



Research Methods and Project Management

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Introduction to Network Analysis and Critical Path

Network analysis

- ❖ There is no clear terminology in the literature and you will see this area referred to by the phrases: network analysis, PERT, CPM, PERT/CPM, critical path analysis and project planning.
- ❖ Network analysis is a vital technique in **Project Management**.
- ❖ It enables us to take a *systematic quantitative structured approach* to the problem of managing a project through to successful completion.
- ❖ Moreover, it has a graphical representation which means it can be understood and used by those with a less technical background.

Various Project Management Tools/Techniques

❖ Gantt Chart

- Tool that can be used to plan and track project activities.

❖ Critical Path Method (CPM)

- A method used for determining the sequence of task activities that directly affect the completion of a project.

❖ Program Evaluation and Review Technique (PERT)

- A technique that uses optimistic, pessimistic, and realistic time to calculate the expected time for a particular task.

❖ Microsoft Project

- Most widely used project management software
- <http://office.microsoft.com/en-us/project/default.aspx>

Example

- ❖ We will illustrate the concepts involved (network analysis) with reference to the following example:
 - Suppose that we are going to carry out a minor redesign of a product and its associated packaging.
 - We intend to test market this redesigned product and then revise it in the light of the test market results, finally presenting the results to the Board of the company.

Example

- ❖ After much *thought* we have identified the following list of separate activities together with their associated completion times (assumed known with certainty).

- ❖ **The key question is:**

How long will it take to complete this project?

Activity number		Completion time (in weeks)
1	Redesign product	6
2	Redesign package	2
3	Order and receive components for redesigned product	3
4	Order and receive components for redesigned packaging	2
5	Assemble product	4
6	Make up packaging	1
7	Package redesigned product	1
8	Test market with redesigned product	6
9	Revise redesigned product	3
10	Revise redesigned packaging	1
11	Present results to the Board	1

Work Breakdown Structure (WBS) - Our Example

❖ Again after much thought (and aided by the fact that we listed the activities in a logical/chronological order) we come up with the following list of immediate precedence relationships.

❖ The key to constructing this table is, for each activity in turn, to ask the question:

"What activities must be finished before this activity can start"

Activity number		Activity number
1	must be finished before	3 can start
2		4
3		5
4		6
5,6		7
7		8
8		9
8		10
9,10		11

Notes on our example

- ❖ Activities 1 and 2 do not appear in the right hand column of the above table, this is **because there are no activities which must finish before they can start**, i.e. both activities 1 and 2 can start immediately.
- ❖ **Two activities (5 and 6) must be finished before activity 7 can start .**
- ❖ It is plain from this table that non-immediate precedence relationships (e.g. "activity 1 must be finished before activity 9 can start") need not be included in the list since they can be deduced from the relationships already in the list.
- ❖ Once we have completed our list of activities and our list of precedence relationships we combine them into a diagram/picture (called a **network** - which is where the name **network analysis** comes from).
- ❖ We asked the key question above:

How long will it take to complete this project?

