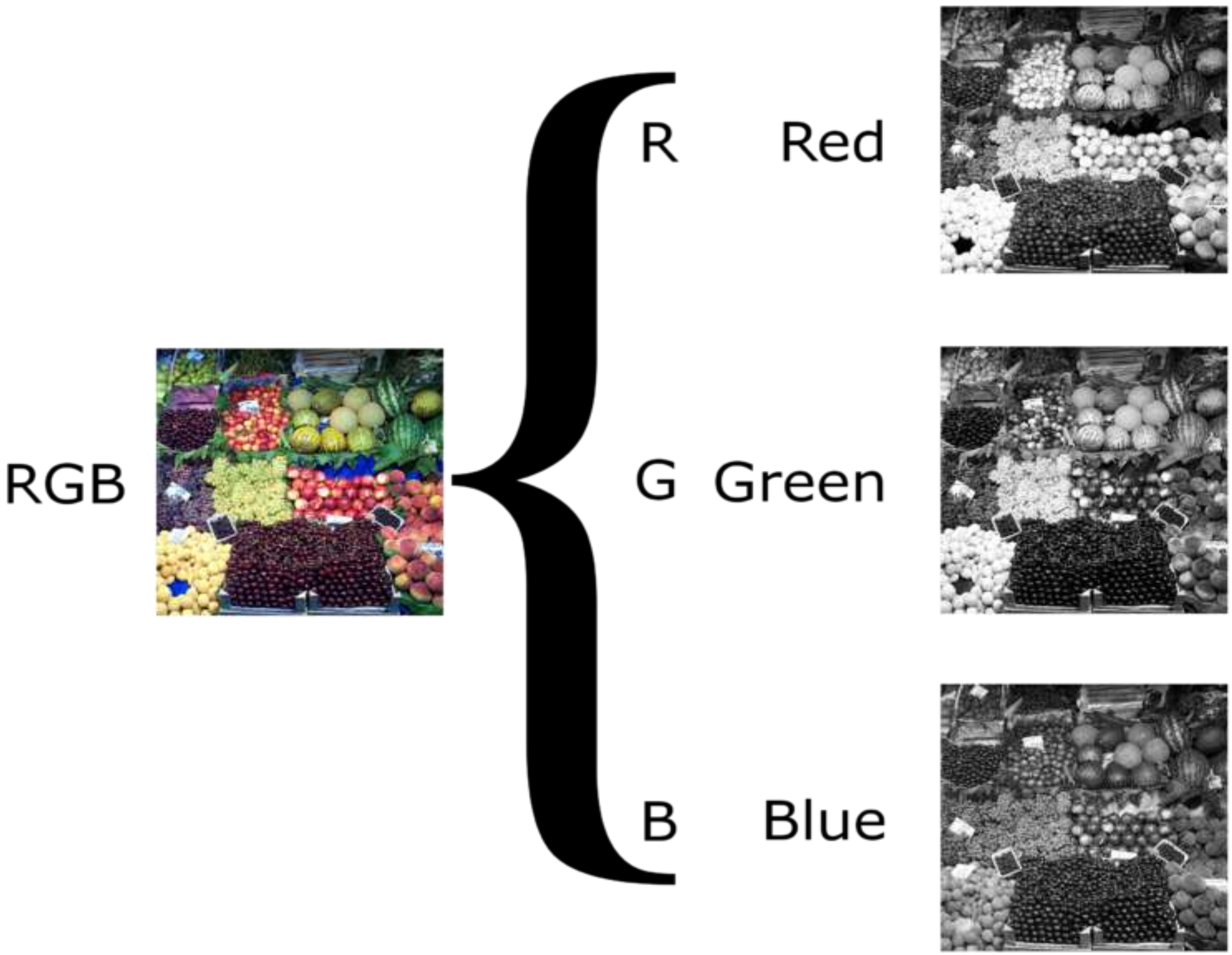
https://en.wikipedia.org/wiki/Grayscale

Types of digital Images

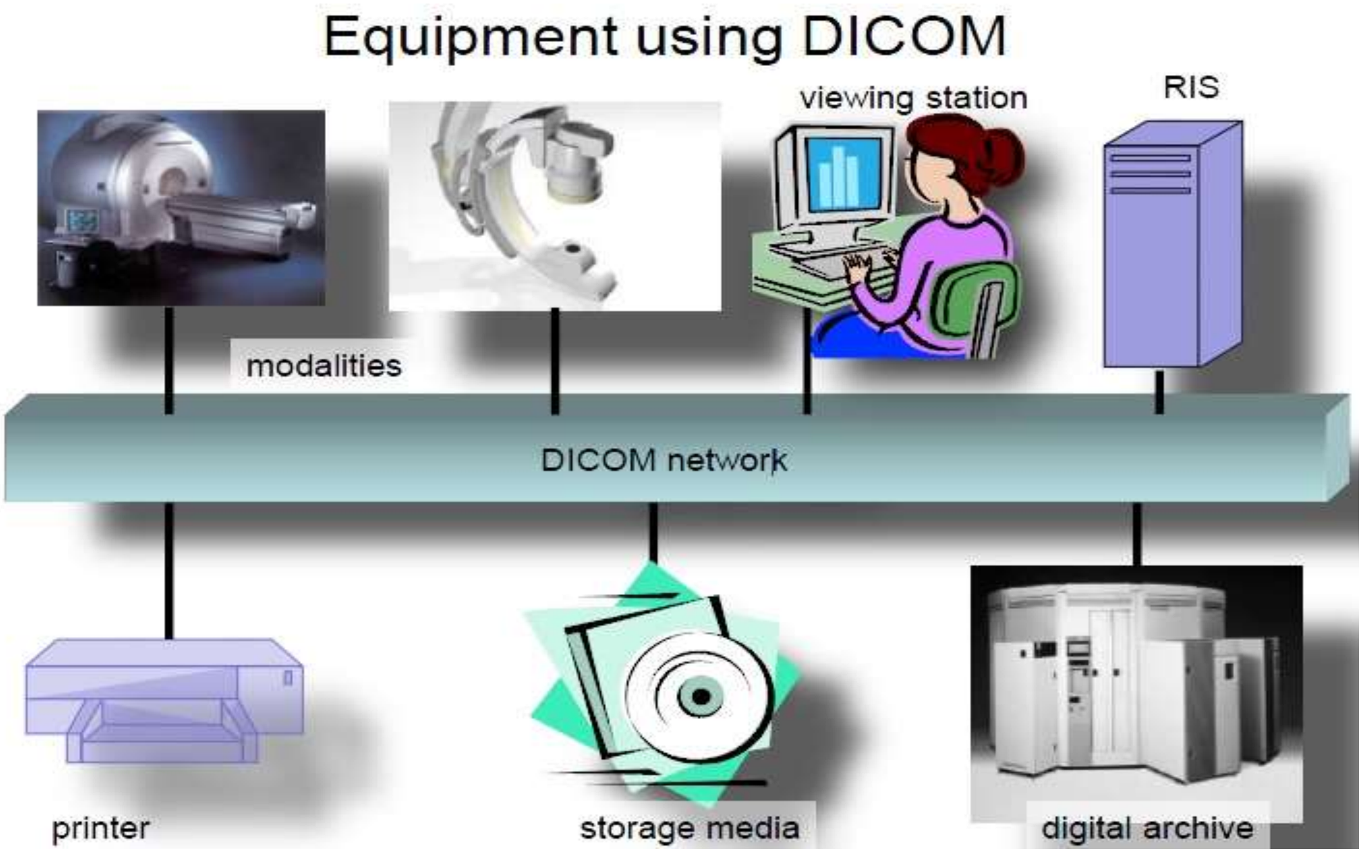
- **<u>RGB color images</u>**: Each pixels has a color described by the quantity of red R, green G and blue B(μπλέ) in it. In essesnce each color image is comprised of 3 matrices one for each color with range 0-255.
- In total we therefore have 255^3=16,777,216 different colors in this representation.
- Since for each pixel's value representation we need 3x8=24bit, RGB images are also known as 24bit color images.

Types of digital Images

- RGB Color Images

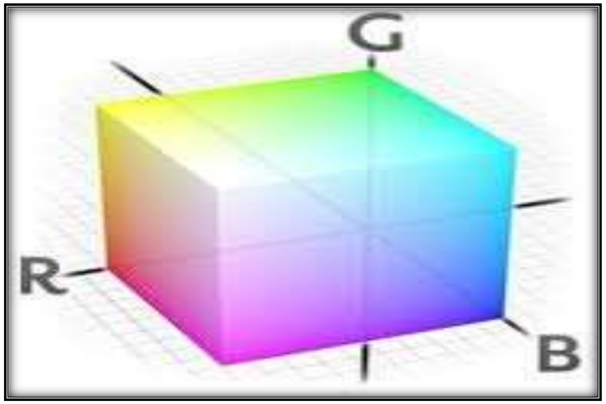


Medical Image Ecosystem



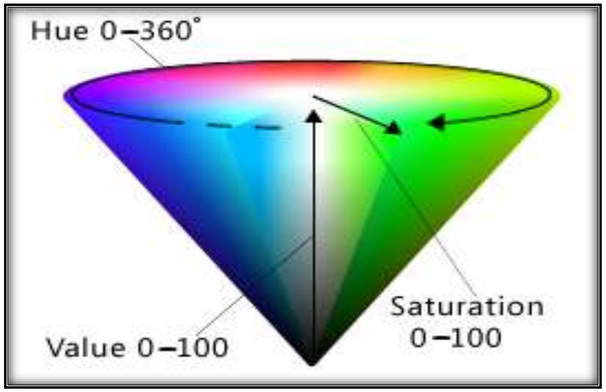
Juergen.Hesser@MedMa.Uni-Heidelberg.De

Image Representation: Color models



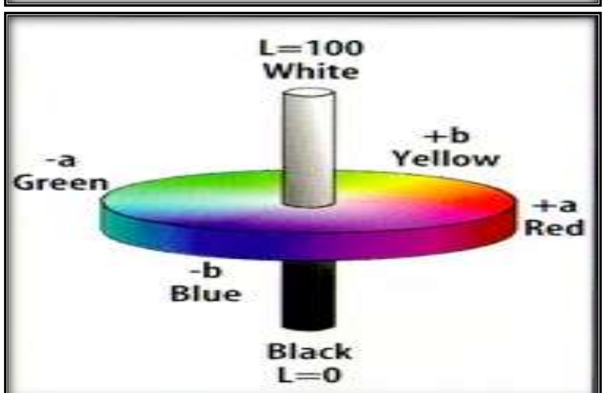
Red
Green
Blue

RGB MODEL
Color originates as a combination of the three color bands



Hue
Saturation
Value

HSV MODEL
Color information originates from the H and S bands



Luminance
A
B

Lab MODEL
Color originates as a combination of the A and B bands

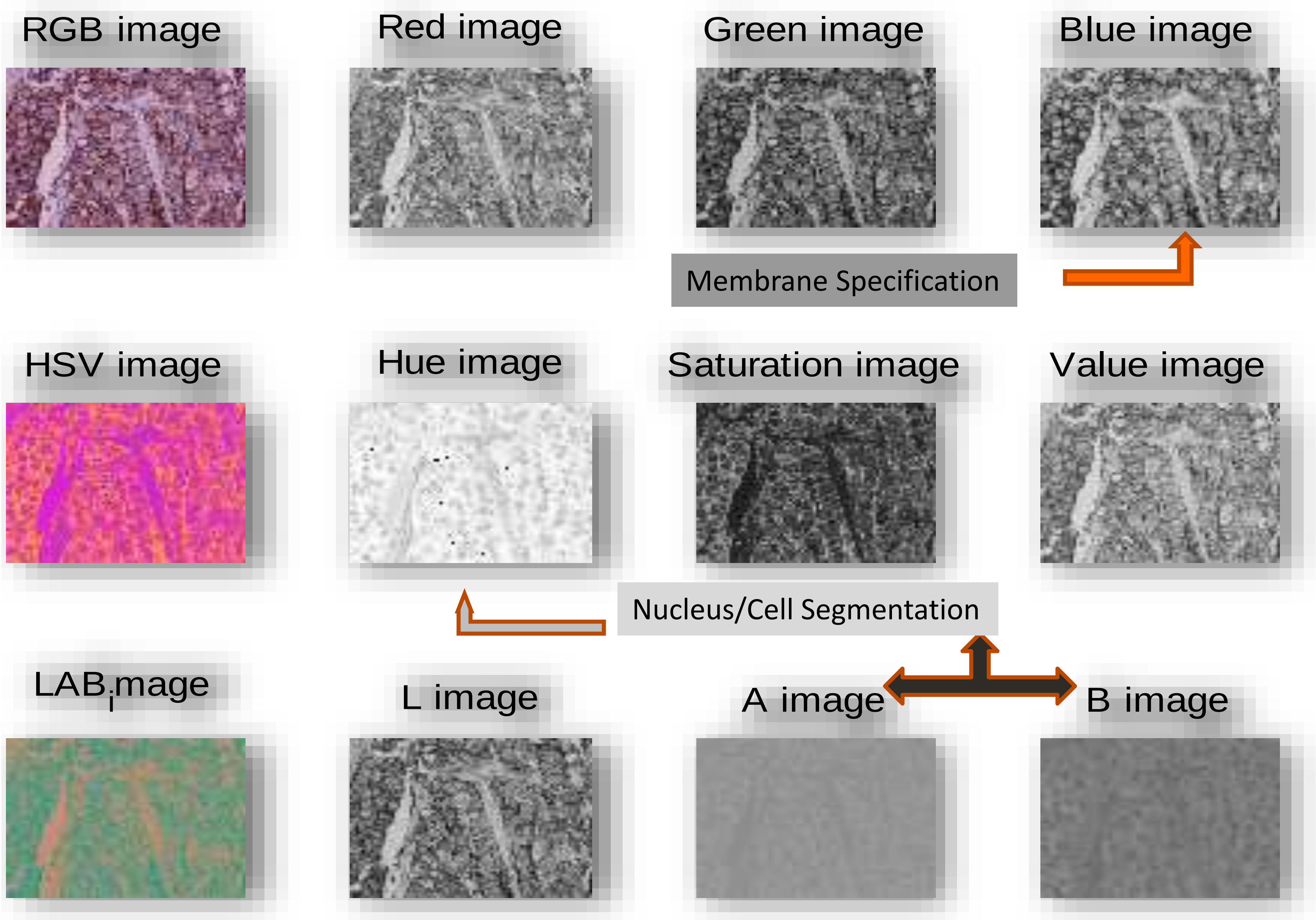

RGB image
Red image
Green image
Blue image
Membrane Specification
HSV image
Hue image
Saturation image
Value image
Nucleus/Cell Segmentation
LAB image
L image
A image
B image

Image Files Format

- There are different formats with advantages or disadvantages depending on the application.
- Matlab easily loads most of them (e.g. GIF, TIFF, PNG), but in any case it is good to know some basic things.
- Of the many available formats, some are designed for specific needs (eg transmission of images on the network), while others are designed e.g. for specific operating systems.

Image Files Format

- 2D image formats
  – TIFF, JPEG, JPEG2000, GIF, BMP, PNG, raw, DICOM
- 3D image formats
  – TIFF (stacked), DICOM (slices), raw (specific formats like vgi)
- 4D image formats – (specific formats like vgi)
- Raw format: write all pixel information into one string
- BMP: bitmap – header: BMP, size x, size y – data

Image Data Format, Storage, and Communication (DICOM)

- Digital Image Generation
- Digital Transfer/Archiving
- Post-Processing
- Cross-Vendor Compatibility
- Communication over Networks

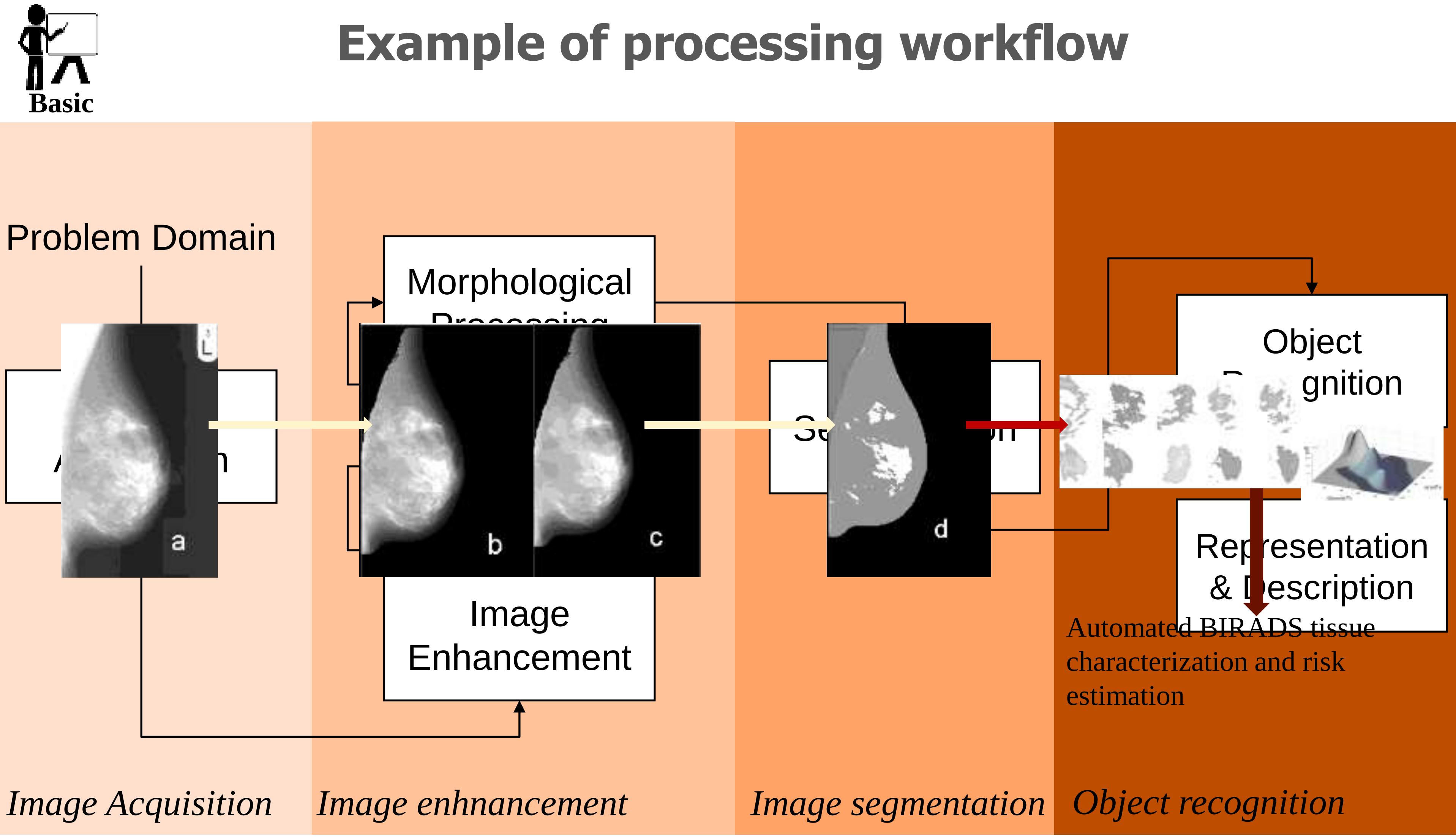
Basic

Image processing and computer vision

- A wide categorization can be made in low, middle and high level processing:

Low Level Processing	Middle Level Processing	High Level Processing
Input: Image **Output:** Image	**Input:** Image **Output:** Features	**Input:** Features **Output:** Understanding, AI
Examples: Noise removal, contrast enhancement	**Examples :** Image segmentations, Object recognition	**Examples : Scene recognition, automated diagnosis**

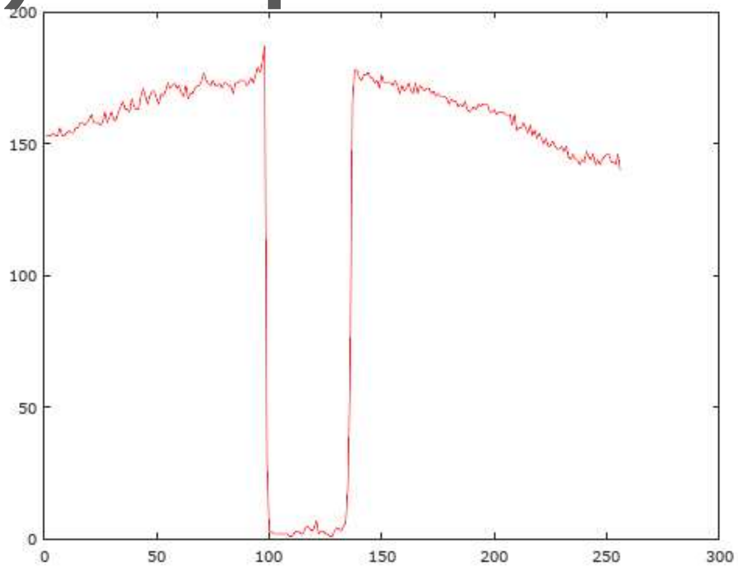
Example of processing workflow



K. Marias, C.P. Behrenbruch, R.P. Highnam, S. Parbhoo, A. Seifalian and Michael Brady: "A mammographic image analysis method to detect and measure changes in breast density". European Journal of Radiology, Volume 52, Issue 3, December 2004, Pages 276-282.