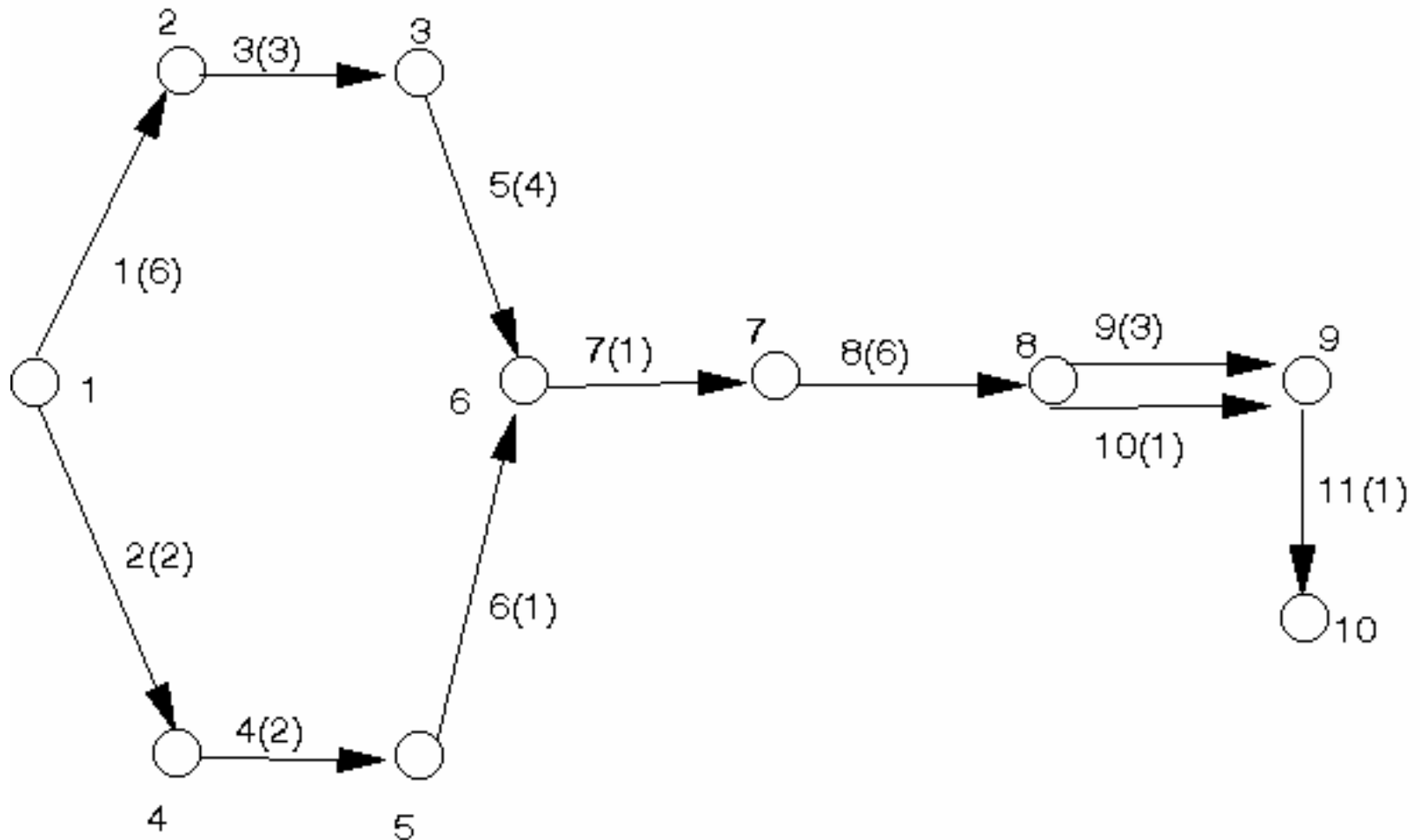
Notes on our example

- ❖ One answer could be if we first do activity 1, then activity 2, then activity 3, ..., then activity 10, then activity 11.
- ❖ This is possible here and the project would then take the sum of the completion times of all activities, *i.e.* 30 weeks.
- ❖ However could we complete the project in less time? It is clear that logically we need to amend our key question to be:

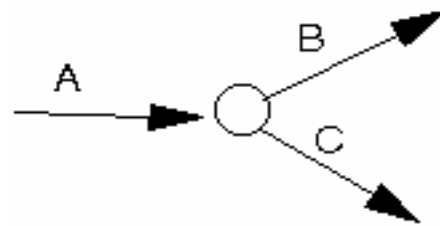
What is the minimum possible time in which we can complete this project?

- ❖ We shall see how the network diagram we construct and its analysis helps us to answer this question.

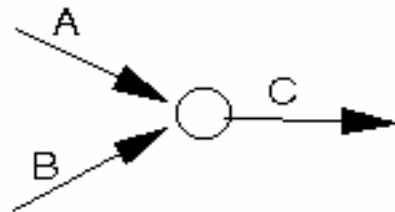
Network diagram (Activity on Arc)



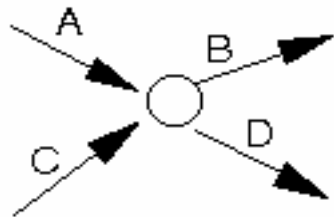
Situations represented in a Network Diagram



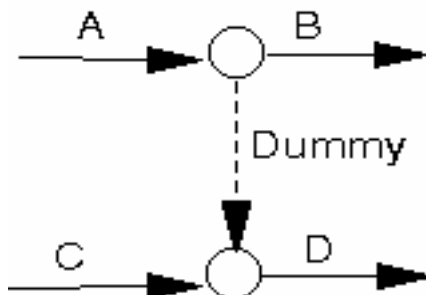
A must finish before either B or C can start



both A and B must finish before C can start



both A and C must finish before either of B or D can start



A must finish before B can start

both A and C must finish before D can start

Building a house



THE NETWORK DIAGRAM IS
ABOUT CREATING
RELATIONSHIPS BETWEEN
ACTIVITIES

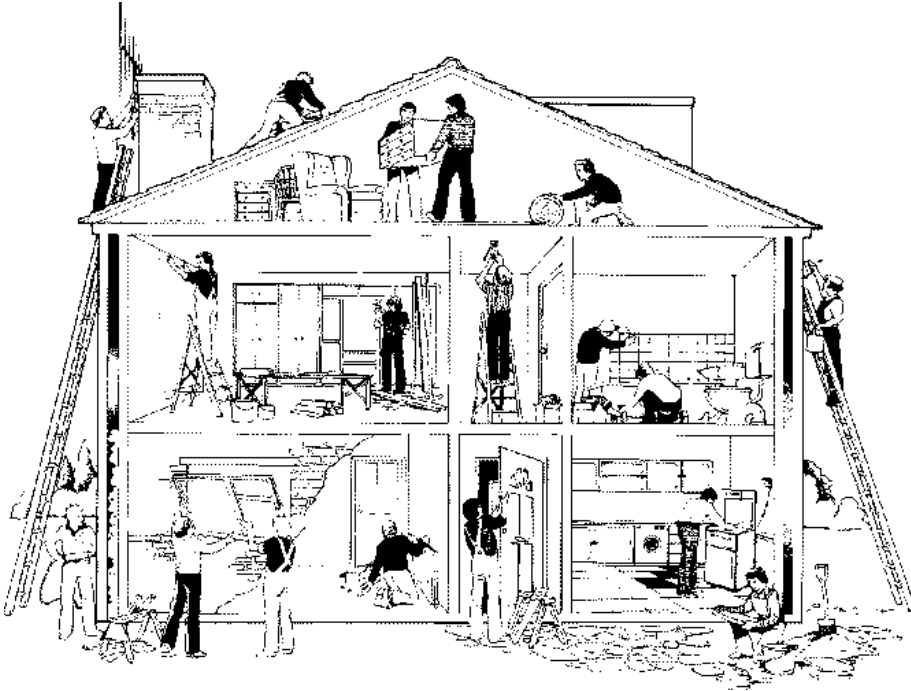
Building a house



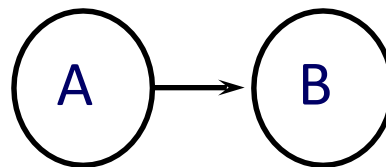
RELATIONSHIPS ARE
ALSO REFERRED TO AS
DEPENDENCIES - THE
MOST BASIC IS “FINISH
TO START”

THUS THE BASIC
QUESTION IS ...
“WHICH ACTIVITIES MUST
BE FINISHED BEFORE
WHICH ACTIVITIES CAN
START”