Basic Image Processing Concepts

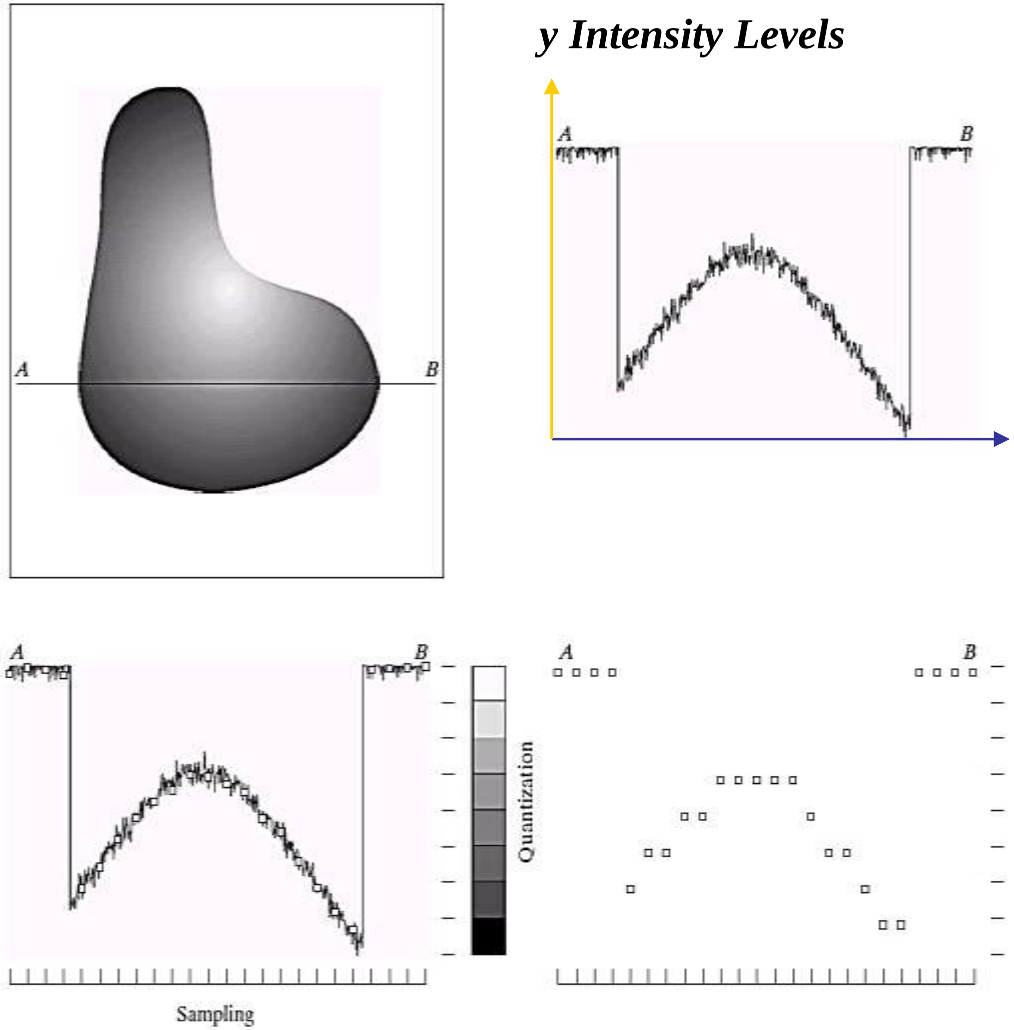
Gray-scale Images (8-bit) example code




```
figure(2);plot(1:256,line50,'r-')
I=imread('cameraman.gif');
figure(1), imshow(I)
line50=I(50,1:255);
line50=I(50,:);
size(line50)
```

Plotting a line of the image as a 1D signal; notice the drop in intensity as we go from the background to the dark hair of cameraman and then the rise in intensity again

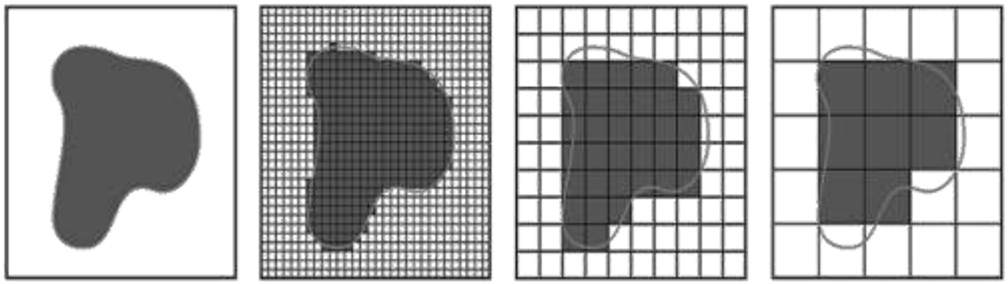
Sampling and quantization



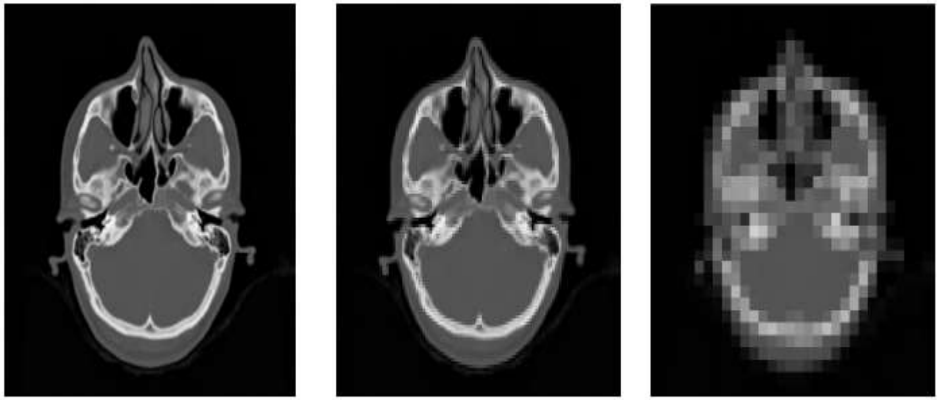
- Digitizing the coordinate values is called *sampling*.
- Digitizing the amplitude values is called *quantization*.
- Sampling is constrained by the Nyquist theorem.

“Digital Image Processing”, Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002

Spatial Analysis of Image- Resolution



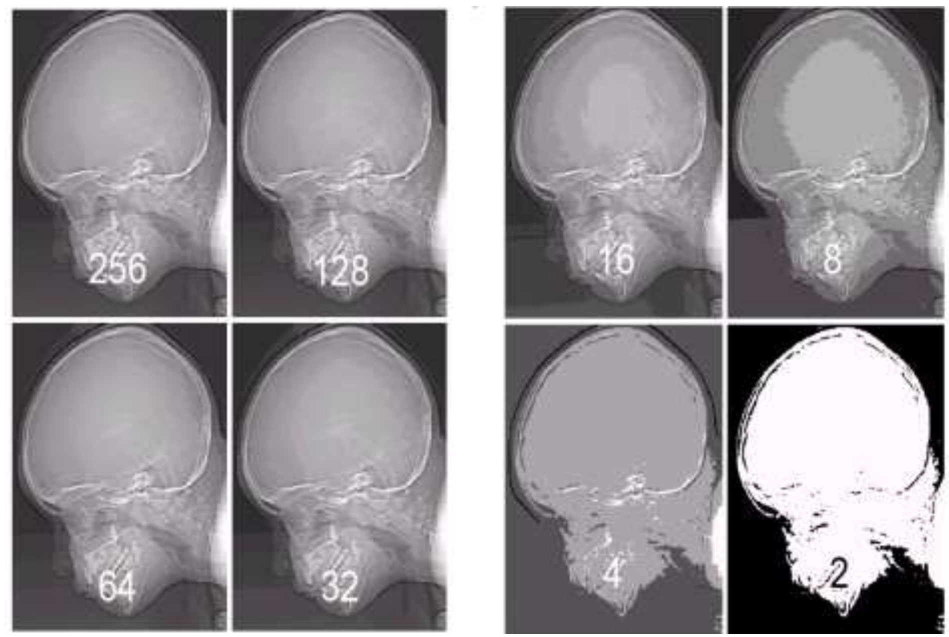
Sampling Rate:
Nyquist Frequency,
Shannon Theorem



512 × 512 128 × 128 32 × 32

Quantization of Image Levels – Dynamic Range

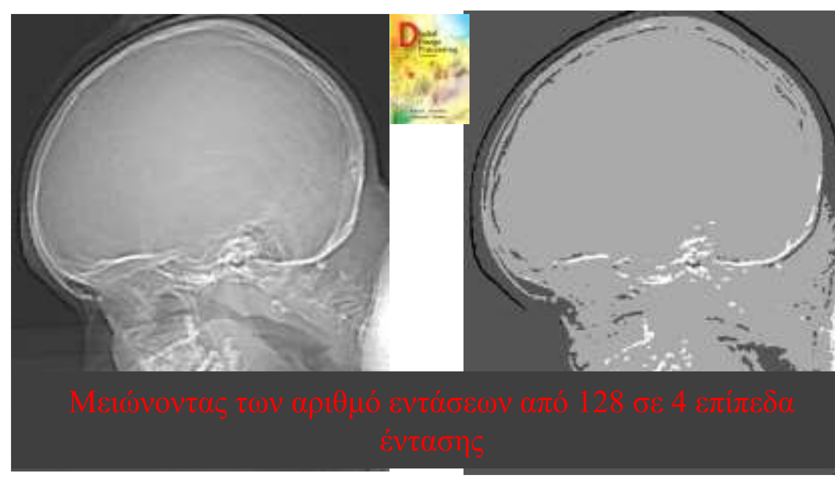
Number of Bits	Number of Intensity Levels	Examples
1	2	0, 1
2	4	00, 01, 10, 11
4	16	0000, 0101, 1111
8	256	00110011, 01010101
16	65,536	1010101010101010



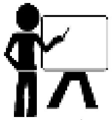
Basic

Sampling and quantization

- The number, b, of bits required to store an image with 2^k intensity levels:
- b = M * N * k (rows * columns * k)
- The intensities ($L = 2^k$) are evenly distributed in the interval [0, L - 1]



“Digital Image Processing”, Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002



Basic

Image size

- Image N×M, Gray Levels $G=2^{\kappa}$

$b = N \times M \times \kappa$

Image Type	N	M	k	bits	bytes
Binary	100	100	1	10.000	1.250
Gray	100	100	8	80.000	10.000
Color	100	100	24	240.000	30.000

- k: pixel color information range

Image size

Image files are generally large. For a 512x512 pixels binary image we need:

512x512pixels x 1bit / pixel = 262144bit

Dividing by 8 equals 32768bytes or 32,768Kb

For a Gray Scale image we need:

512x512pixels x 1byte / pixel = 262144bytes or 262,144Kb

For an RGB color image each pixel needs 3 bytes (RGB = 3 tables from 0-256) so we need:

512x512pixels x 3byte / pixel = 786432bytes or 786,432Kb

Down Sampling

- Whenever a low pass filter is applied, it may be possible to discard alternating pixels without much loss of information (down-sampling, or decimation)
- If down-sampling is desired, it may be best to do some low pass filter to avoid aliasing–Reasonable LPF to use: 1/16[1,4,6,4,1]

Burt Filter

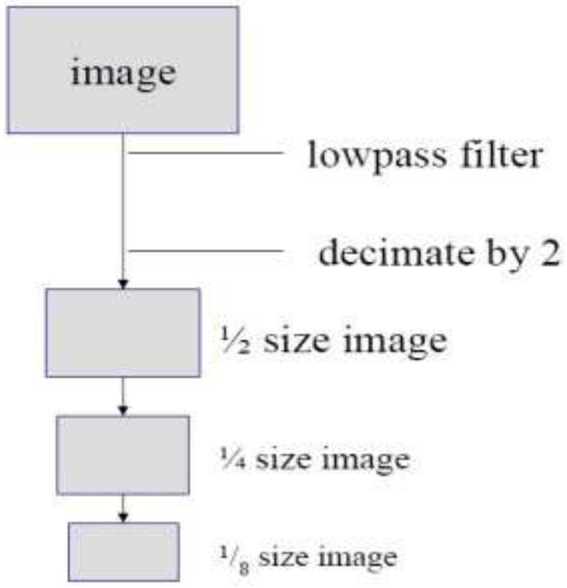
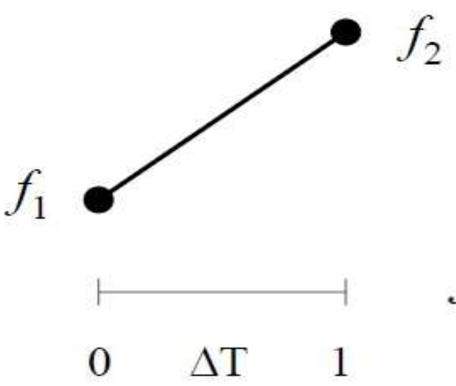


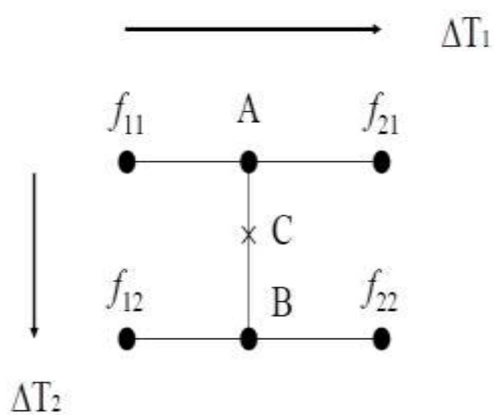
Image interpolation



Linear Interpolator (1D)

$$f(\Delta T) = f_1 + \Delta T\,(f_2 - f_1)$$

- 2D linear interpolator : use 1D linear interpolation for A and B, use 1D linear interpolation among A and B to get C



Bilinear Interpolator

$$f(\Delta T_1, \Delta T_2) = f_{11} + \Delta T_1\,(f_{21} - f_{11}) + \Delta T_2\,(f_{12} - f_{11}) + \Delta T_1\,\Delta T_2\,(f_{22} - f_{21} - f_{12} + f_{11})$$

Interpolation – comparison

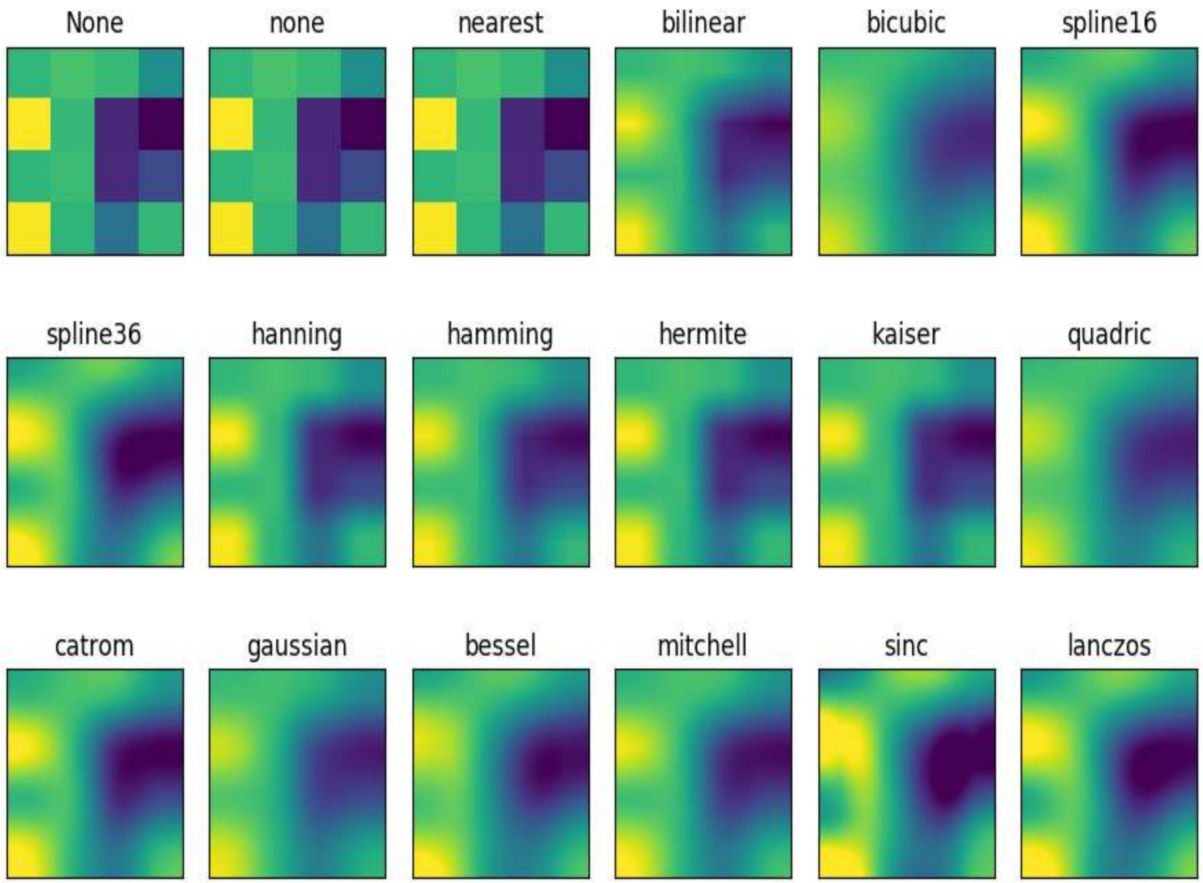


Image Artifacts

- **Noise**
  - MRI (ex: Gaussian,)
  - PET / SPECT (ex: Poisson, mixed Poisson-Gaussian)
  - CT (ex: Gaussian) DTI,
  - DWI, ...
- **Intensity inhomogeneity**
  - MRI
- **Intensity Non-Standardness**
  - MRI
- **Partial Volume**
  - MRI, PET,…

Image Histogram

- The histogram h of Image I depicts the frequency of each intensity of gray g in the image
- The value of the histogram h (g) for intensity g is equal to the number of pixels in image I that have intensity g.
- For an 8-bit image the histogram h has g = 1..256 frequency values from 0-255.
- $h_I(g)$ = *number of pixels in Image I with value g-1*

In Matlab an array of length *n* has indices from 1 to *n*. In many computer languages, *e.g.* "C" or "C++" an n-element array is indexed from 0 to *n*-1.

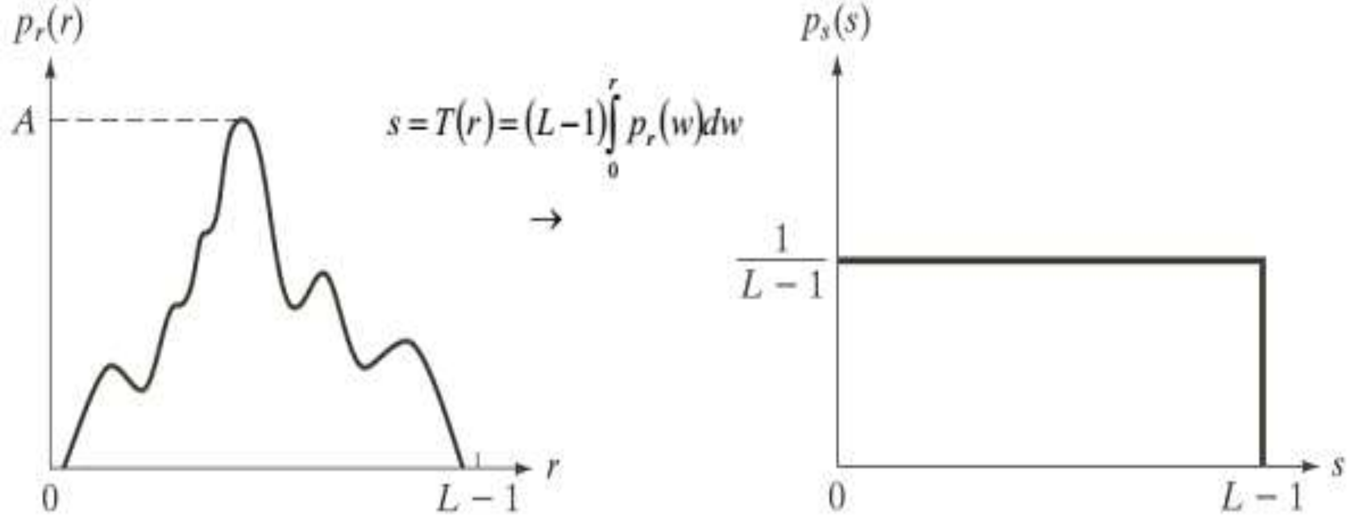
Matlab

Image Histogram Code

Basic

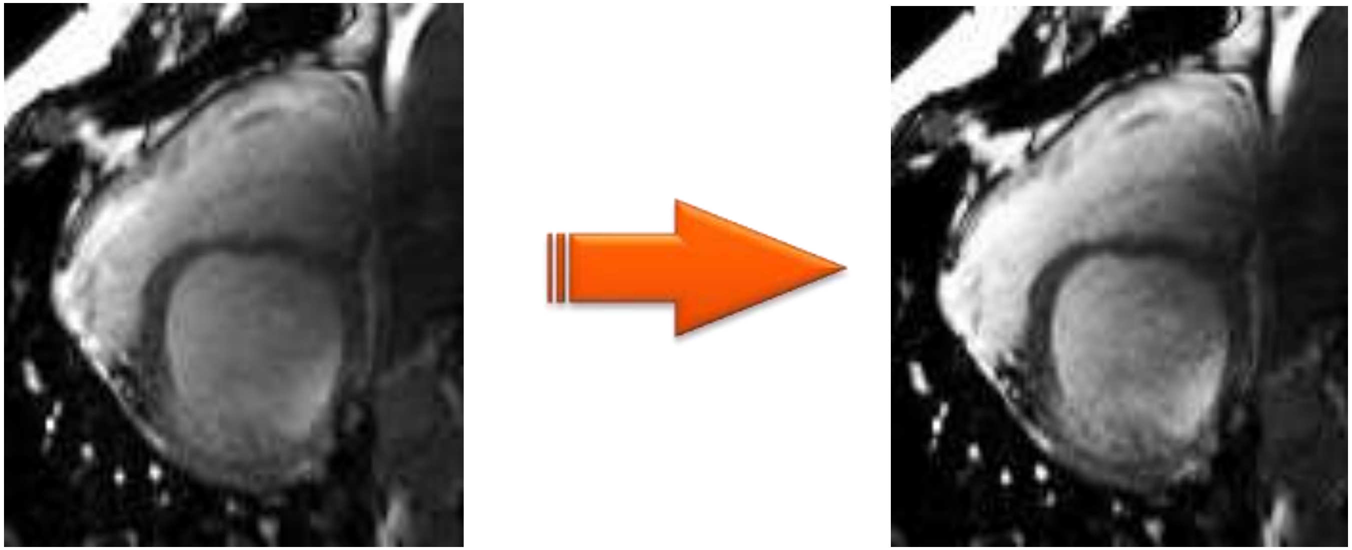
```
%RGB Image Histogram
function h=histogram(I)
[R C B]=size(I);
% allocate the histogram
h=zeros(256,1,B);
% range through the intensity values
for g=0:255
   h(g+1,1,:) = sum(sum(I==g)); % accumulate
end
return;
```

Histogram Equalization

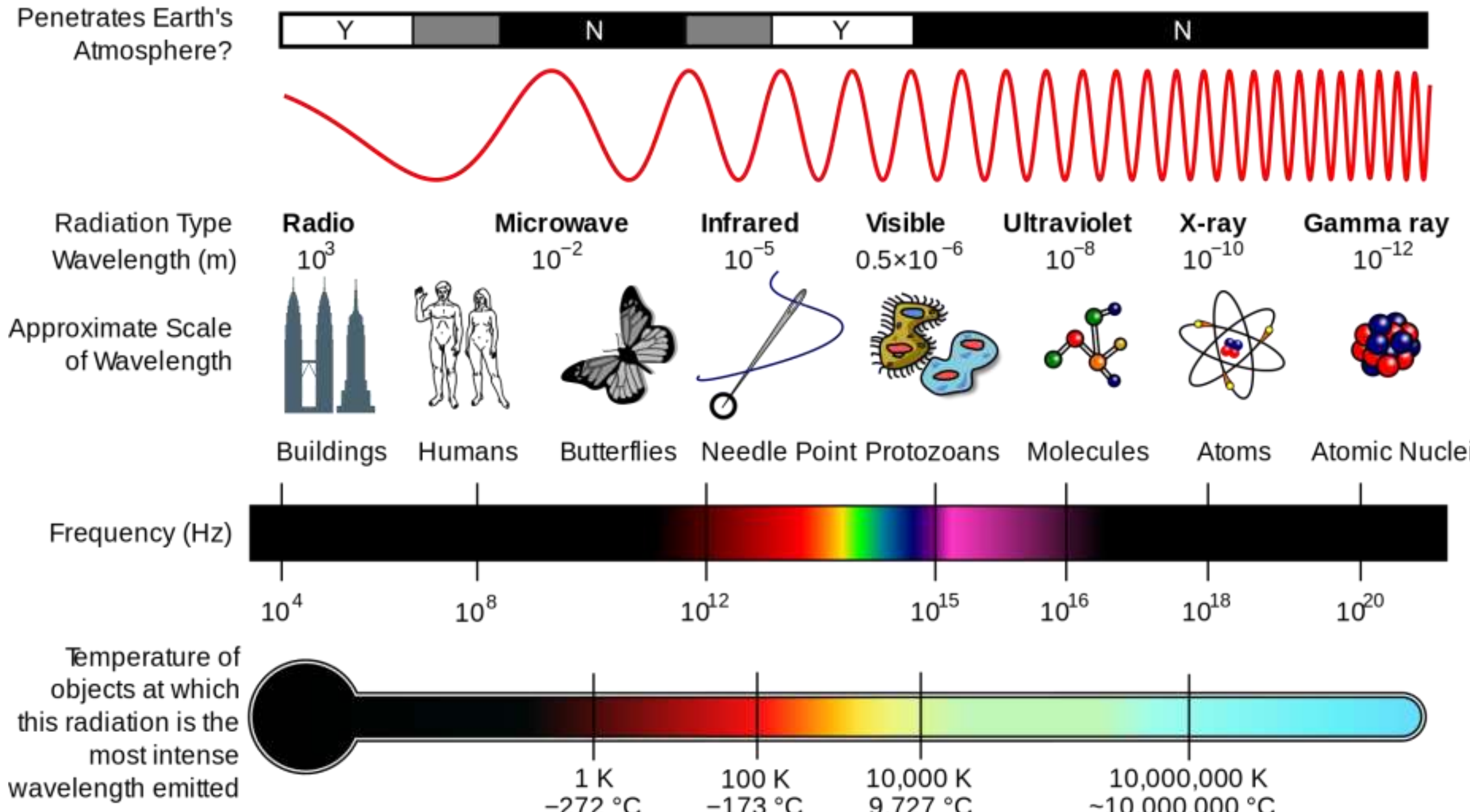


$$S_k = T(r_k) = (L-1)\sum_{i=0}^{k} p_r(r_i) = \frac{L-1}{M \cdot N}\sum_{i=0}^{k} n_i$$

Histogram Equalization: Point Operation - Enhancement



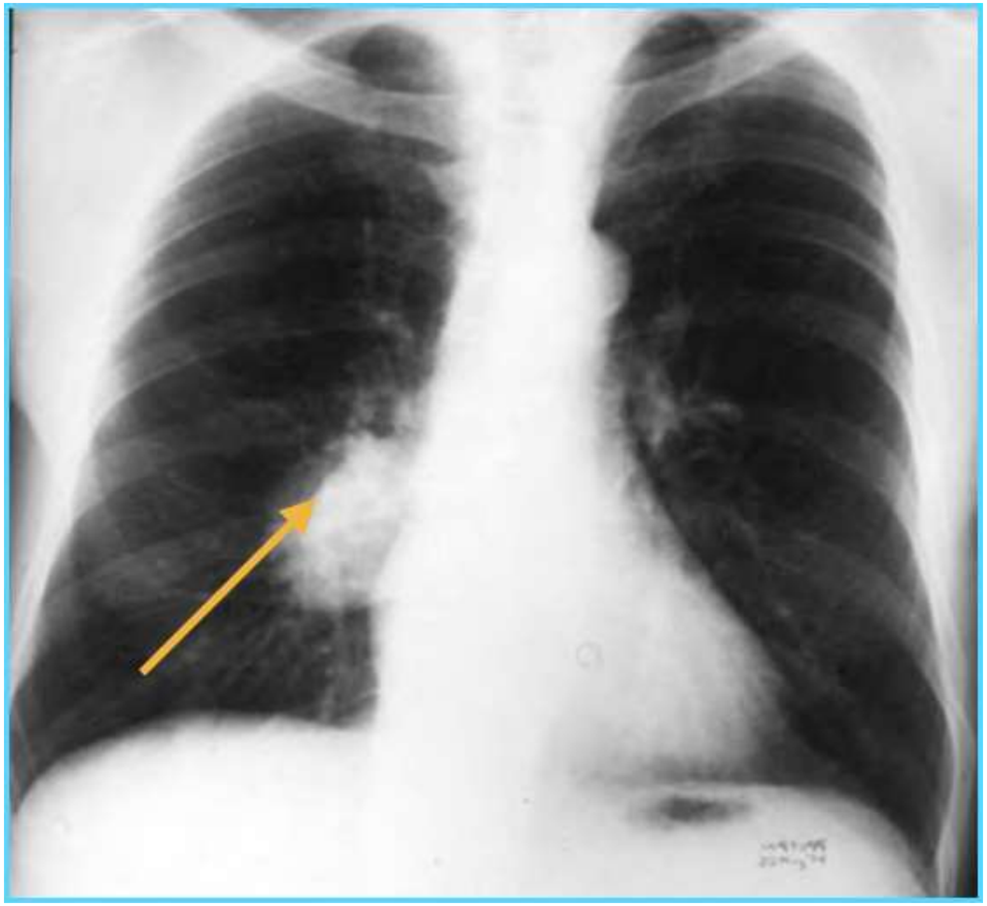
Brief Introduction to Imaging Modalities



A diagram of the electromagnetic spectrum, showing various properties across the range of frequencies and wavelengths

[EM Spectrum Properties edit - Electromagnetic spectrum - Wikipedia]

How Radiologists Search Abnormal Patterns in Chest X-Rays?



Computer algorithms can solve/simplify these problems for improved healthcare

Radiologists often report the following

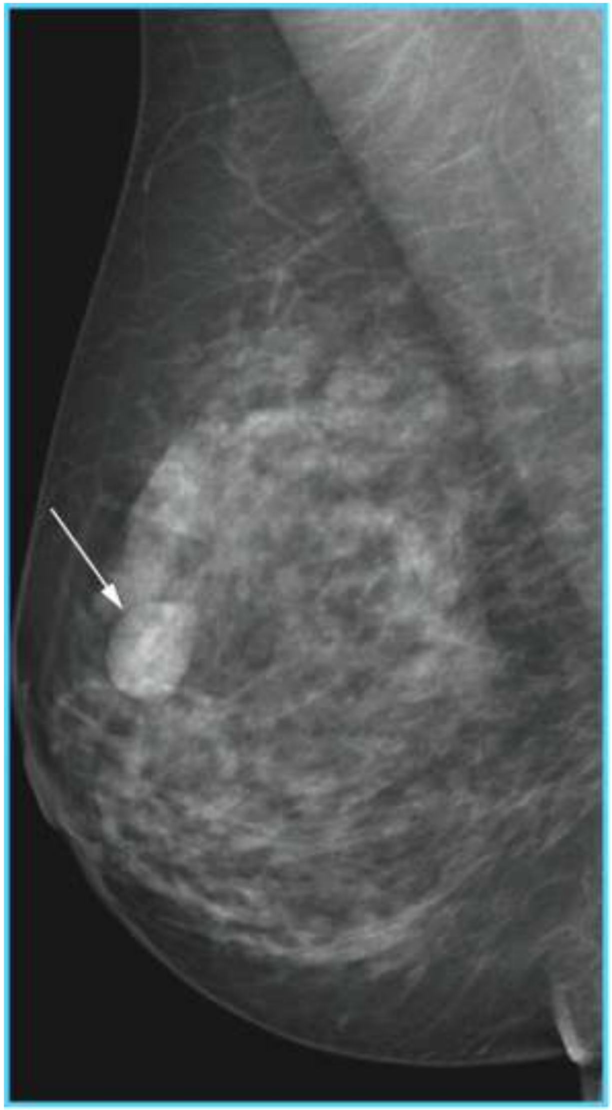
- Size, dimension, volume
- Pattern description,
- Location,
- Interaction with Nearby structures,
- Intensity distribution
- Shape
- …

Difficulties

- Noise
- vessels can be seen as small nodules
- radiologists may miss the pattern
- patterns may not be diagnostic
- CT often required for better diagnosis
- size estimation is done manually in 2D
- Shadowing
- total lung capacity computation

Another Example for X-ray Imaging

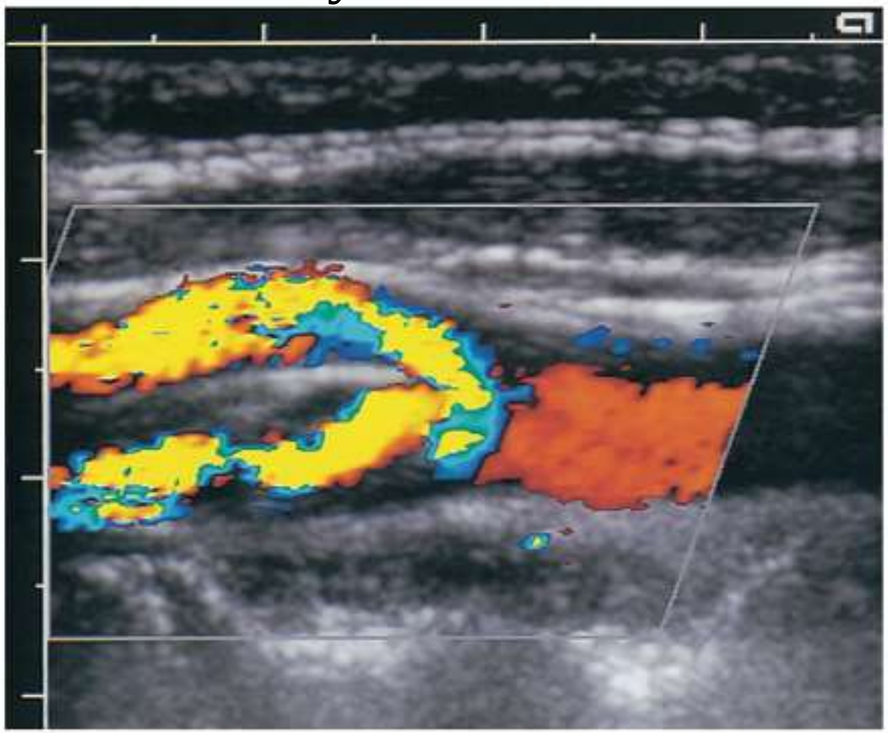
Benign

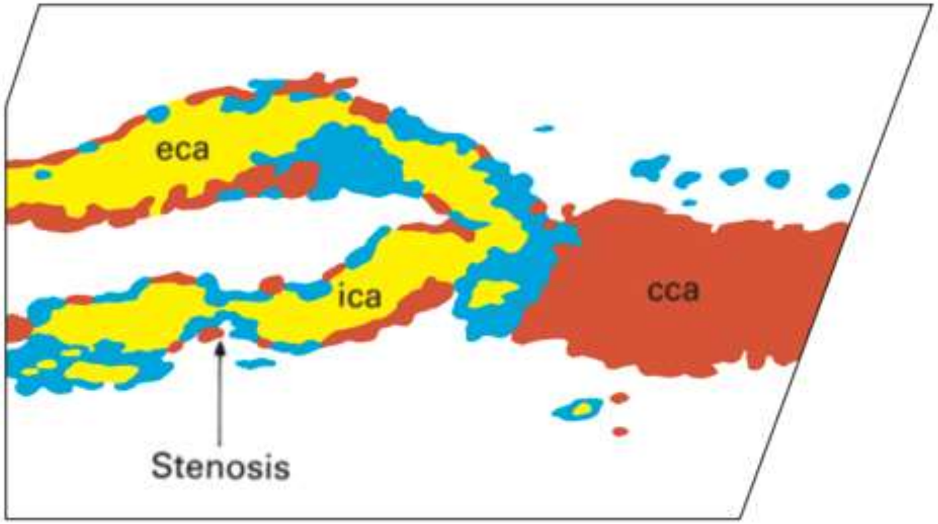


Malignant

Clinical Use of US Imaging

Renal Artery Blood Flow

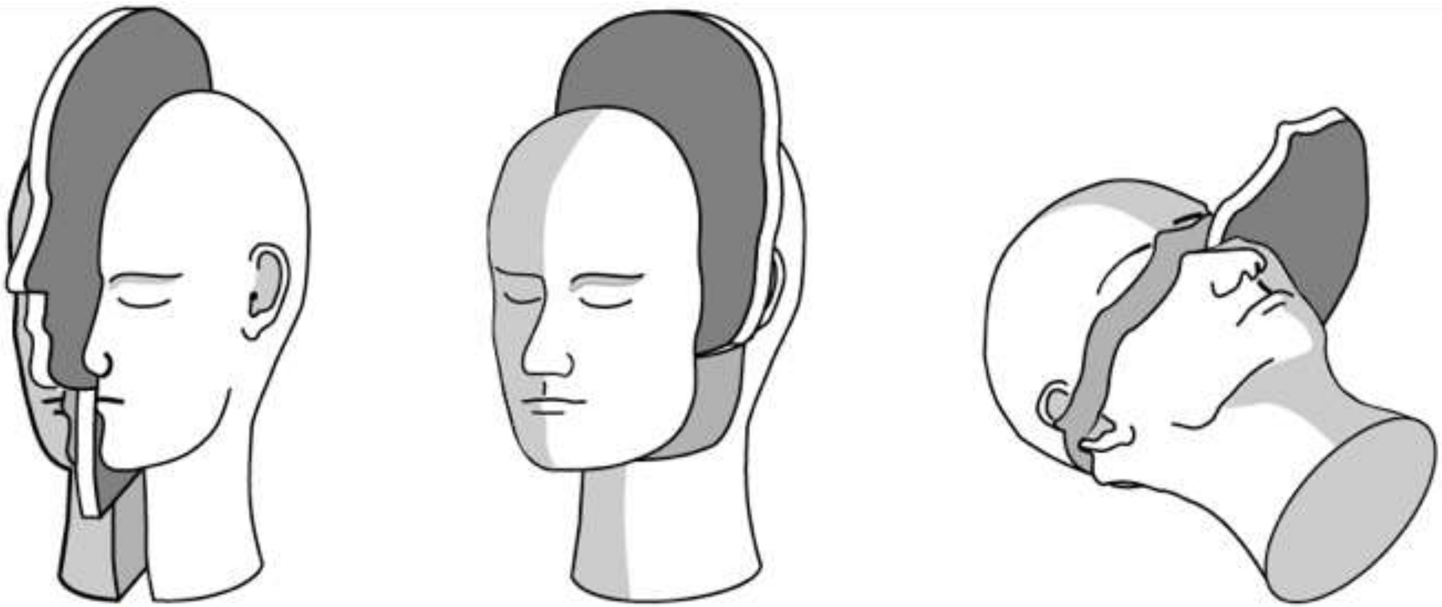




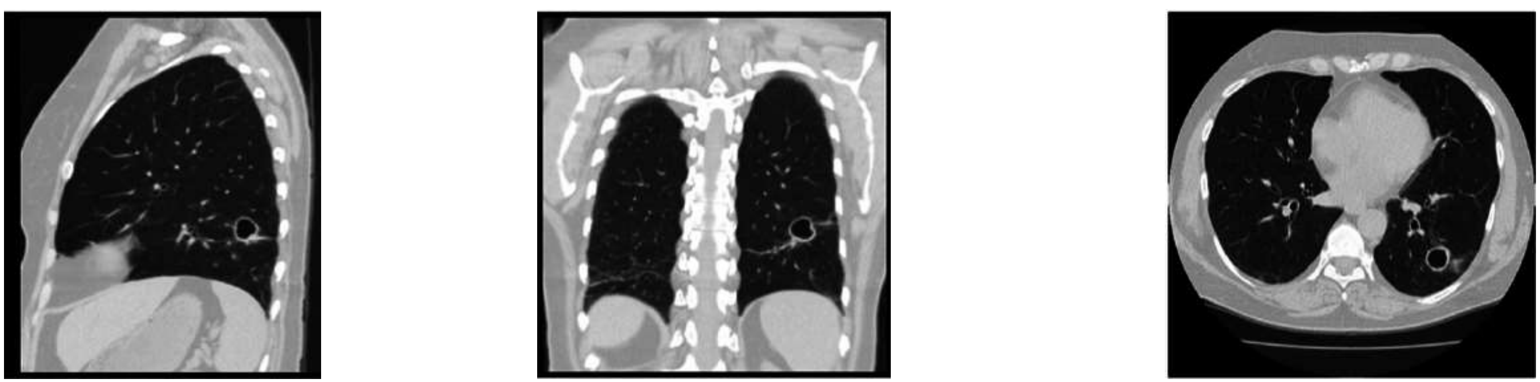
manual measurements?
<u>can computer help calculating all blood flow and identify automatically the abnormal regions? (See Next Lecture, afternoon)</u>

stenosis is seen
eca: external carotid artery
cca: common carotid artery
ica: internal carotid artery