Building a house

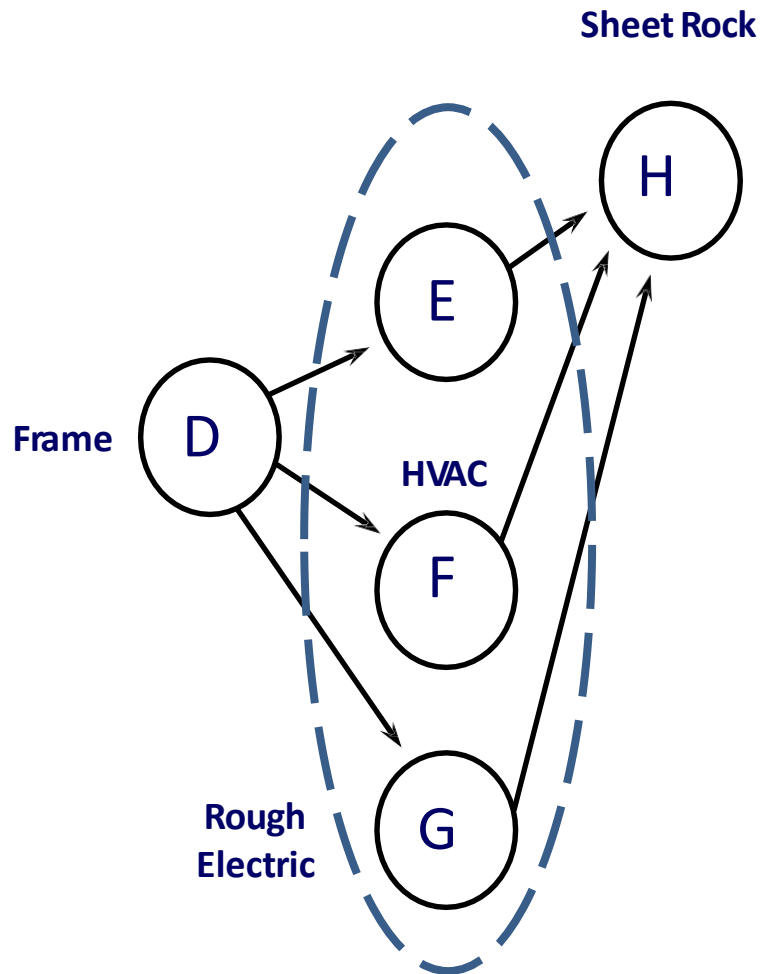


THE USE OF AN ARROW
(SPECIFICALLY IN THE
NOTATION USED IN THIS
EXAMPLE) SHOWS THIS
RELATIONSHIP



Activity A must be finished before B can start

Building a house

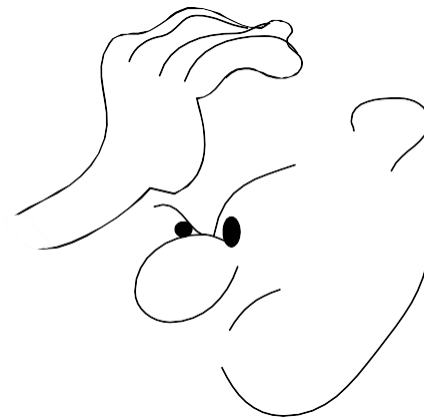


- ❖ But activity E, F and G don't have to wait for each other.
- ❖ Note that E, F and G don't have to start and finish at the same time.
- ❖ They however need to be finished before H can start.

1st Task is to create these
Dependencies by
indicating the Predecessors
for each Activity

Summary of activities

Activity	Description	Time Required (in days)	Immediate Predecessor Activities
A	Excavate		
B	Lay foundation		
C	Rough plumbing		
D	Frame		
E	Finish exterior		
F	Install HVAC (Heating, Ventilation and Air Conditioning)		
G	Rough electric		
H	Sheet rock		
I	Install cabinets		
J	Paint		
K	Final plumbing		
L	Final electric		



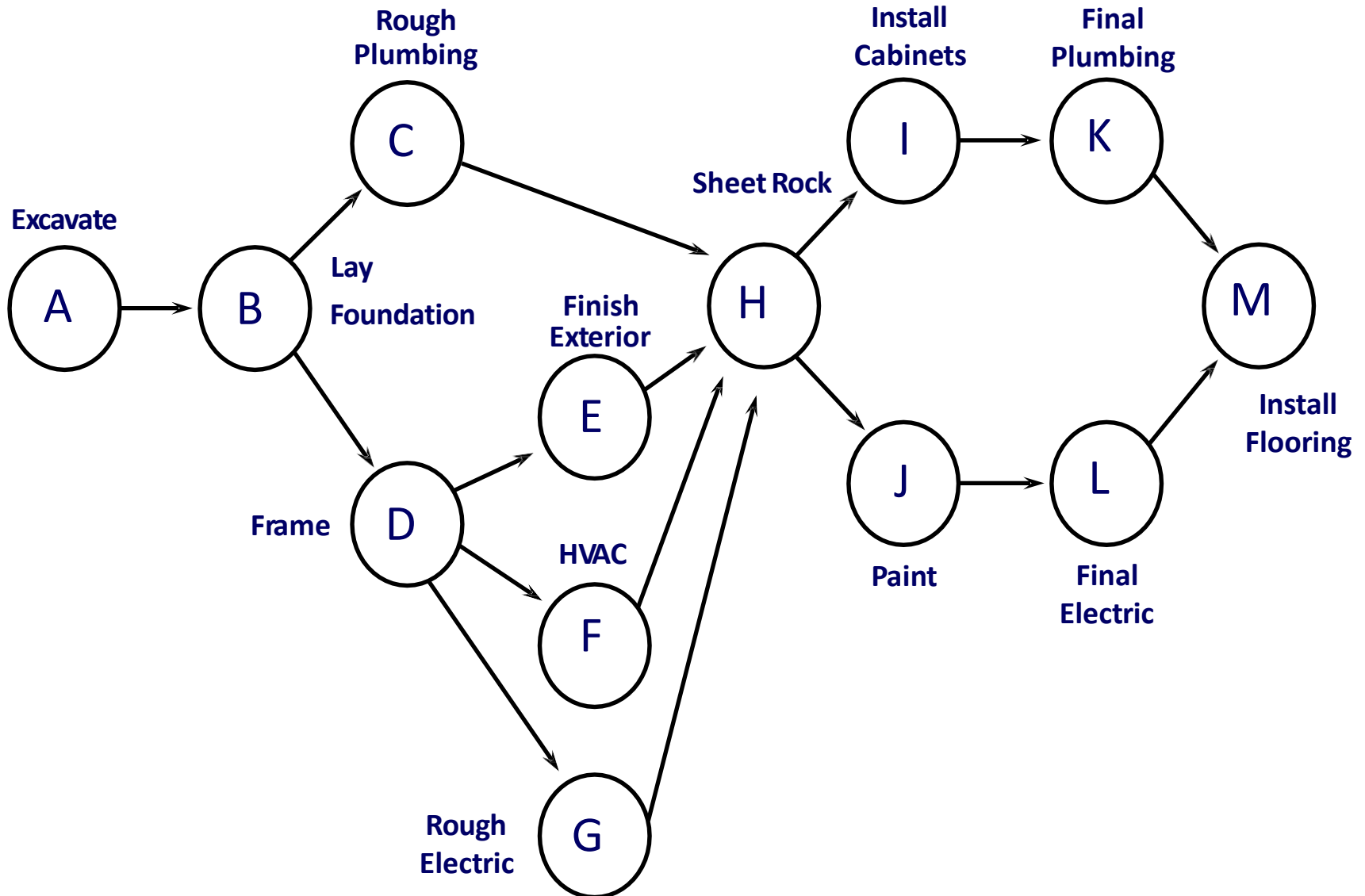
Summary of activities

Activity	Description	Time Required (in days)	Immediate Predecessor Activities
A	Excavate		--
B	Lay foundation		A
C	Rough plumbing		B
D	Frame		B
E	Finish exterior		D
F	Install HVAC		D
G	Rough electric		D
H	Sheet rock		C, E, F, G
I	Install cabinets		H
J	Paint		H
K	Final plumbing		I
L	Final electric		J
M	Install flooring		K, L

Task 2

Test the logic by constructing the
Network Diagram

An Activity-On-Arrow (AOA) Network



Basic Rules for Constructing the Network Diagram

- ❖ Networks typically flow from left to right;
- ❖ An activity cannot begin until all of its preceding activities are complete;
- ❖ Arrows indicate precedence and flow and can cross over each other;
- ❖ Identify each activity with a unique number; this number must be greater than its predecessors;
- ❖ Looping is not allowed;
- ❖ Conditional statements are not allowed;
- ❖ Use unique start and stop nodes.