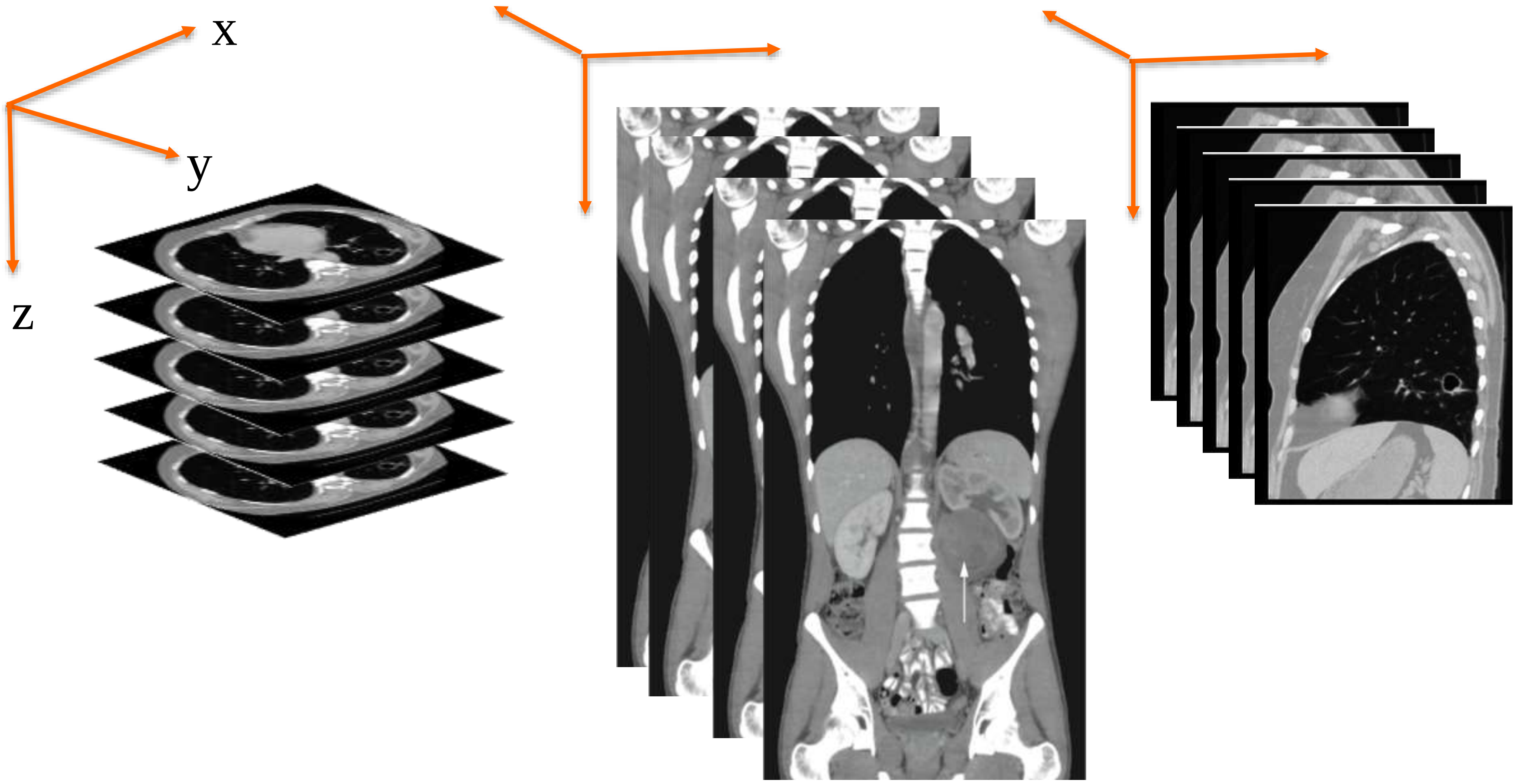
Remark: 3D View Terminology



A Sagittal **B** Coronal **C** Axial



3D Images



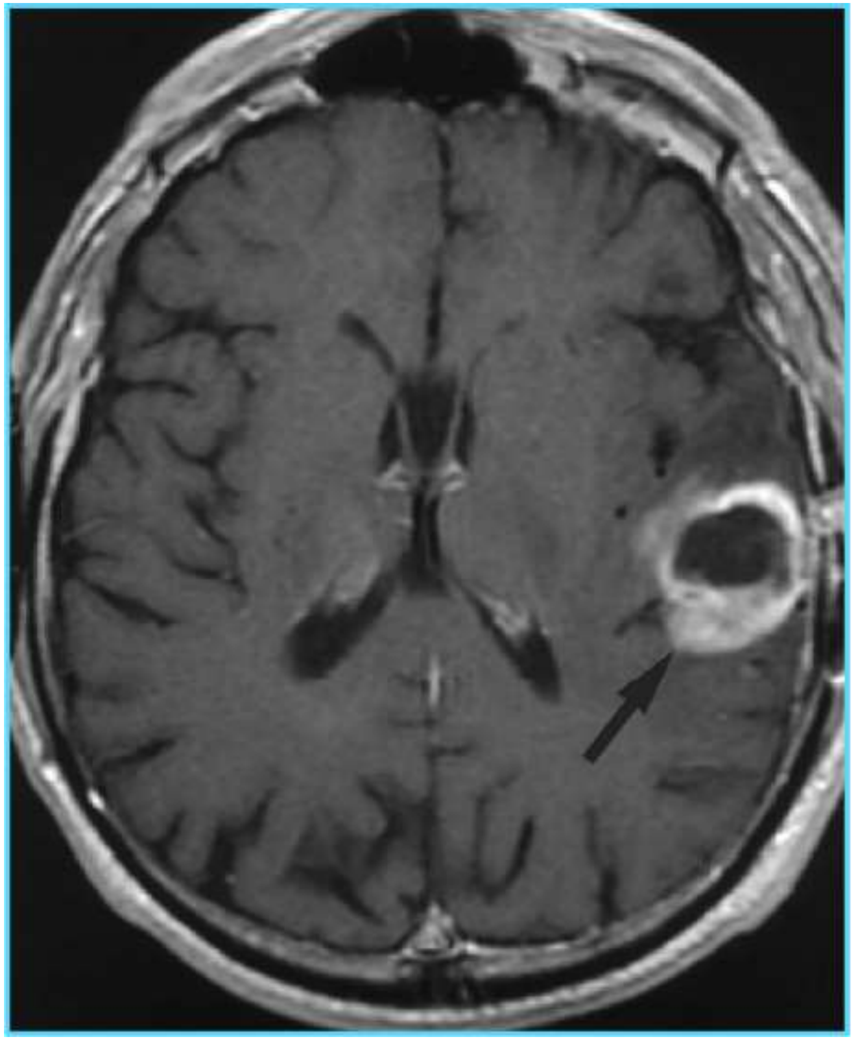
I: Image

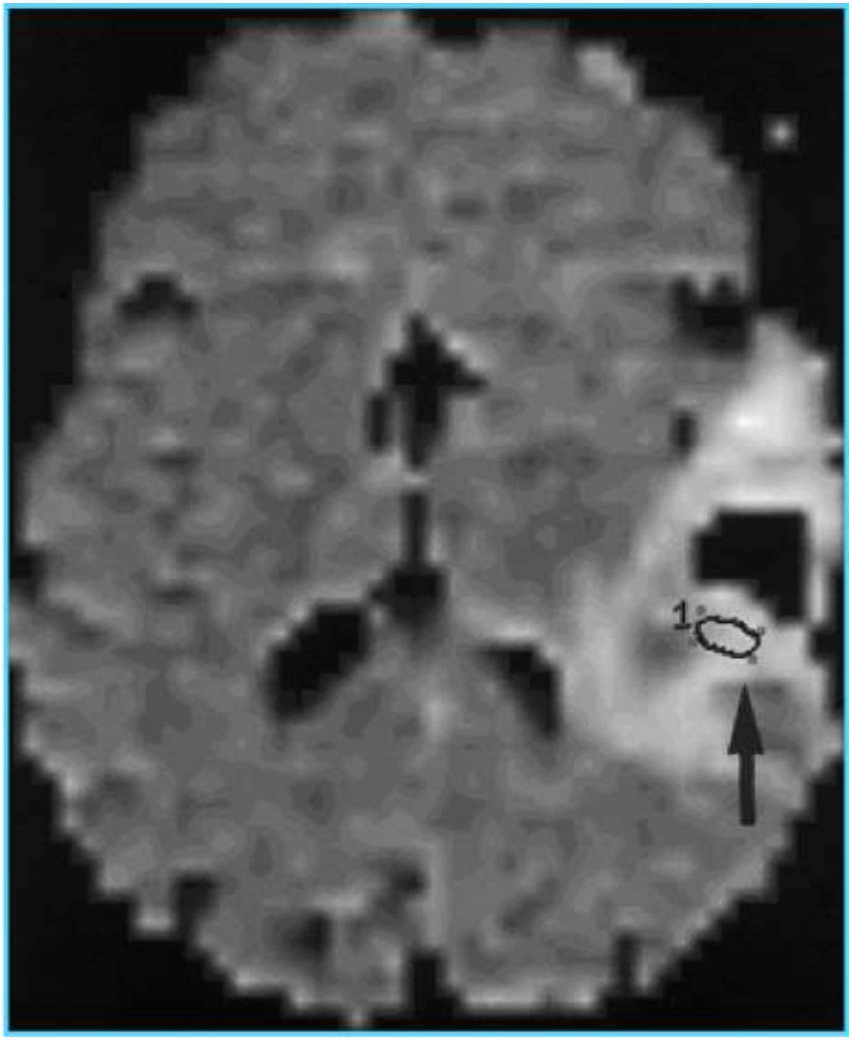
I(x,y,z) denotes intensity value at pixel location x,y,z

Clinical Use of CT Imaging

- Standard imaging technique in many organs, particularly gold standard for lung imaging
- Fast
- Radiation exposure
- Often used in surgery rooms
- Show anatomy and pathology
- Intensity values are (more-or-less) fixed, read as HU (Hounsfield Unit)

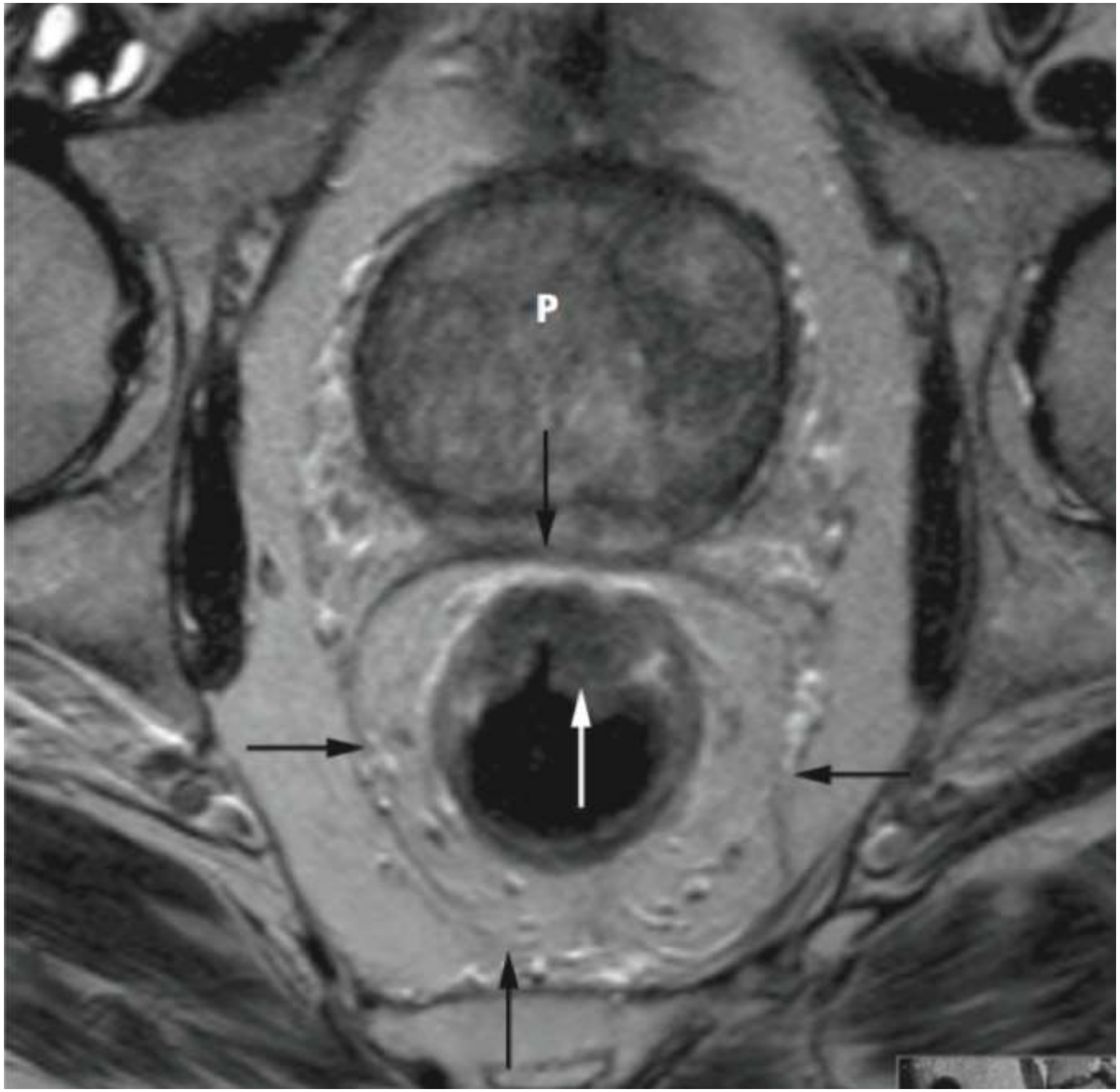
Diffusion Weighted Imaging (DWI)





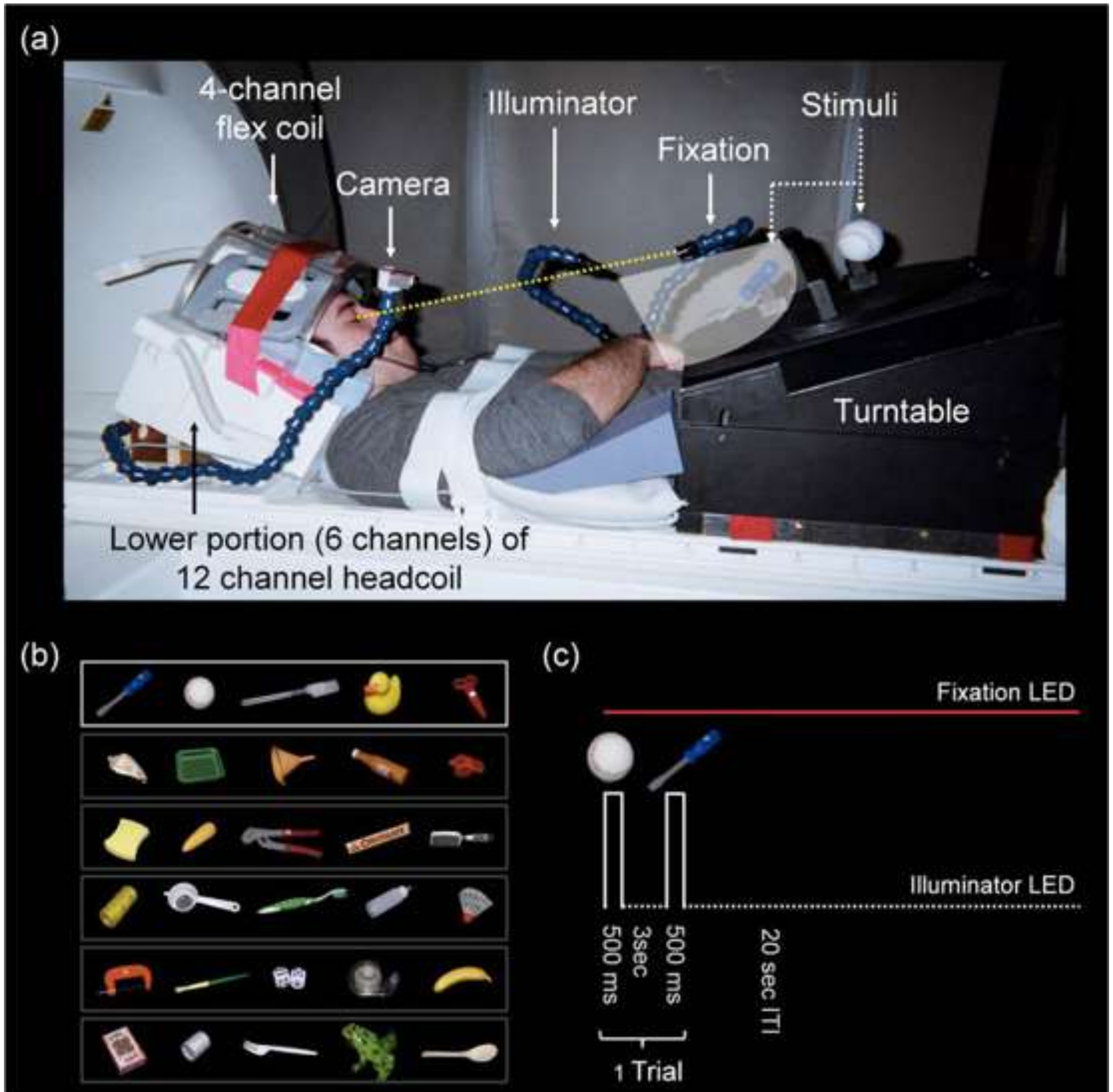
Glioblastoma Tumor

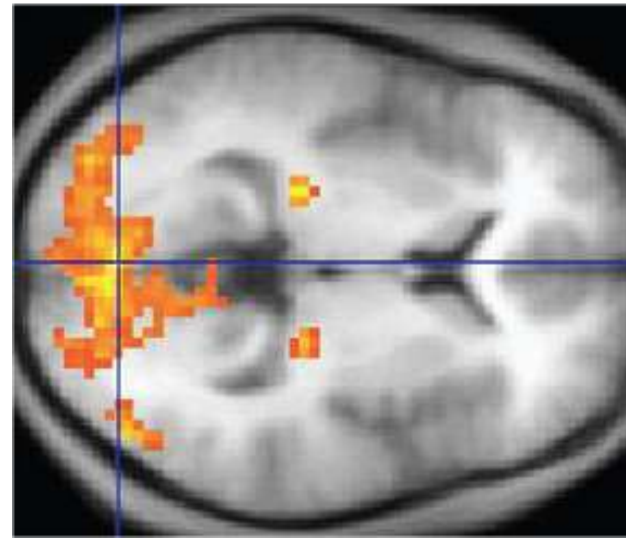
Clinical Use: Example

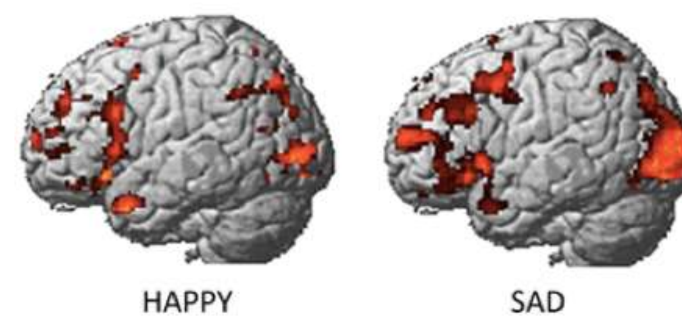


rectal tumor

fMRI Settings

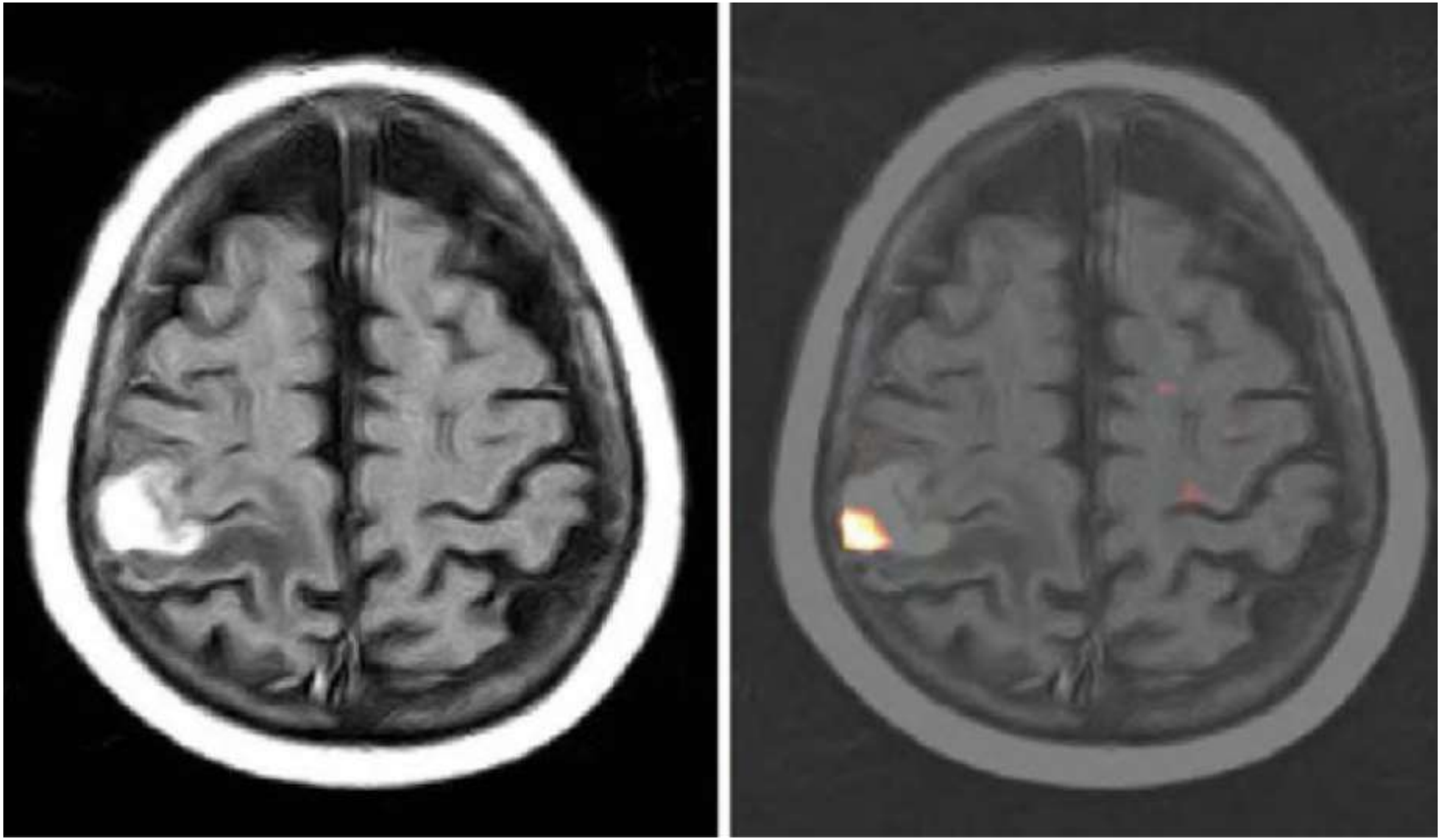






Active Regions

Clinical Use of PET: Example

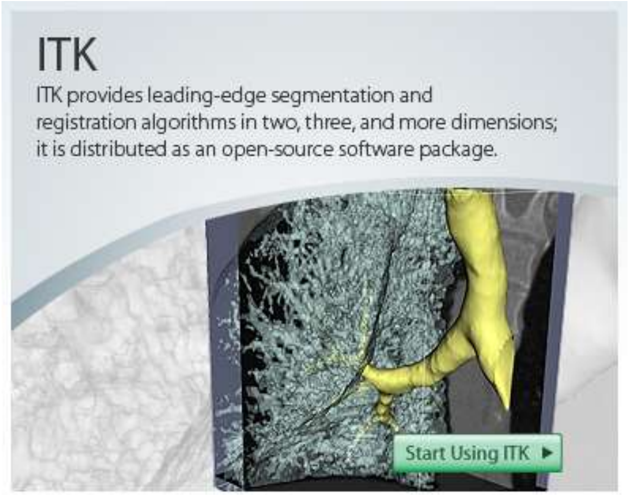


Shallow Comparison of Imaging Methods

	Chest	Abdomen	Head/Neck	Cardiovascular	Skeletal/muscular
CT	gold standard	Need contrast for excellency, widely used	Good for trauma	Gold standard	Gold standard
US	no use except heart or Perfusion	Problems with gas	Poor	Poor	Elastography
Nuclear	Extensive use in heart and therapy in lung	CT or MRI is merged	PET	Perfusion	bone marrow
MRI	growing cardiac applications	Increased role of MRI	Gold standard	Will replace ct in near future	Excellent

Software to Use, and Coding Standards

- Free software (with GUI) you can use for analysis of medical images
  - ITKSnap, Slicer, MITK, ImageJ/Fiji, MIPAV, Osirix, FSL, SPL, Mango, and many others can be found in IdoImaging.com and NITRC website.
  - Preference: Slicer, ITKSnap, ImageJ/Fiji
- Coding (self): ITK/VTK libraries will be used
  - C/C++ and Python can be used to call libraries
  - SimpleITK with Python is simpler
  - Octave is a good environment for starters
- Image Format
  - DICOM
  - Analyze (.img/hdr)
  - Nifti
  - …



Medical Image Preprocessing & Enhancement: Spatial Domain

Image Processing Areas

Application of signal processing to medical imaging:

- **Linear** signal processing
  - Image reconstruction (tomography, MRI)
  - Image enhancement
  - Noise and Artifact Reduction
  - Edge Detection

- **Non-linear** signal processing
  - Non-linear, adaptive filters
    - Tube enhancing filters
  - Quantization and Down Sampling
  - Segmentation and beyond

Image Filtering

<u>**Purpose**</u>**:** To suppress unwanted (non-object) info.
To enhance wanted (object) information.

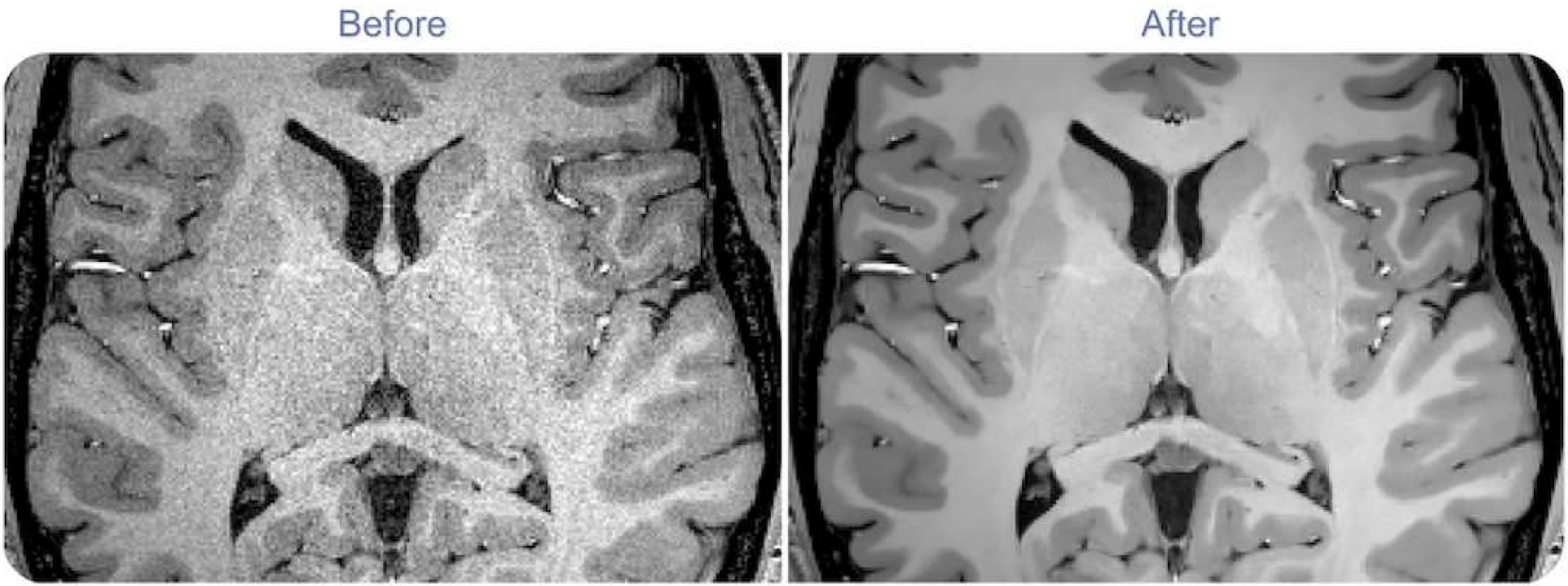
Enhancive: For enhancing edges, regions.
For intensity scale standardization.
For correcting background variation.