Task 3

Determine durations for each activity

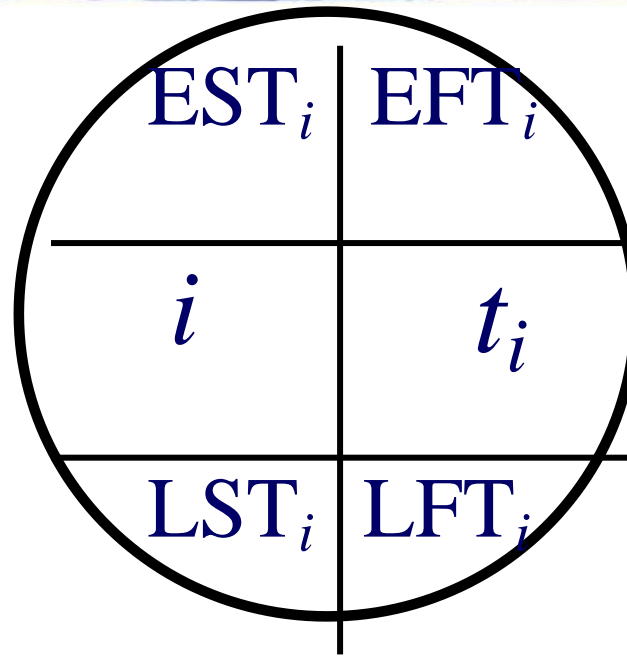
Summary of activities

Activity	Description	Time Required (in days)	Immediate Predecessor Activities
A	Excavate	3	--
B	Lay foundation	4	A
C	Rough plumbing	3	B
D	Frame	10	B
E	Finish exterior	8	D
F	Install HVAC	4	D
G	Rough electric	6	D
H	Sheet rock	8	C, E, F, G
I	Install cabinets	5	H
J	Paint	5	H
K	Final plumbing	4	I
L	Final electric	2	J
M	Install flooring	4	K, L

Task 4

Fill each node as follows

Information Recorded for each Node



t_i = DURATION required to perform activity i

EST_i = earliest possible start for activity i

EFT_i = earliest possible finish for activity i

LST_i = latest possible start for activity i

LFT_i = latest possible finish for activity i

Task 5

Calculate the forward and backward pass

Forword and Backward pass

- ❖ A Forward Pass through the network determines:
 - the **earliest times** each activity can start and finish and also
 - determines the **total duration** of the project.
- ❖ A Backward Pass through the network determines
 - the **latest times** each activity can start and finish **without delaying** completion of the project
 - with this information we can determine where we can **delay activities (have slack)** and where we cannot.

The Forward pass

- ❖ The earliest start(EST) for the initial activity in a project is “time zero”;
- ❖ The EST of an activity is equal to **the latest (or maximum) finish** of the activities directly preceding it;
- ❖ The EFT of an activity is equal to its **EST plus the duration** required to perform the activity.

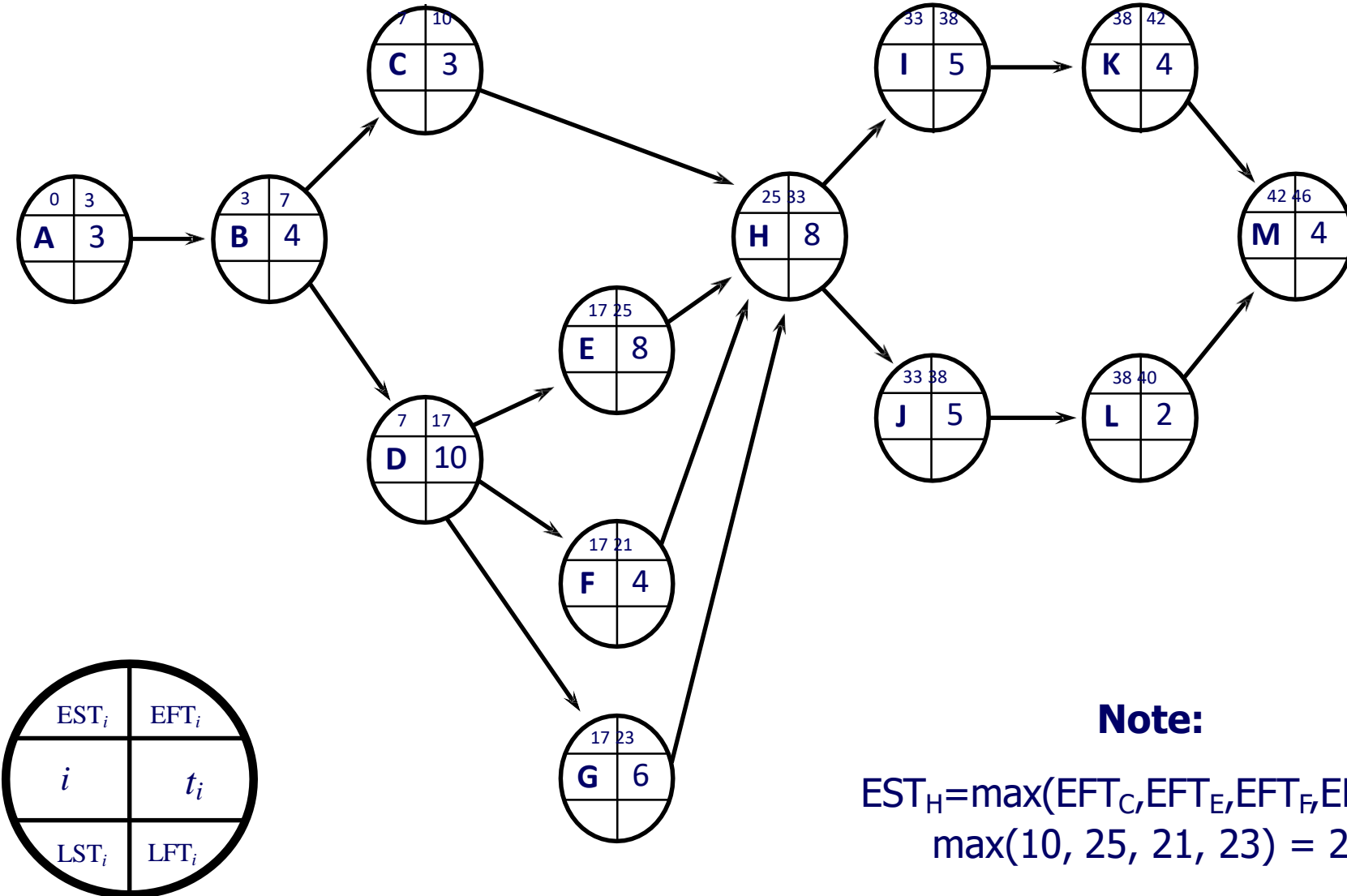
Earliest time

- ❖ Note here that the formal definition of the earliest times is given by:

$$EST_j = \max[EST_i + T_{ij} \mid i \text{ one of the nodes linked to } j \text{ by an arc from } i \text{ to } j]$$

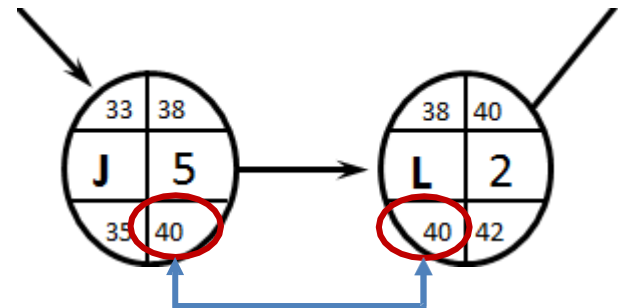
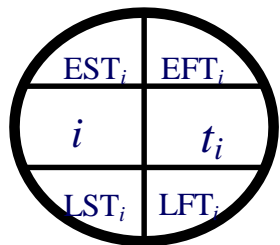
- ❖ This equation for calculating E_j is actually a formal statement of the dynamic programming algorithm for the problem.

Results of the Forward pass



The Backward pass

- ❖ The latest finish (LFT) for the final activity in a project is equal to its EFT as determined by the forward pass;
- ❖ The LFT for any other activity is equal to the earliest (or minimum) LST of the activities directly following (or succeeding) it;
- ❖ The LST of an activity is equal to its LFT minus the time required to perform the activity.