Suppressive: Mainly for suppressing random noise.

Inappropriate use of Enhancement Methods

- Enhancement methods themselves may increase noise while improving contrast!
- They may eliminate small details and edge sharpness while removing noise
- They may produce artifacts in general.

Smoothing MRI



Credit to: © Dr Pierrick Coupé

P. Coupé et al. An Optimized Blockwise NonLocal Means Denoising Filter for 3-D Magnetic Resonance Images. IEEE TMI 2008

- **Spatial resolution**
  - determines the smallest structure that can be represented in a digital image.
- **Contrast resolution**
  - Local change in brightness and defined as the ratio between average brightness of an object and background
  - is an indirect measure of the perceptibility of structures. The number of intensity levels has an influence on the likelihood with which two neighboring structures with similar but not equal appearance will be represented by different intensities.

$$Contrast_{global} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Another method for measuring contrast

- **RMS (root mean square) contrast:**

$$Contrast_{RMS} = \sqrt{\frac{1}{MN-1}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\left(I(i,j) - avg(I)\right)^2}$$

The measure takes all pixels into account instead of just the pixels with maximum and minimum intensity values (M and N are size of the image, avg means mean operation over the entire image I).

Image Artifacts

- **Noise**
  - MRI (ex: Gaussian,)
  - PET / SPECT (ex: Poisson, mixed Poisson-Gaussian)
  - CT (ex: Gaussian)
  - DTI, DWI, ...
- **Intensity inhomogeneity**
  - MRI
- **Intensity Non-Standardness**
  - MRI
- **Partial Volume**
  - MRI, PET,…

Image Artifacts

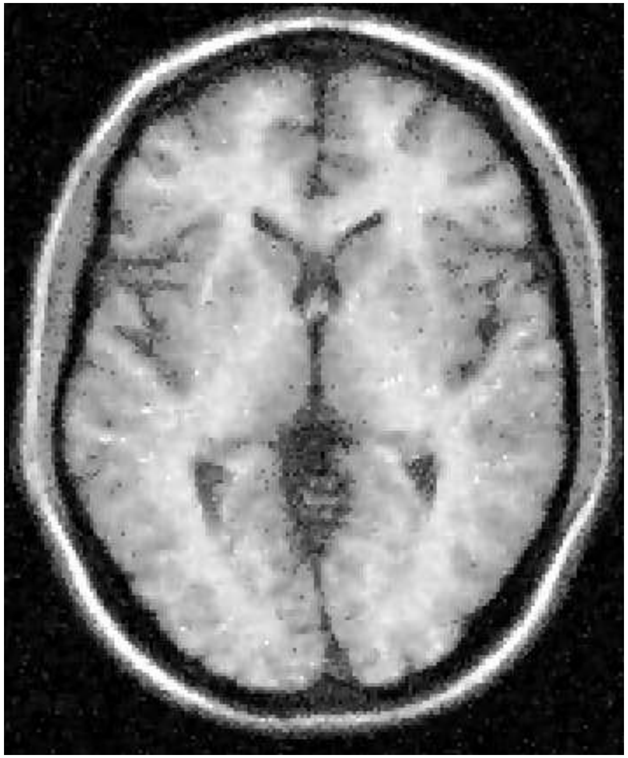
- **Noise**
  - MRI (ex: Gaussian,)
  - PET / SPECT (ex: Poisson, mixed Poisson-Gaussian)
  - CT (ex: Gaussian)
  - DTI, DWI, ...
- **Intensity inhomogeneity**
  - MRI
- **Intensity Non-Standardness**
  - MRI
- **Partial Volume**
  - MRI, PET,…

Noise

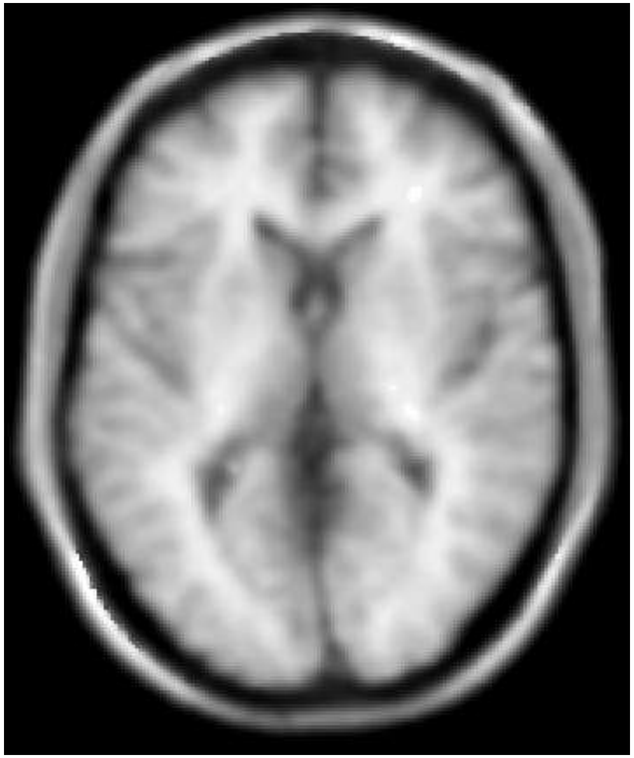
Noise is corrupting the image information, and it is unwanted.

- **Signal independent noise**
  - $g = f + n$
    - *Gaussian*
- **Signal dependent noise**
  - $g = f*n$
    - *Poisson*
- *Often, medical images are considered to have Gaussian noise, however PET/SPECT images have mixed Poisson/Gaussian, and MRI have Rician type noise.*

Noise Suppression



Higher noise, higher contrast



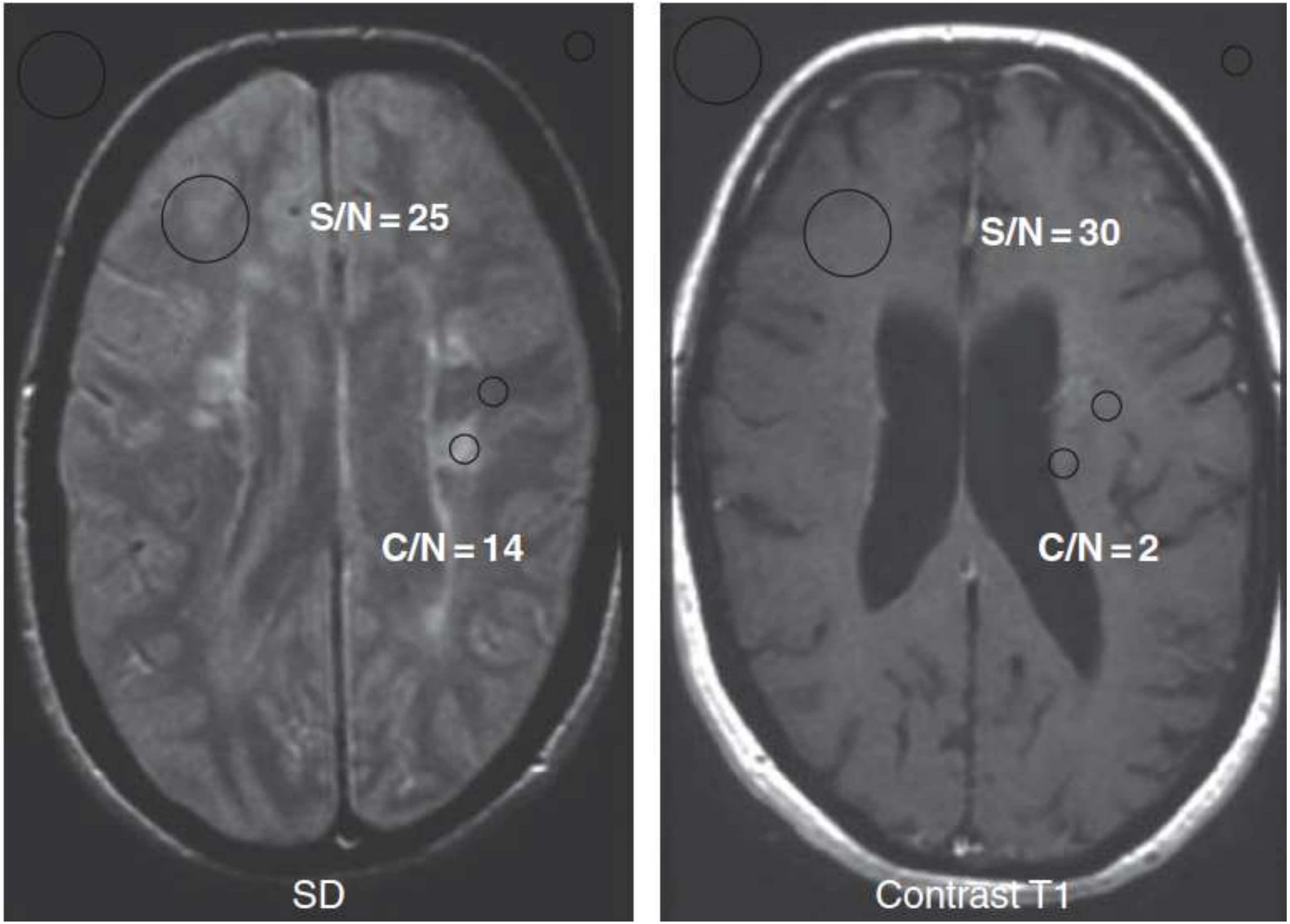
Lower noise, lower contrast

For best results, we need lower noise and higher contrast.

How to measure for evaluating noise removal algorithms?

- SNR (signal-to-noise ratio): basic measure of image quality

SNR in an image is simply determined by averaging signal intensity within similar-sized regions of interest (ROIs) inside and outside the sample (background).



SNR (S/N) (of normal brain) and CNR (C/N) of multiple sclerosis plaques to normal brain on spin-density and T_1 magnetic resonance images.

Basic Filters

Basic

Spatial domain filtering

- The term ‘spatial’ field refers to the image itself, and image filtering methods in this category are based on the direct manipulation of pixels in an image.
- Two main categories of processing are intensity transformations and spatial filtering.
- Intensity transformations operate on individual pixels of an image, mainly for the purpose of adjusting the contrast and thresholding of the image.
- Spatial filtering involves performing tasks such as sharpening the image, working in a neighborhood of each pixel in the image.

Basic

Spatial domain filtering

Spatial filtering operations can be described by the general relation:

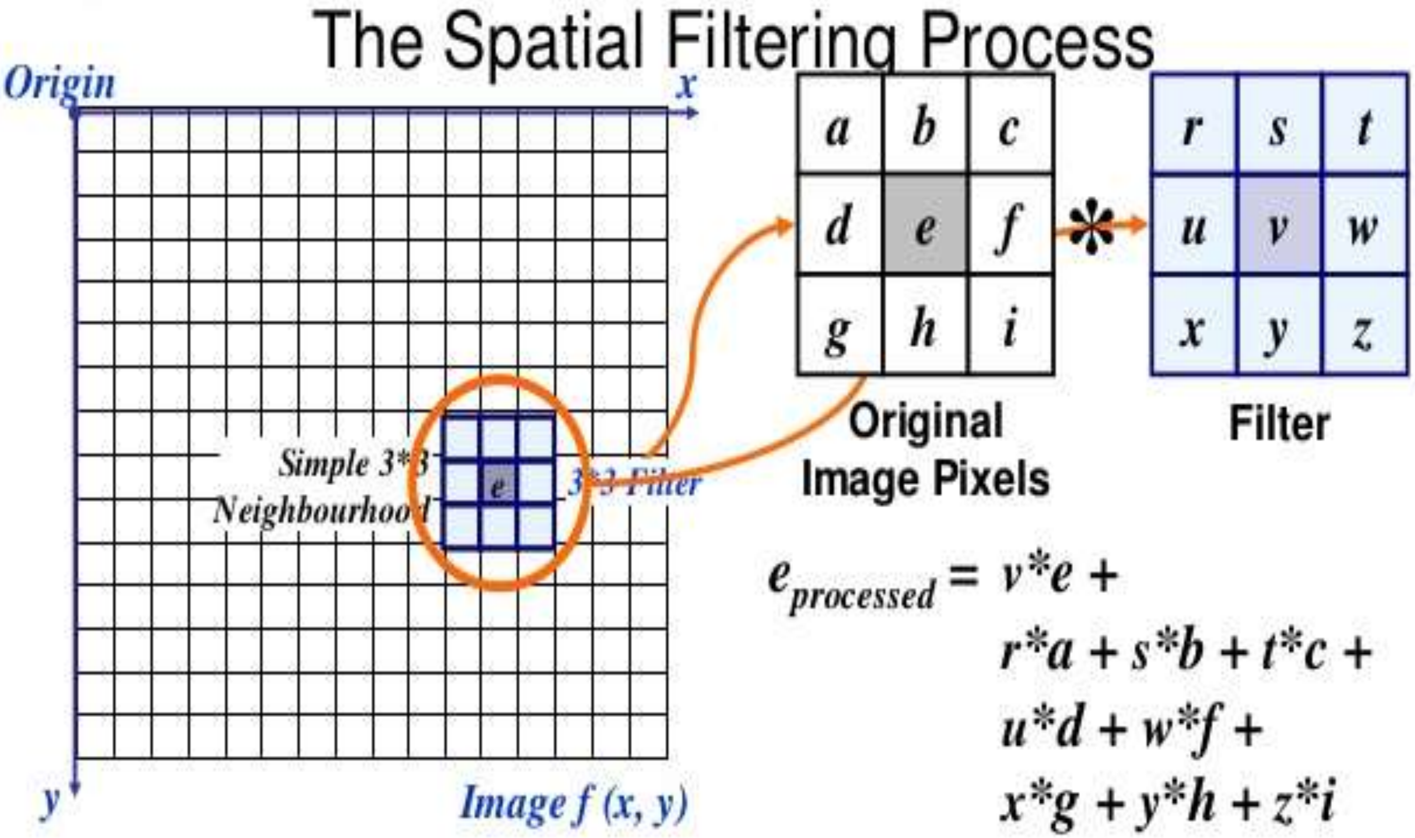
$$g(x,y)=T\,[f(x,y)]$$

The input image is f (x, y) while the output image is g (x, y) which is obtained through the operator T which is defined in the neighborhood of point (x, y) in one or more images (π .x. average for noise removal).

Convolution

- Convolution is a filtering operation, expresses the amount of overlap of one function as it is shifted over another function
- Convolution and correlation are similar but we'll explain the difference!

Spatial filtering: Sliding window masks



$$e_{processed} = v*e + r*a + s*b + t*c + u*d + w*f + x*g + y*h + z*i$$

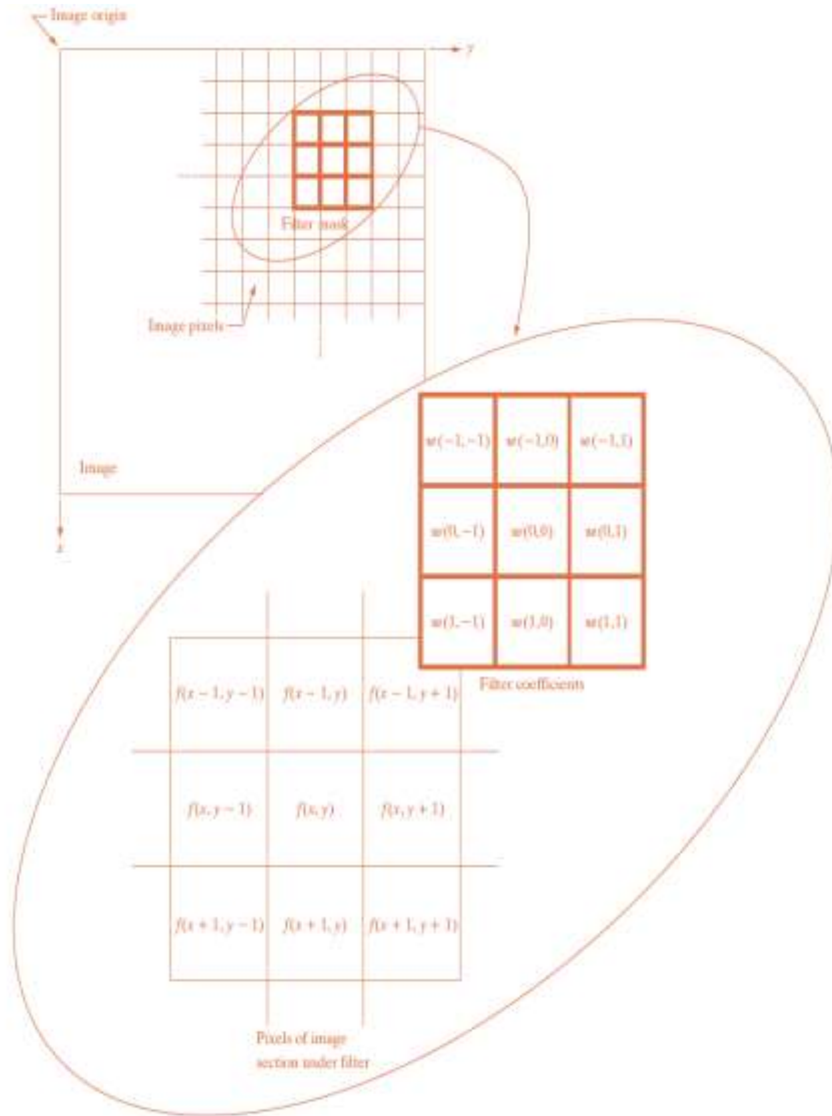
- Filter: Moving window operation

$$g(x,y) = w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + w(-1,1)f(x-1,y+1) +$$
$$w(0,-1)f(x,y-1) + w(0,0)f(x,y) + w(0,1)f(x,y+1) +$$
$$w(1,-1)f(x+1,y-1) + w(+1,0)f(x+1,y) + w(1,1)f(x+1,y+1)$$

or in the convolution form

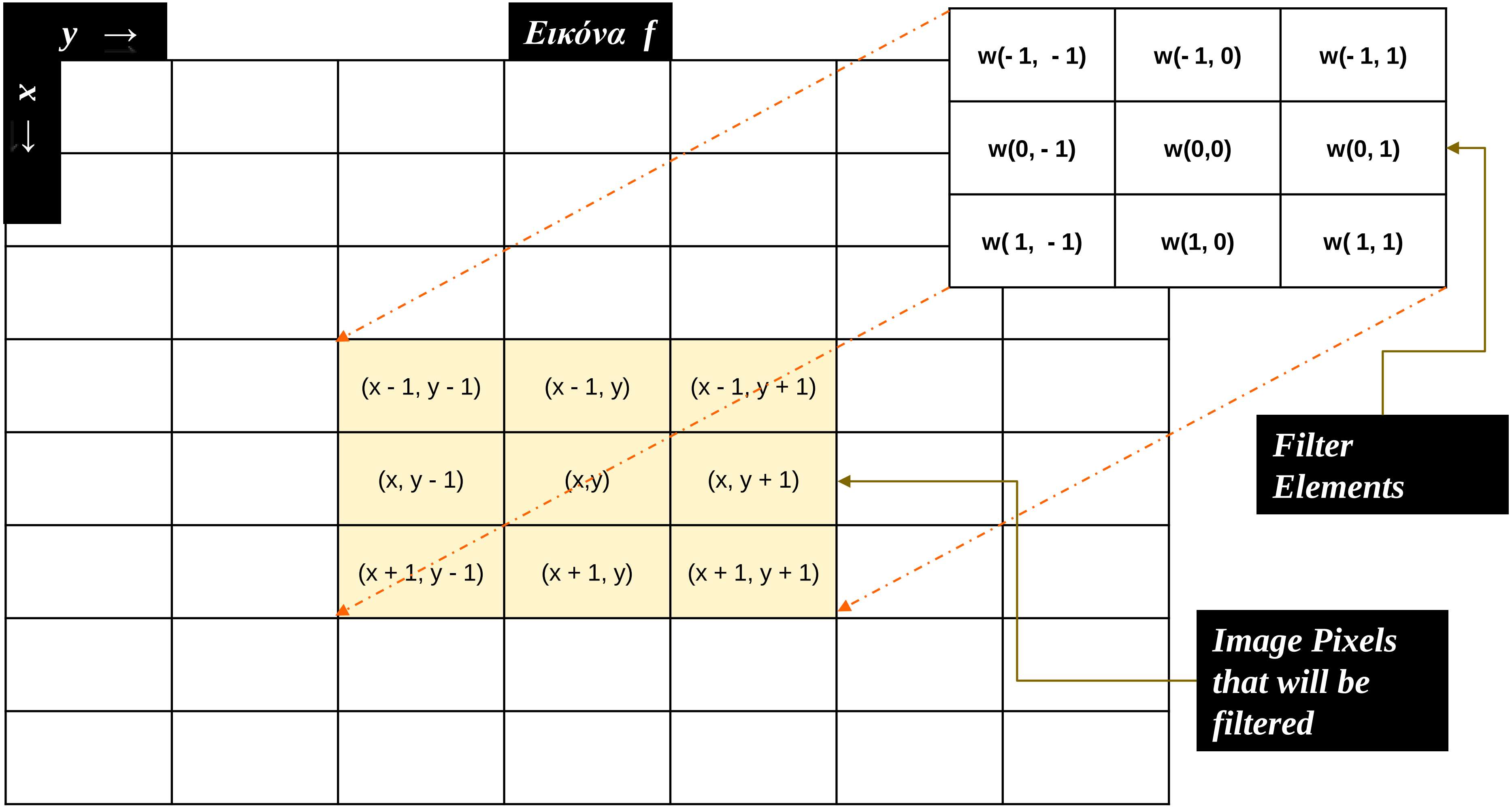
$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} h(s,t)\, f(x-s, y-t)$$

where x, y, s, t are integers
and $h(s,t)$ is the filter



Spatial domain filtering through correlation-convolution

- Linear spatial filtering principle:



Basic

Spatial domain filtering

- Linear spatial filtering principle:

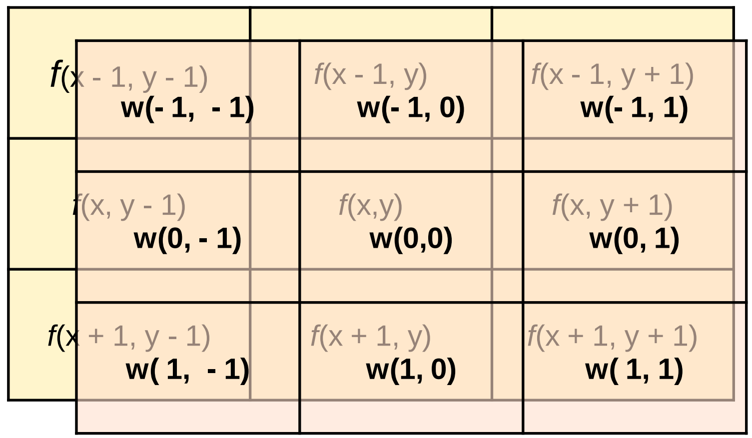
$f(x-1, y-1)$	$f(x-1, y)$	$f(x-1, y+1)$
$f(x, y-1)$	$f(x,y)$	$f(x, y+1)$
$f(x+1, y-1)$	$f(x+1, y)$	$f(x+1, y+1)$

w(- 1, - 1)	**w(- 1, 0)**	**w(- 1, 1)**
w(0, - 1)	**w(0,0)**	**w(0, 1)**
w(1, - 1)	**w(1, 0)**	**w(1, 1)**

Basic

Spatial domain filtering

- Linear spatial filtering principle through correlation-convolution:



$$
\begin{aligned}
g(x, y) = {} & w(-1, -1)\, f(x - 1, y - 1) + w(-1, 0)\, f(x - 1, y) + w(-1, 1) f(x - 1, y+1) + \\
& w(0, -1)\, f(x, y - 1) + w(0, 0)\, f(x, y) + w(0, 1)\, f(x , y+1) + \\
& w(1,-1)\, f(x+1, y - 1) + w(1, 0)\, f(x+1, y) + w(1, 1)\, f(x +1, y+1)
\end{aligned}
$$

Basic

Spatial domain filtering

- The simplified relationship for applying a filter to a 3x3 pixel neighborhood:

p1	*p2*	*p3*
p4	*p5*	*p6*
p7	*p8*	*p9*

⇐

w1	*w2*	*w3*
w4	*w5*	*w6*
w7	*w8*	*w9*

- And generalizing about mxn dimension filters:

$$\mathrm{g(x, y)} = \sum_{k=1}^{mn} w_k p_k$$

Basic

Spatial Filtering: Correlation/Convolution

- In order to have convolution the mask w must be rotated 180º:

p1	*p2*	*p3*
p4	*p5*	*p6*
p7	*p8*	*p9*

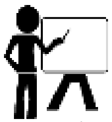
w9	*w8*	*w7*
w6	*w5*	*w4*
w3	*w2*	*w1*

- Correlation and convolution are close concepts and are used for linear filtering.
- Correlation is what we described in the previous slides where we move the filter mask over the image and calculate the sum of the products in each position.
- The convolution is done in the same way but the filter mask needs to be rotated 180 °. So we do that before but in the relation g(x, y) = $\sum_{k=1}^{mn} w_k p_k$ where w1 was swapped with w9, w2 with w8 etc. If the filter is symmetrical, convolution is identical with correlation.

Basic

Common Filters

- mean FILTER
- median FILTER and ranked filters
- GAUSS filters


Basic

Filtering Example

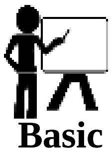
Initial Image:

1 1/9	6 1/9	3 1/9	2	9
2 1/9	(x,y) 11 1/9	3 1/9	10	0
5 1/9	10 1/9	6 1/9	9	7
3	1	0	2	8
4	4	2	9	10

Filtered Image (3x3 smoothing filter)

0	0	0	0	0
0	5			0
0				0
0				0
0	0	0	0	0

- f(x,y)=f(2,2)=11
- New Value g(x,y)=T[f(x,y)] = 1·1/9 + 6·1/9 + 3·1/9 + 2·1/9 + 11·1/9 + 3·1/9 + 5·1/9 + 10·1/9 + 6·1/9 = 47/9 = 5.222



Filtering Example

Initial Image:

1 1/9	6 1/9	3 1/9	2	9
2 1/9	11 1/9	3 1/9	10	0
5 1/9	10 1/9	6 1/9	9	7
3	1	0	2	8
4	4	2	9	10

Filtered Image (3x3 smoothing filter)

0	0	0	0	0
0	5			0
0				0
0				0
0	0	0	0	0

- New Value = $1 \cdot 1/9 + 6 \cdot 1/9 + 3 \cdot 1/9 + 2 \cdot 1/9 + 11 \cdot 1/9 + 3 \cdot 1/9 + 5 \cdot 1/9 + 10 \cdot 1/9 + 6 \cdot 1/9 = 47/9 = 5.222$

Basic

Filtering Example

Initial Image:

1	6 1/9	3 1/9	2 1/9	9
2	11 1/9	3 1/9	10 1/9	0
5	10 1/9	6 1/9	9 1/9	7
3	1	0	2	8
4	4	2	9	10

Filtered Image (3x3 smoothing filter)

0	0	0	0	0
0	5	7		0
0				0
0				0
0	0	0	0	0

- New Value= $6 \cdot 1/9 + 3 \cdot 1/9 + 2 \cdot 1/9 + 11 \cdot 1/9 + 3 \cdot 1/9 + 10 \cdot 1/9 + 10 \cdot 1/9 + 6 \cdot 1/9 + 9 \cdot 1/9 = 60/9 = 6.667$

Filtering Example

Basic

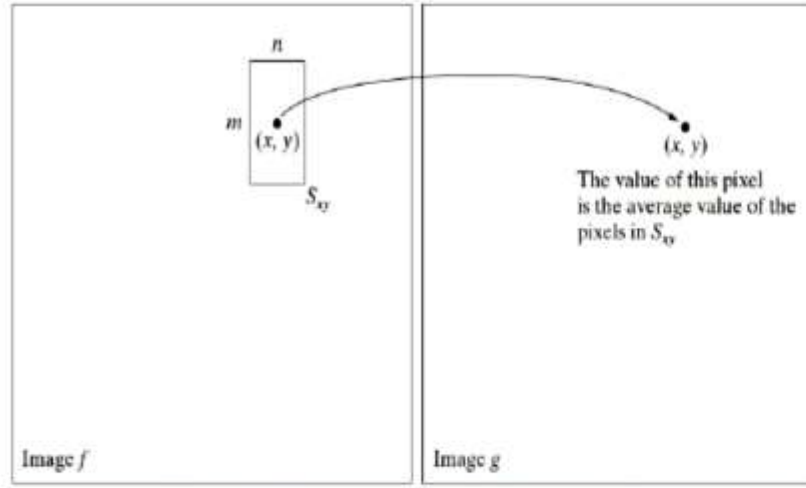
Initial Image:

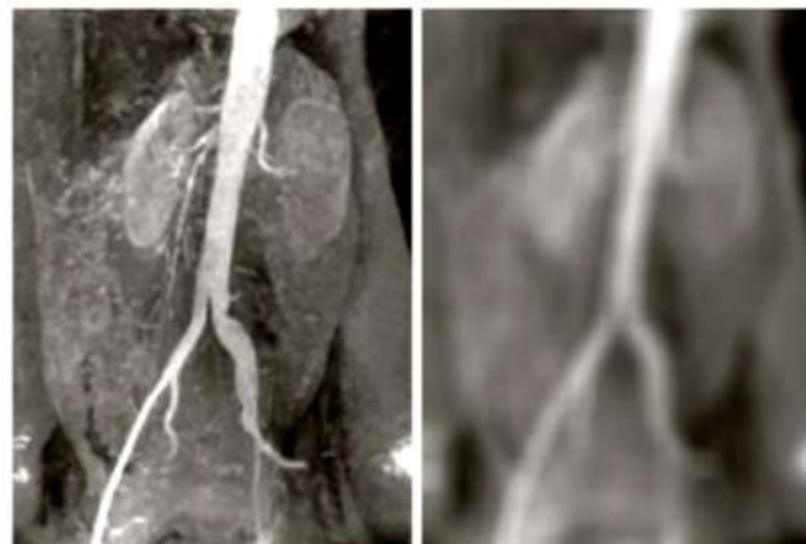
1	6	3	2	9
2	11	3	10	0
5	10	6	9	7
3	1	0	2	8
4	4	2	9	10

Filtered Image (3x3 smoothing filter)

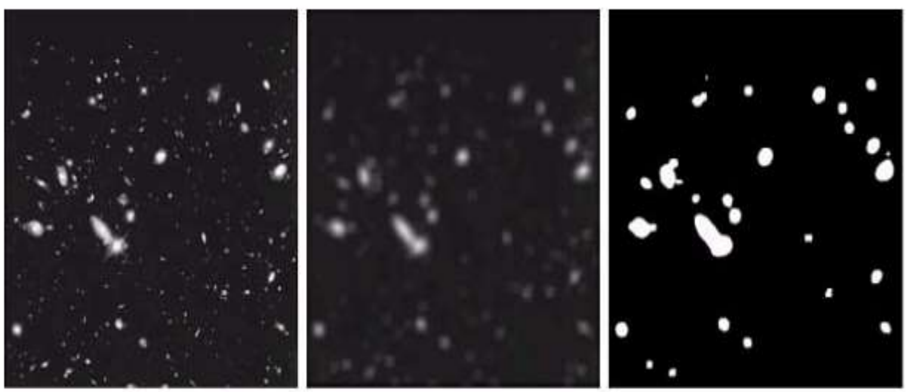
0	0	0	0	0
0	5	7	5	0
0	5	6	5	0
0	4	5	6	0
0	0	0	0	0

mean FILTER





Sliding Window: Moving average



Αρχική Smoothed Thresholded

mean FILTER

- The function of the average value filter is to replace the brightness in each pixel with the average brightness in a neighborhood.
- They are lowpass filters since we replace the value of the pixel with the average value of its neighborhood, at which time we gradually reduce abrupt changes in the intensity of the pixels.
- While we reduce random noise, we usually lose sharpness at the edges of the image (edge blurring).

Basic

mean FILTER

- The function of the average value filter is to replace the brightness in each pixel with the average brightness in a neighborhood.
- If N is the neighborhood of the pixel (i, j) of an image I, then the value of the pixel (i, j) is replaced by :
- $I'(x,y) = \sum_{(x,y)\in N} I(x,y)$
- where M the number of pixels of the neighborhood N.

Basic

mean FILTER

- The neighborhood N is usually defined for each
- processing and usually corresponds to square masks.
- So for a radius equal to one we have essentially a neighborhood of dimensions 3 × 3.
- An average 3 × 3 filter can be practically implemented with a mask of the form:

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

ή 1/9

1	1	1
1	1	1
1	1	1

Basic

mean FILTER

- The mean value filter can be considered as a low pass filter.
- If we want to emphasize the contribution of the pixels depending on their distance, then we can use smoothing masks such as the following

1/16

1	2	1
2	4	2
1	2	1

Basic

mean FILTER

- In its general form for linear filtering mean filtering smoothing of an MxN image with filter m x n is given by the relation:

$$g(x,y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)}$$

Filtering Operation (Spatial Domain)

Example: **<u>Box Filtering (smoothing)</u>**

What does it do?

- Replaces each pixel with an average of its neighborhood
- Achieve smoothing effect (remove sharp features)

$g[\cdot\,,\cdot\,]$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Filtering Operation (Spatial Domain)

$$g[\cdot\,,\cdot\,] \quad \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$$f[.,.]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$h[.,.]$$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$h[m,n] = \sum_{k,l} g[k,l]\; f[m+k, n+l]$$

Credit: S. Seitz

Basic

median filtering

- Median filtering is used to smoothing the edges and reduce the noise of an image.
- Filtering with a medium value filter is a non-linear technique. The median value of a set A is equal to the mean value of the set.
- Specifically, if
- $A=\{\alpha_1,\alpha_2,\alpha_3,..,\alpha_n\}$
- Is a set with ordered values $\alpha_1 \leq \alpha_2 \leq \ldots \leq \alpha_n \in R$.

Basic

median filtering

- The median of *A* is equal to

$$median(A) = \begin{cases} a_{\frac{n+1}{2}} \lfloor n, & odd \\ \frac{1}{2}\left(a_{\frac{n}{2}} + a_{\frac{n}{2}+1} \right) \lfloor n, & even \end{cases}$$

- Properties:
  - median(k+A) = k+median(A)
  - median(k.A) = k.median(A)
  - median(A+B)≠median(A)+ median(B) → **μη γραμμικότητα!**

Basic

median filtering examples

- For example median{4,3,5,8,2}→

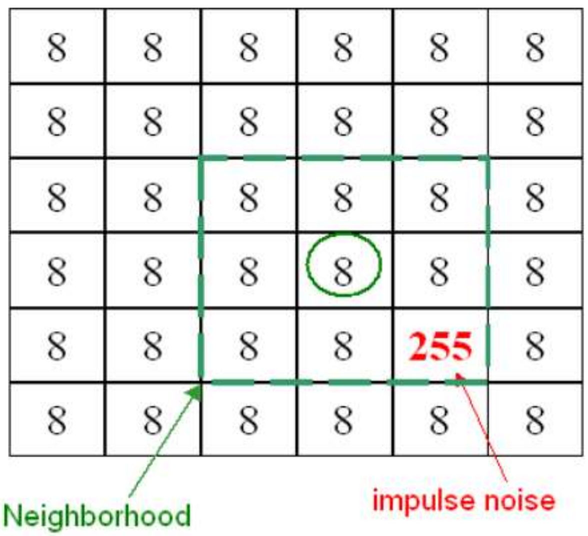
n=5 (odd)→ median{2,3,4,5,8}= $\alpha_{(5+1)/2} = \alpha_3 = 4$

- median{4,3,5,8,2,6} →

n=6 (even) → median{2,3,4,5,6,8}= $\frac{1}{2}\{\alpha_{\frac{n}{2}}+\alpha_{\frac{n}{2}+1}\}=$

$\frac{1}{2}\{\alpha_3+\alpha_4\}= 4.5$

Median Filtering (Details)



[8 8 8 8 8 8 8 8 255]

median

Basic

Gaussian Filtering

- They are image blurring filters that use the Gaussian function (which expresses the normal distribution in statistics) to calculate the filter coefficients to transform each pixel:

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- Where x, y are the distances from the beginning of the axes and σ is the standard deviation of the Gauss distribution.
- In the 2 dimensions this equation gives a surface whose contours are concentric circles with Gaussian distribution from the central point.

$$\frac{G(i,j)}{c} G(i,j) = \frac{1}{2\pi\sigma^2} e^{-\frac{i^2+j^2}{2\sigma^2}}$$

Basic

Gaussian Filtering

- With the constant c, choosing a value for σ^2, we can compute it in an n × n window to get a mask for which the value in [0,0] is 1.

[i,j]	-3	-2	-1	0	1	2	3
-3	0.011	0.039	0.082	0.105	0.082	0.039	0.011
-2	0.039	0.135	0.287	0.368	0.287	0.135	0.039
-1	0.082	0.287	0.606	0.779	0.606	0.287	0.082
0	0.105	0.368	0.779	1	0.779	0.368	0.105
1	0.082	0.287	0.606	0.779	0.606	0.287	0.082
2	0.039	0.135	0.287	0.368	0.287	0.135	0.039
3	0.011	0.039	0.082	0.105	0.082	0.039	0.011

$\sigma^2 = 2$ $\quad$ $n=7$.

Noise suppression: Image Filtering

- Enhance or restore data by removing noise without significantly blurring the structures in the images.
- Literature is vast! We will cover only a few of them.
- Gaussian Filtering:

$$S = (C, f) \longrightarrow S_F = (C, f_F)$$

Noise suppression: Image Filtering

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$$S = (C, f) \longrightarrow S_F = (C, f_F)$$

$$f_F(\nu) = \sum_{\nu_i \in N(\nu)} w_i f(\nu_i)$$

Noise suppression: Image Filtering

	0.01	0.08	0.01
Gaussian	0.08	0.64	0.08
	0.01	0.08	0.01

- Gaussian Filtering:

$$S = (C, f) \longrightarrow S_F = (C, f_F)$$

$$f_F(\nu) = \sum_{\nu_i \in N(\nu)} w_i f(\nu_i)$$

  - f_F is a Gaussian weighted average of f in a neighborhood of N of voxel v

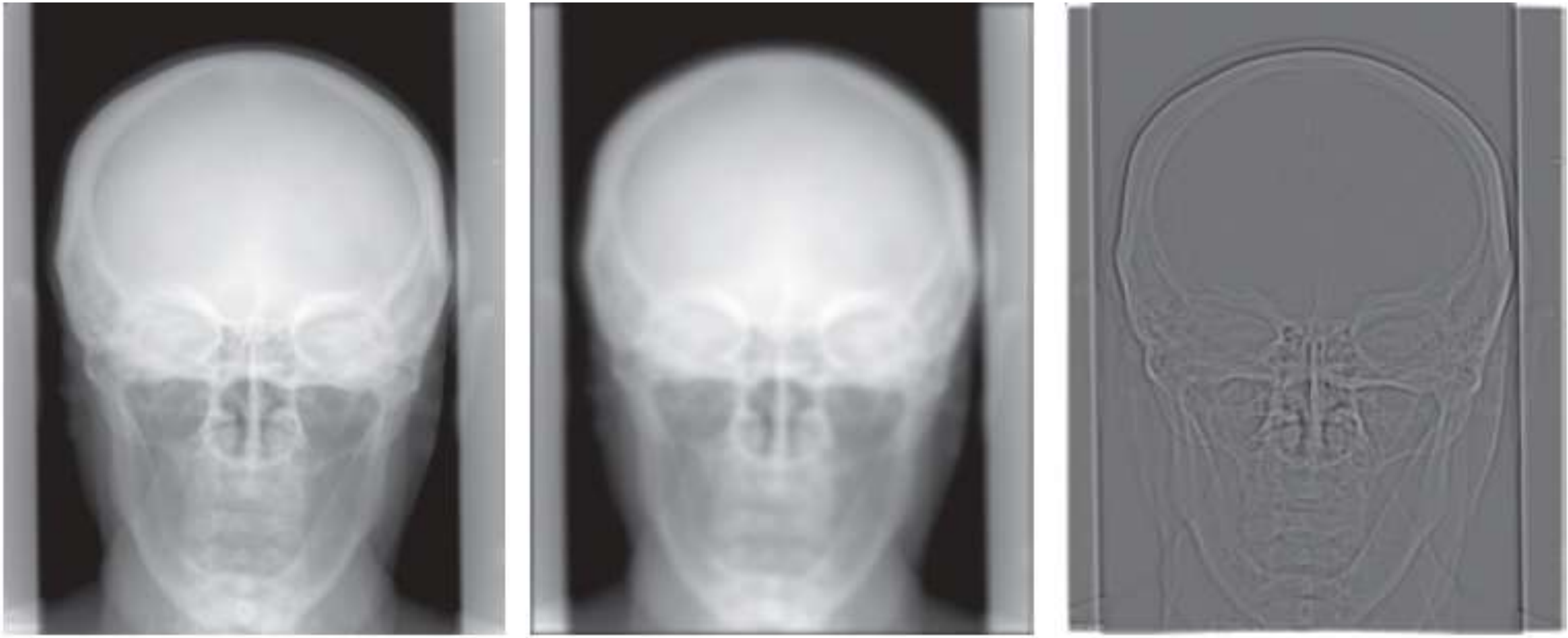
Noise suppression: Image Filtering

- Enhance or restore data by removing noise <u>without significantly blurring</u> the structures in the images.
- Literature is vast! We will cover only a few of them.
- Median Filtering:

$$S = (C, f) \longrightarrow S_F = (C, f_F)$$

  - f_F is median intensity in a neighborhood of voxel v.

Filtering X-ray



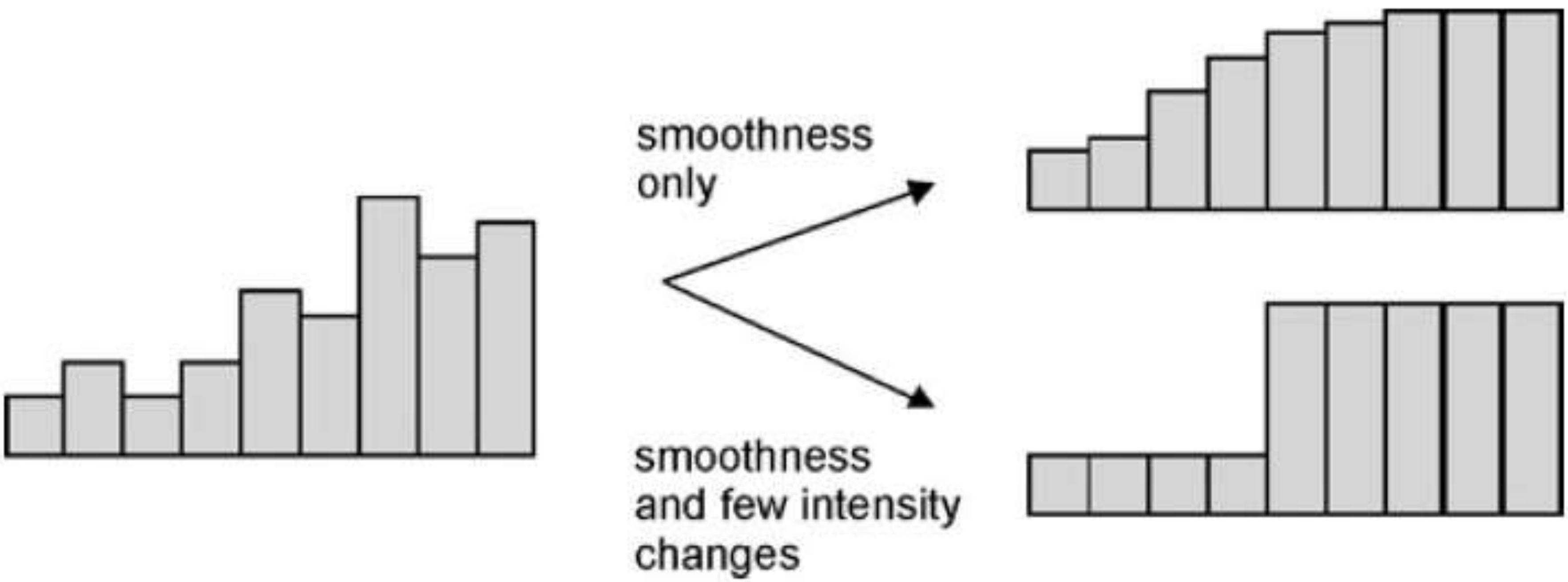
(a) (b) (c)

a. Radiography of the skull, b. low-pass filter with a Gaussian filter (std=15, 20 x 20), c. high-pass Filter obtained from subtracting b from a.

Remark: Why Gaussian Assumption?

- Most common natural model
- Smooth function, it has infinite number of derivatives
- It is Symmetric
- Fourier Transform of Gaussian is Gaussian.
- Convolution of a Gaussian with itself is a Gaussian.
- Gaussian is separable; 2D convolution can be performed by two 1-D convolutions
- There are cells in eye that perform Gaussian filtering.

Composite Filters: Unsharp Masking
Lower Noise, Higher Contrast



histogram histogram

Unsharp Masking

- Not only noise removal, but edge enhancement is necessary!

$$I = g * I + (I - g * I)$$

Unsharp Masking

- Not only noise removal, but edge enhancement is necessary!

$$I = \underbrace{g * I}_{\text{Smoothed image (low pass)}} + \underbrace{(I - g * I)}_{\text{Edge enhanced image (high pass)}}$$

$$\alpha > 0$$

$$I' = g * I + (1 + \alpha)(I - g * I)$$

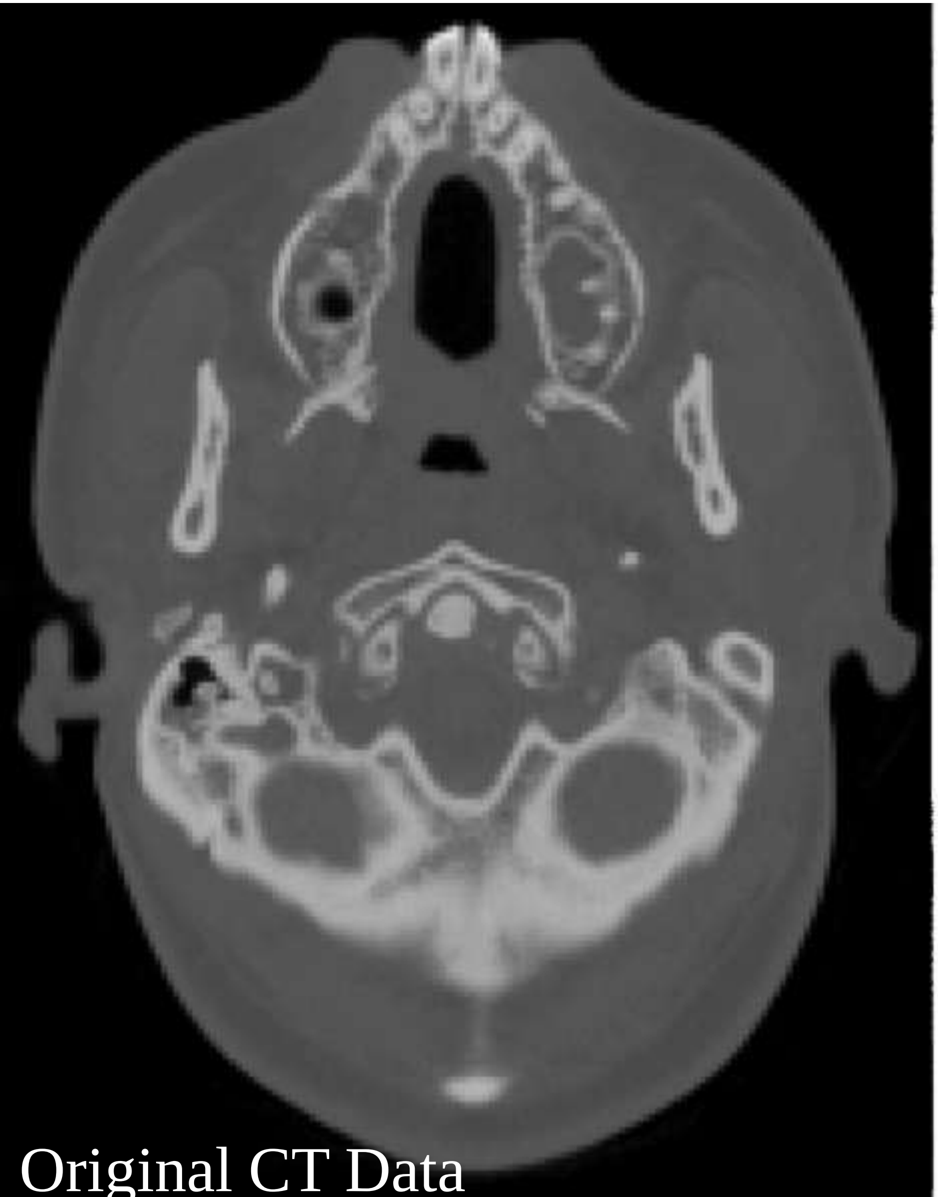
Reminder: Edges are located in high frequency of the images!

Hand X-ray Unsharp Masking (alpha=0.5)

Original Image

Enhanced Image

Unsharp Masking: Example CT (head, axial)

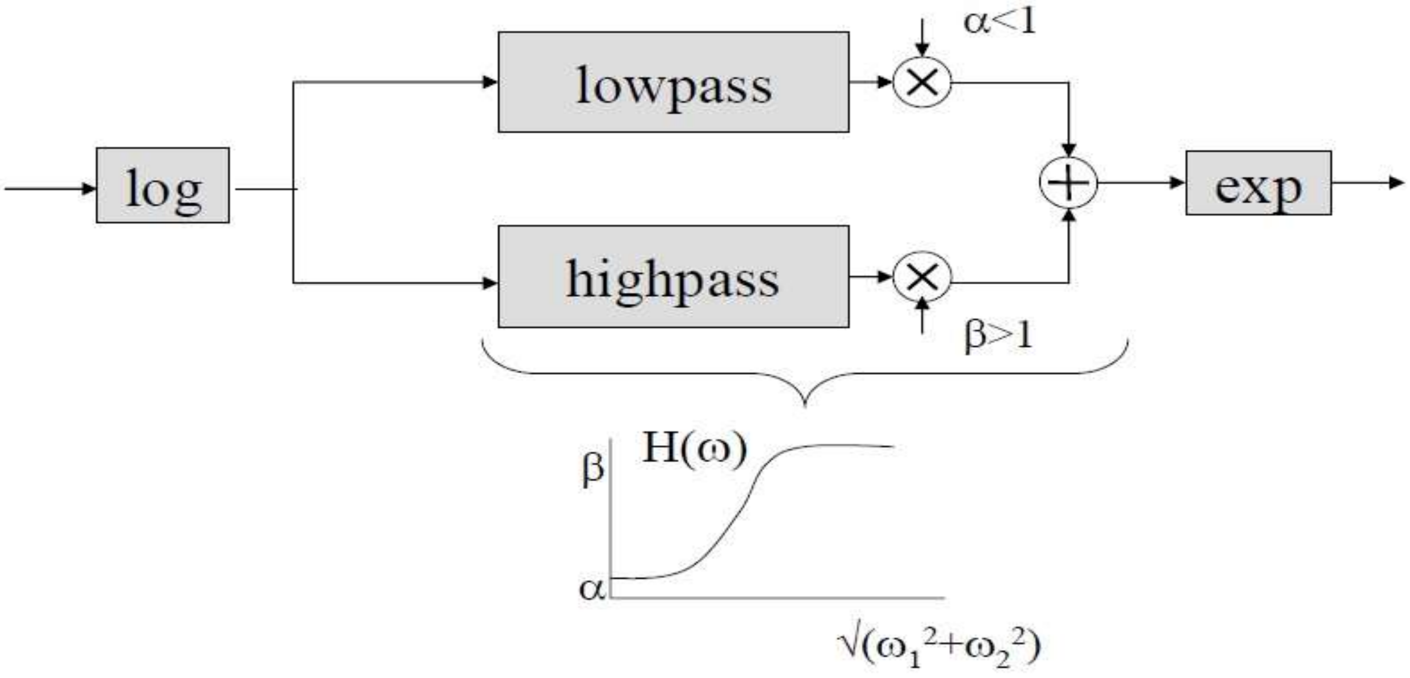


Original CT Data

Filtered CT

Homomorphic filtering -Adaptive Filtering

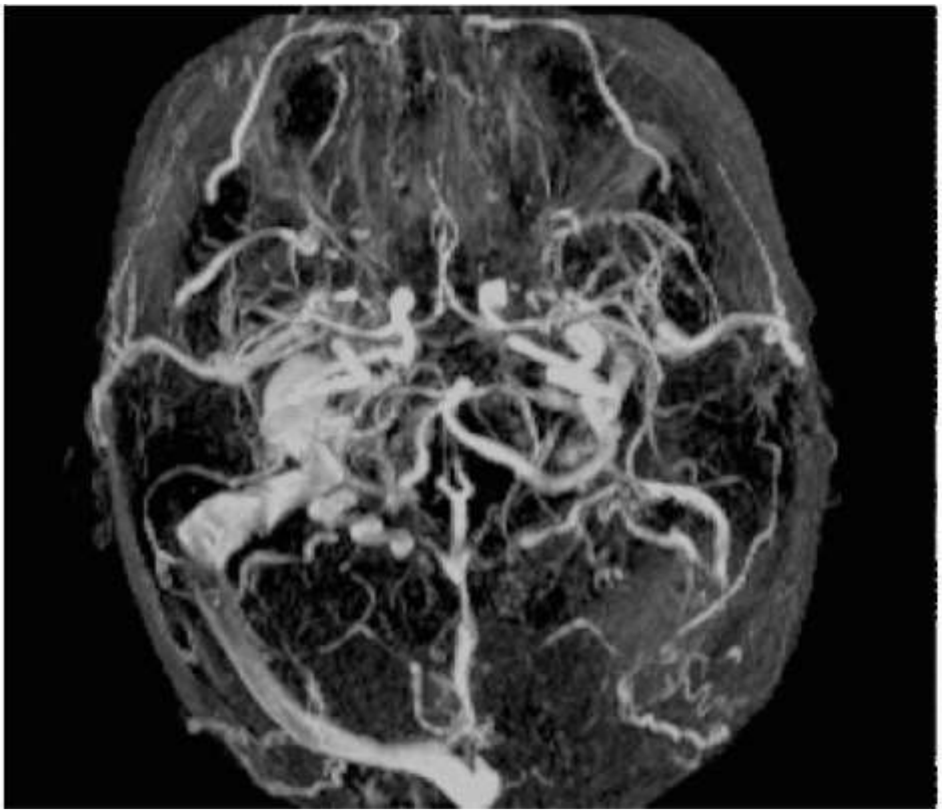
An automatic method for spatially varying contrast adjustment



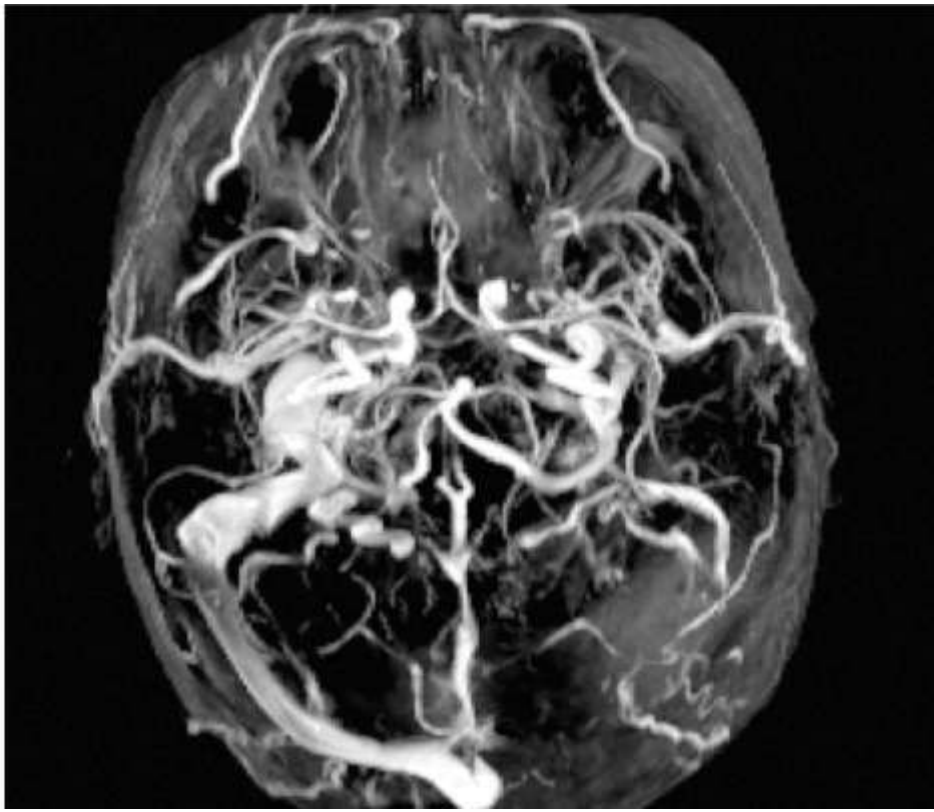
- Homomorphic Enhancement of Angiography

Αρχική Κατωφλίωση Ενίσχυση

Adaptive Filtering: Example head MRA

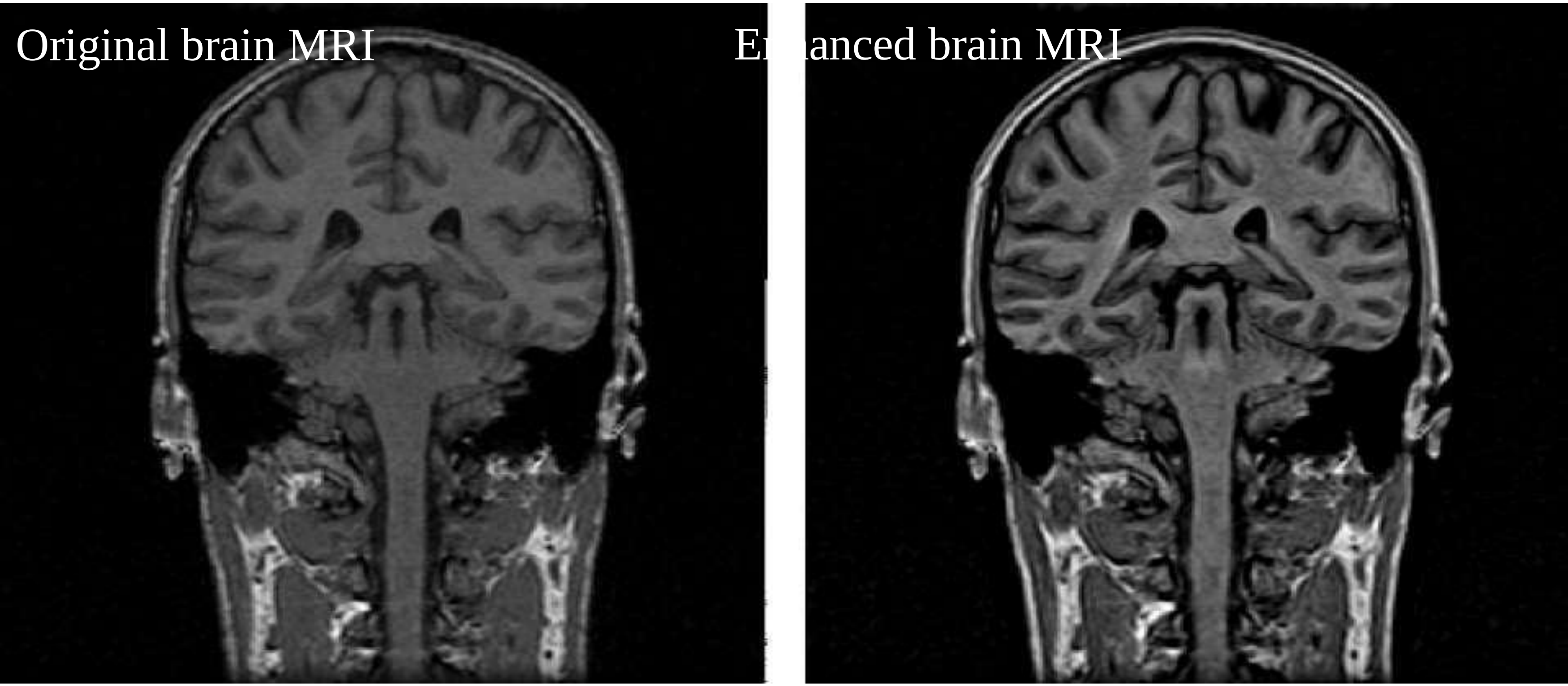


MIP of MRA data before filtering



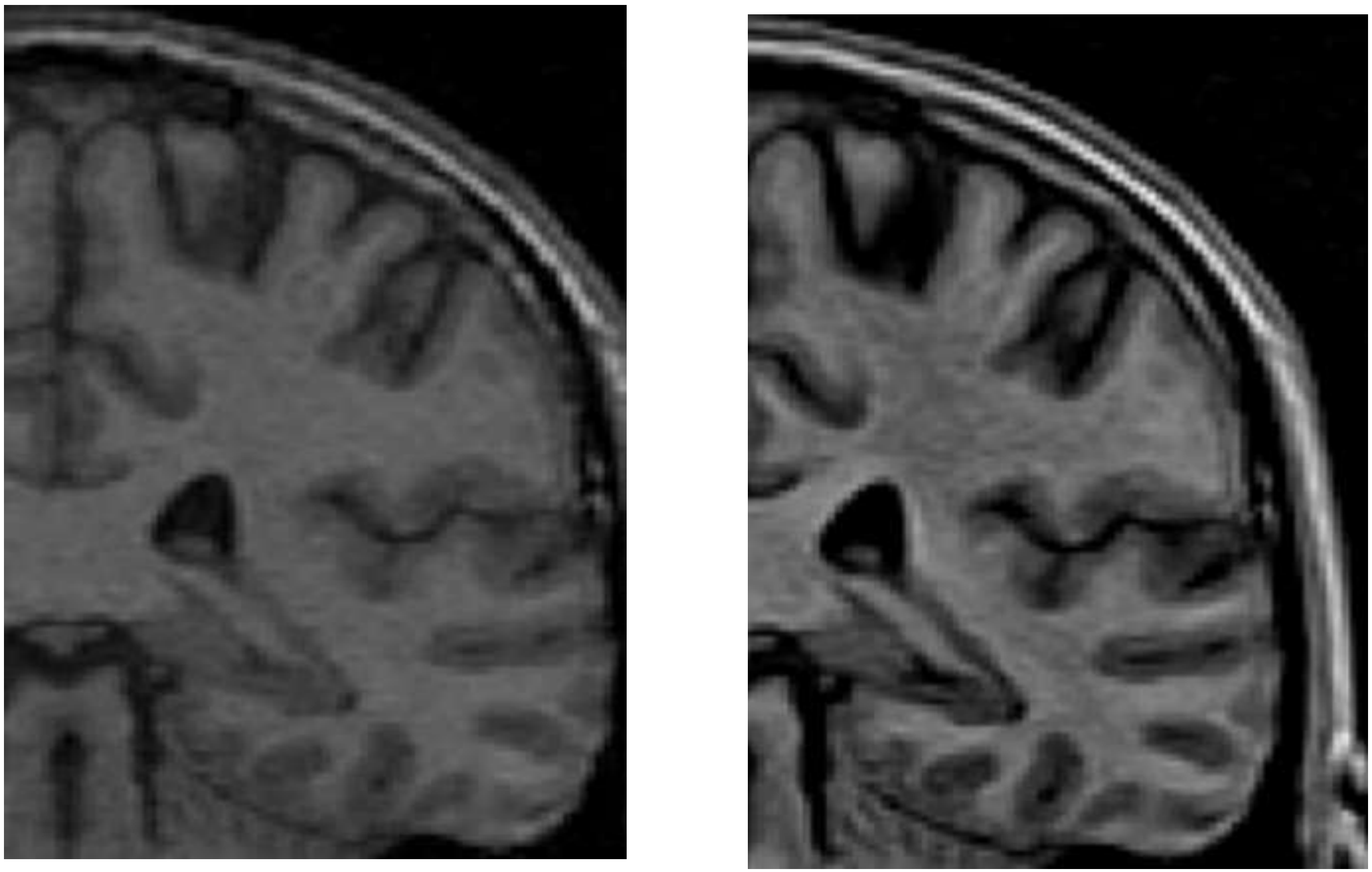
MIP of MRA data after filteri

Adaptive Filtering: Example brain MRI



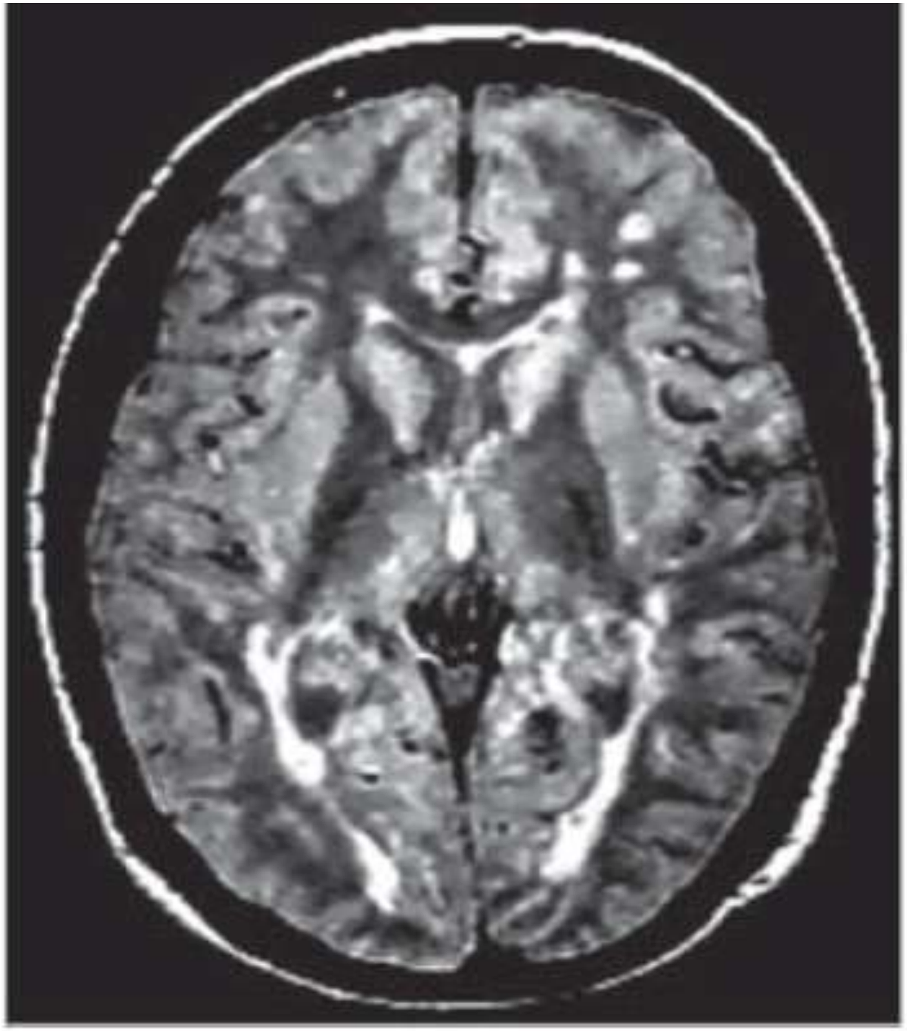
Note the improved contrast between brain and CSF (cerebrospinal fluid)

Adaptive Filtering: Example brain MRI (zoomed)



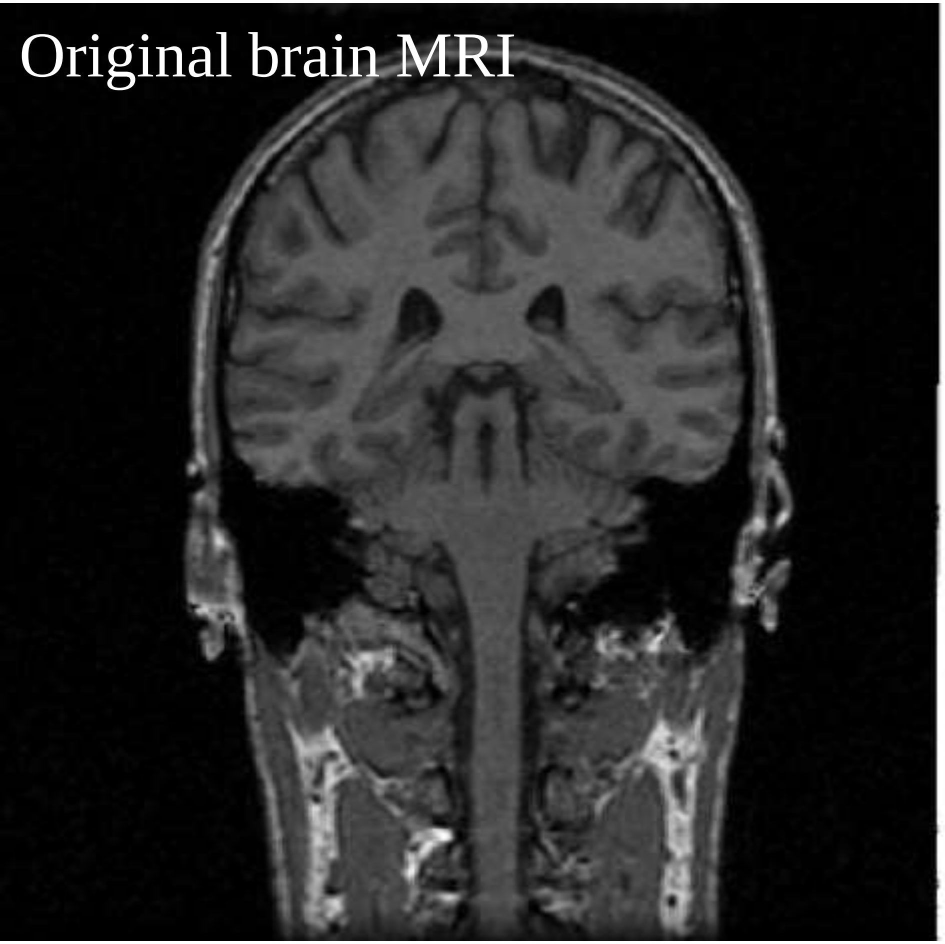
Note the improved contrast between brain and CSF (cerebrospin

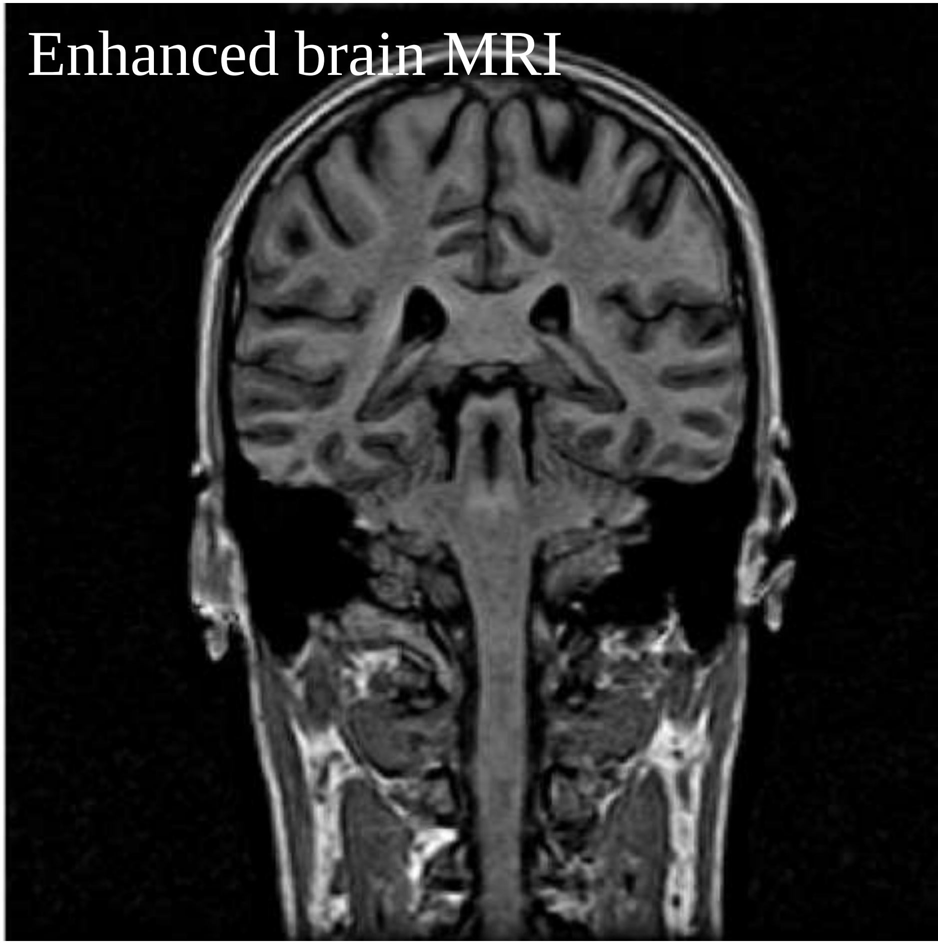
Original



Noise Suppression

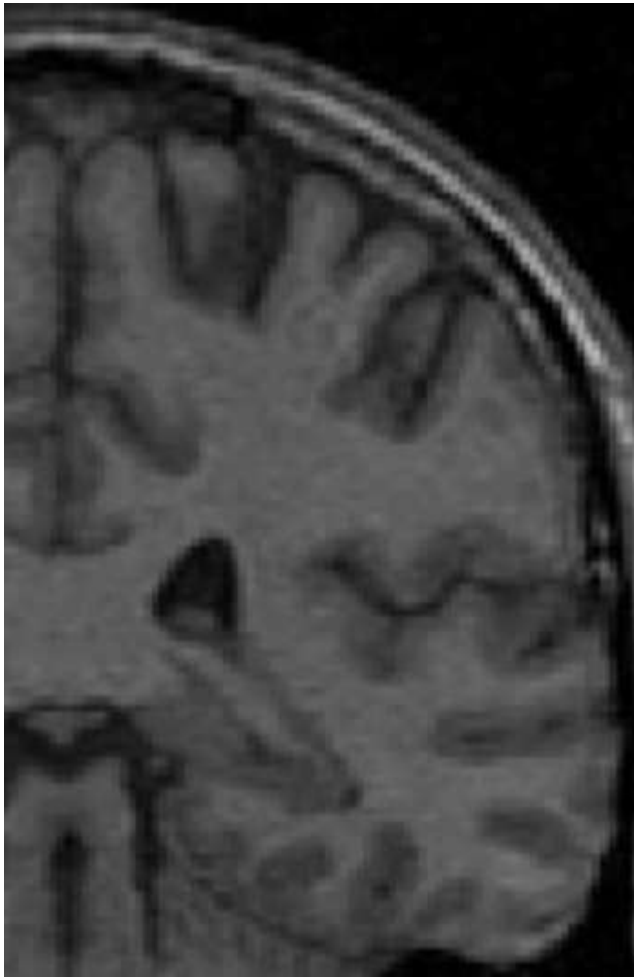
Adaptive Filtering: Example brain MRI

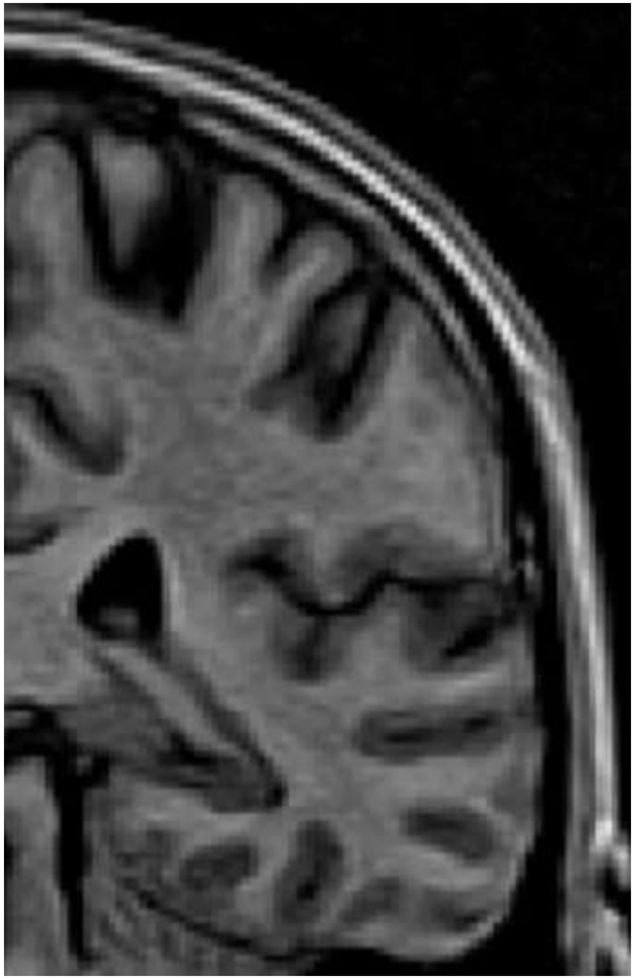




Note the improved contrast between brain and CSF (cerebrospinal fluid)

Adaptive Filtering: Example brain MRI (zoomed)



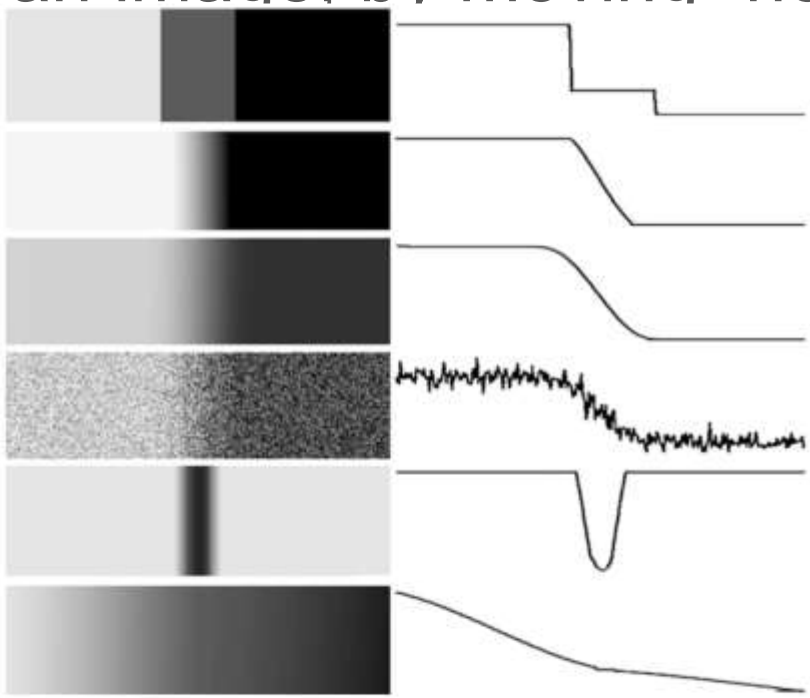


Note the improved contrast between brain and CSF (cerebrospina

Edges and edge detection

Edges

- Discontinuities in images are features that are often useful for initializing an image analysis procedure.
- Edges are important information for understanding an image; by moving "non-edge" data we also



Edges → rate of change

Rate of change → differentiation

Differentiation → difference in dig

Edges

- Discontinuities in images are features that are often useful for initializing an image analysis procedure.
- Edges are important information for understanding an image; by moving “non-edge” data we also simplify the data.

- Goal: Identify sudden changes (discontinuities) in an image
  - Most semantic and shape information from the image can be encoded in the edges
  - More compact than pixels
  - Marks the border of an object

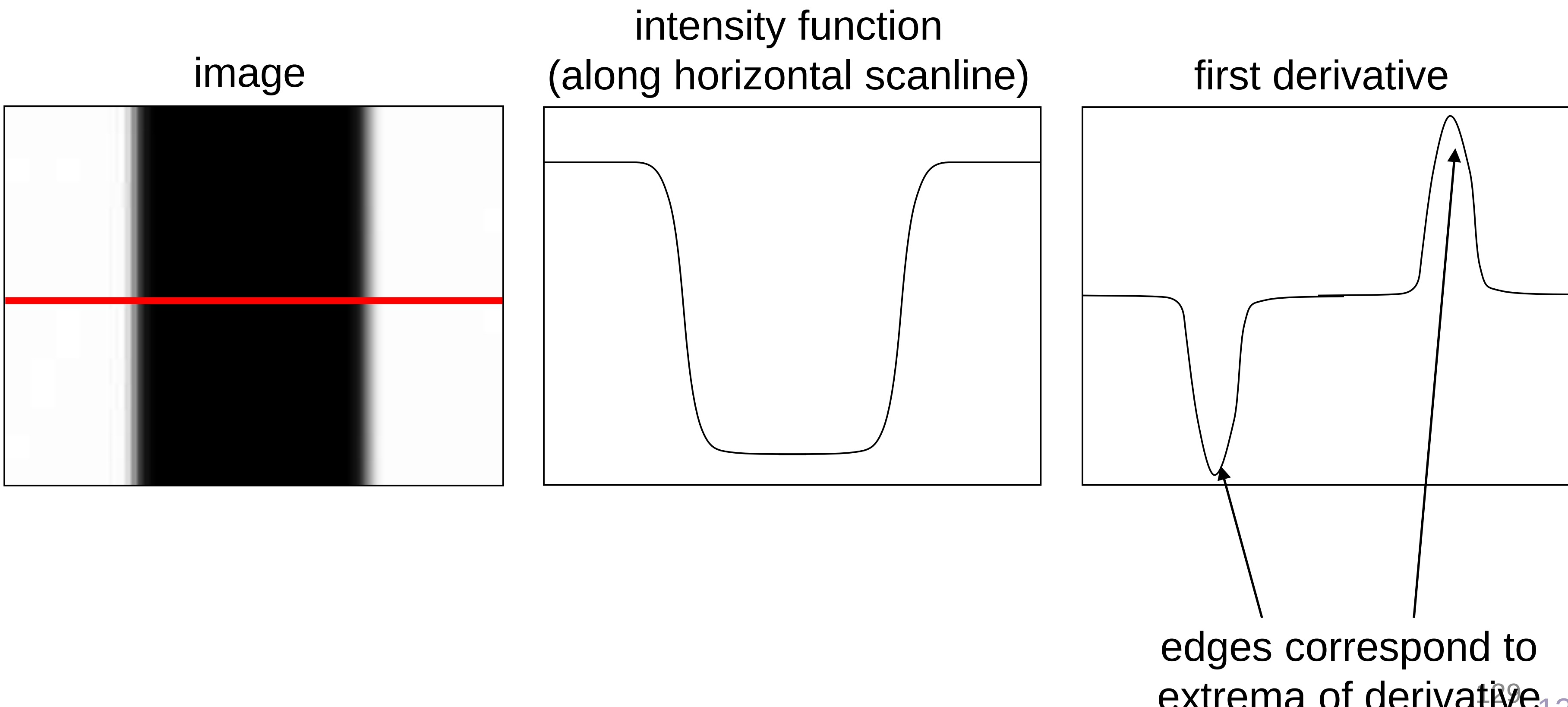
Edges ➔ rate of change

Rate of change ➔ differentiation

Differentiation ➔ difference in dig

Characterizing Edges

- An edge is a place of rapid change in the image intensity function



Laplacian Mask

$$\nabla^2 f(x,y) = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2} =$$

$$= \boldsymbol{f(x+1,y)} + \boldsymbol{f(x-1,y)} + \boldsymbol{f(x,y-1)} + \boldsymbol{f(x,y+1)} - 4\boldsymbol{f(x,y)}$$

f(x - 1, y - 1)	*f*(x - 1, y)	*f*(x - 1, y + 1)
f(x, y - 1)	*f*(x,y)	*f*(x, y + 1)
f(x + 1, y - 1)	*f*(x + 1, y)	*f*(x + 1, y + 1)

0	1	0
1	-4	1
0	1	0

Laplacian Mask (isotropic)

$$\nabla^2 f(x,y) = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2} =$$

$$= f(x+1,y) + f(x-1,y) + f(x,y-1) + f(x,y+1) + f(x+1,y+1) + f(x-1,y-1) + f(x+1,y-1) + f(x-1,y+1) - 8f(x,y)$$

f(x - 1, y - 1)	*f*(x - 1, y)	*f*(x - 1, y + 1)
f(x, y - 1)	*f*(x,y)	*f*(x, y + 1)
f(x + 1, y - 1)	*f*(x + 1, y)	*f*(x + 1, y + 1)

1	1	1
1	-8	1
1	1	1

All the mask coefficients sum to zero, as expected of a derivative operator.

Gradient of Image (first derivative)

- gradient image: $M(x,y) = mag(\nabla f) = \sqrt{g_x^2 + g_y^2} \approx |g_x| + |g_y|$

$$|g_x| = |\boldsymbol{f(x+1, y-1) + 2f(x+1, y) + f(x+1, y+1)} - \boldsymbol{f(x-1, y-1) + 2f(x-1, y) + f(x-1, y+1)}|$$

***f*(x - 1, y - 1)**	***f*(x - 1, y)**	***f*(x - 1, y + 1)**
***f*(x, y - 1)**	***f*(x,y)**	***f*(x, y + 1)**
***f*(x + 1, y - 1)**	***f*(x + 1, y)**	***f*(x + 1, y + 1)**

-1	**-2**	**-1**
0	**0**	**0**
1	**2**	**1**

Sobel operator g_x

Gradient of Image (first derivative)

- gradient image: $M(x,y) = mag(\nabla f) = \sqrt{g_x^2 + g_y^2} \approx |g_x| + |g_y|$

$$|g_y| = |f(x-1, y+1) + 2f(x, y+1) + f(x+1, y+1) - f(x-1, y-1) - 2f(x, y-1) - f(x+1, y-1)|$$

f(x - 1, y - 1)	*f*(x - 1, y)	*f*(x - 1, y + 1)
f(x, y - 1)	*f*(x,y)	*f*(x, y + 1)
f(x + 1, y - 1)	*f*(x + 1, y)	*f*(x + 1, y + 1)

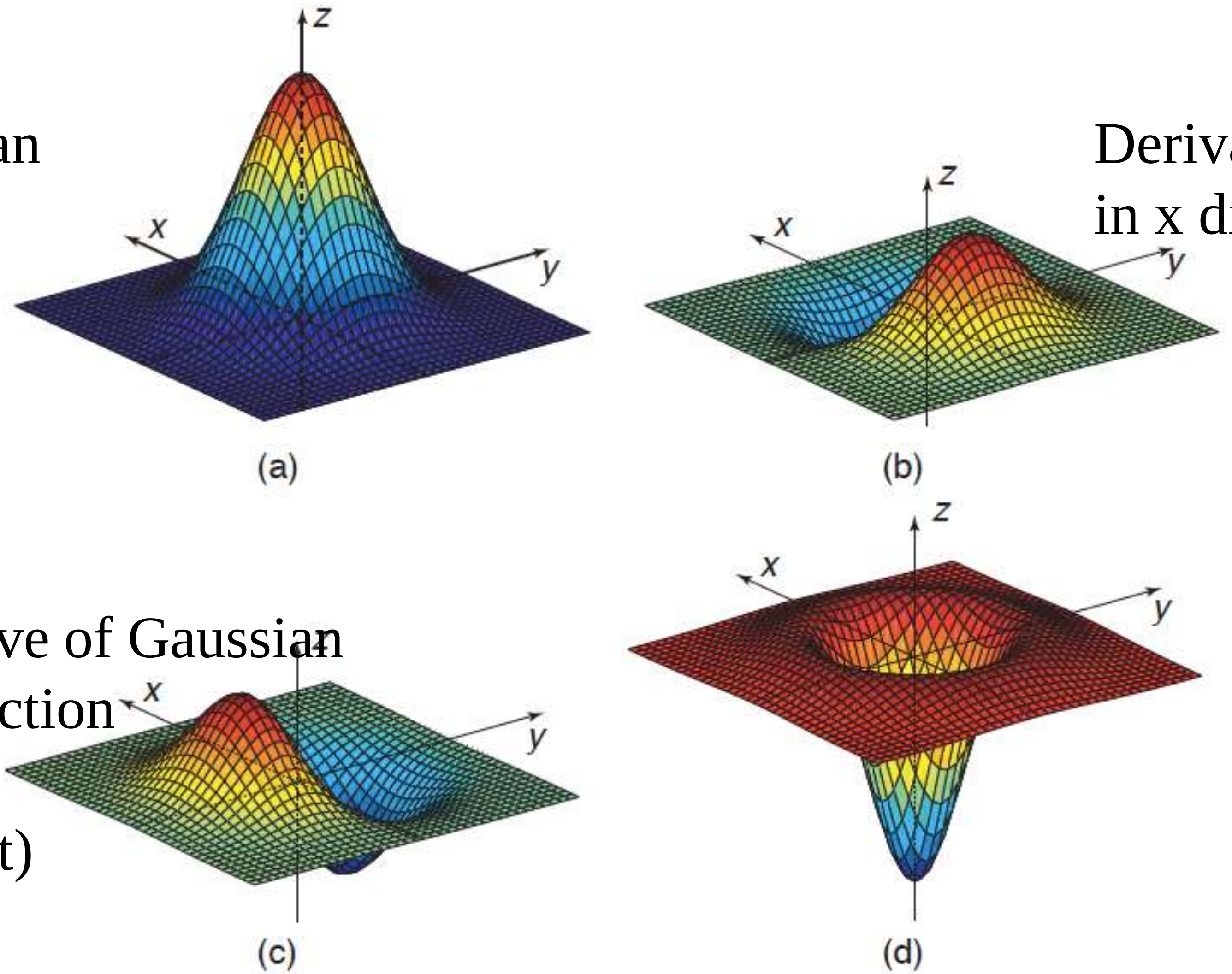
-1	0	1
-2	0	2
-1	0	1

Sobel operator g_y

Laplacian: Difference of Gaussians

Gaussian

Derivative of Gau
in x direction
(gradient)



Derivative of Gaussian
in y direction

(gradient)

Laplacian o
Gaussian

Difference of Gaussians ~ Laplacian

Gaussian

0.01	0.08	0.01
0.08	0.64	0.08
0.01	0.08	0.01

$\frac{\partial}{\partial x}$

0.05	0	−0.05
0.34	0	−0.34
0.05	0	−0.05

$\frac{\partial}{\partial y}$

0.05	0.34	0.05
0	0	0
−0.05	−0.34	−0.05

∇^2

0.3	0.7	0.3
0.7	−4	0.7
0.3	0.7	0.3

End of today's lecture

Thank you for your attention!

References and Slide Credits

- Many thanks to Ulas Bagci (Northwestern University)

for sharing his experience and course material (most slides with title in red, blue color)

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- N. Bryan, Intro. to the science of medical imaging, Cambridge Univ. Press.
- CAP 5415 Computer Vision (Fall 2017) Lecture Presentations