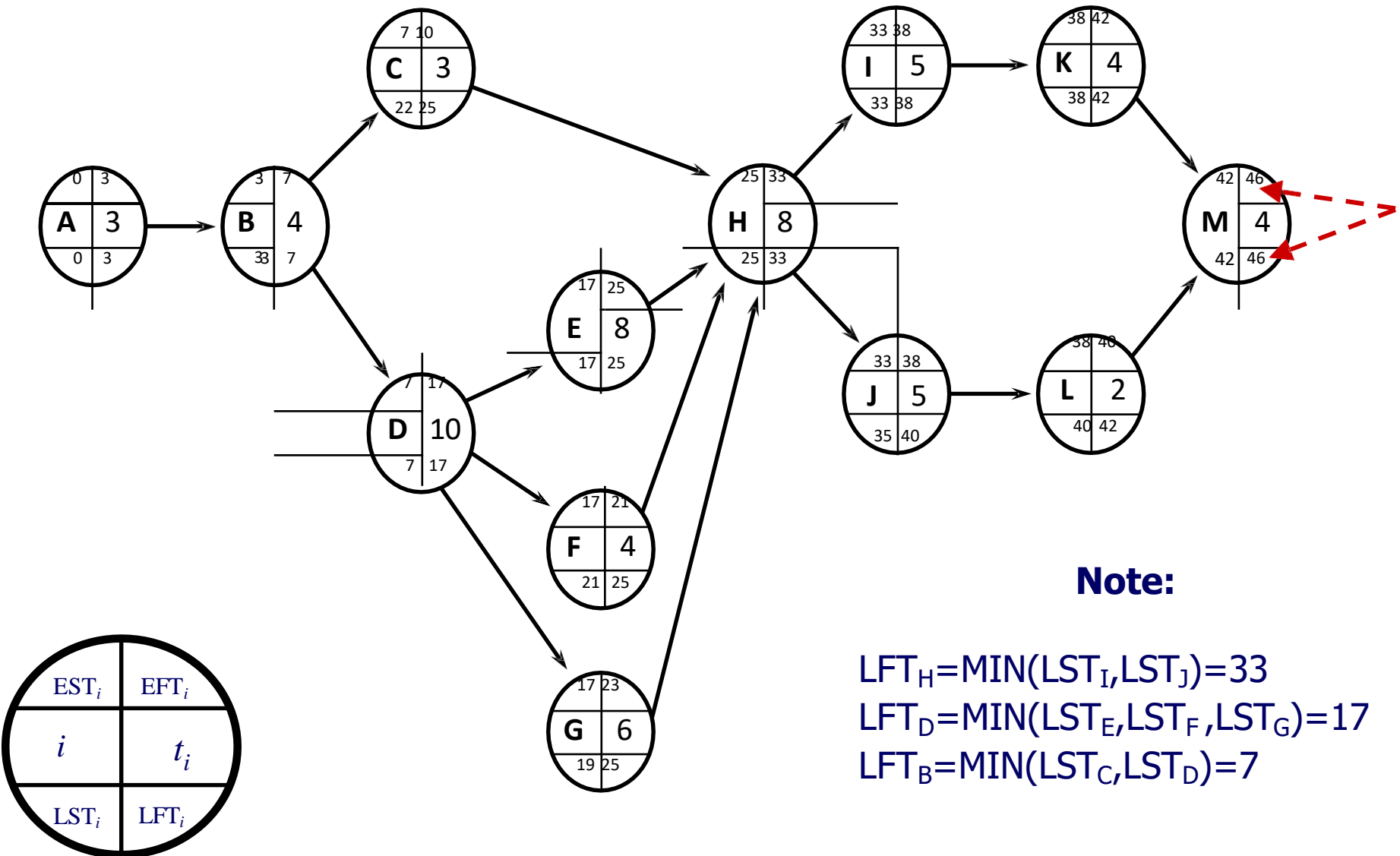
Latest time

- ❖ The formal definition of the latest times is given by:

$$LST_i = \min[L_j - T_{ij} \mid j \text{ one of the nodes linked to } i \text{ by an arc from } i \text{ to } j]$$

- ❖ As a check, that we have done both the earliest start times and latest start times calculations correctly, we must have:
 - all latest start times must be ≥ 0
 - at least one node must have a latest start time of zero.

Results of the Backward pass



Task 6

Determine the critical path

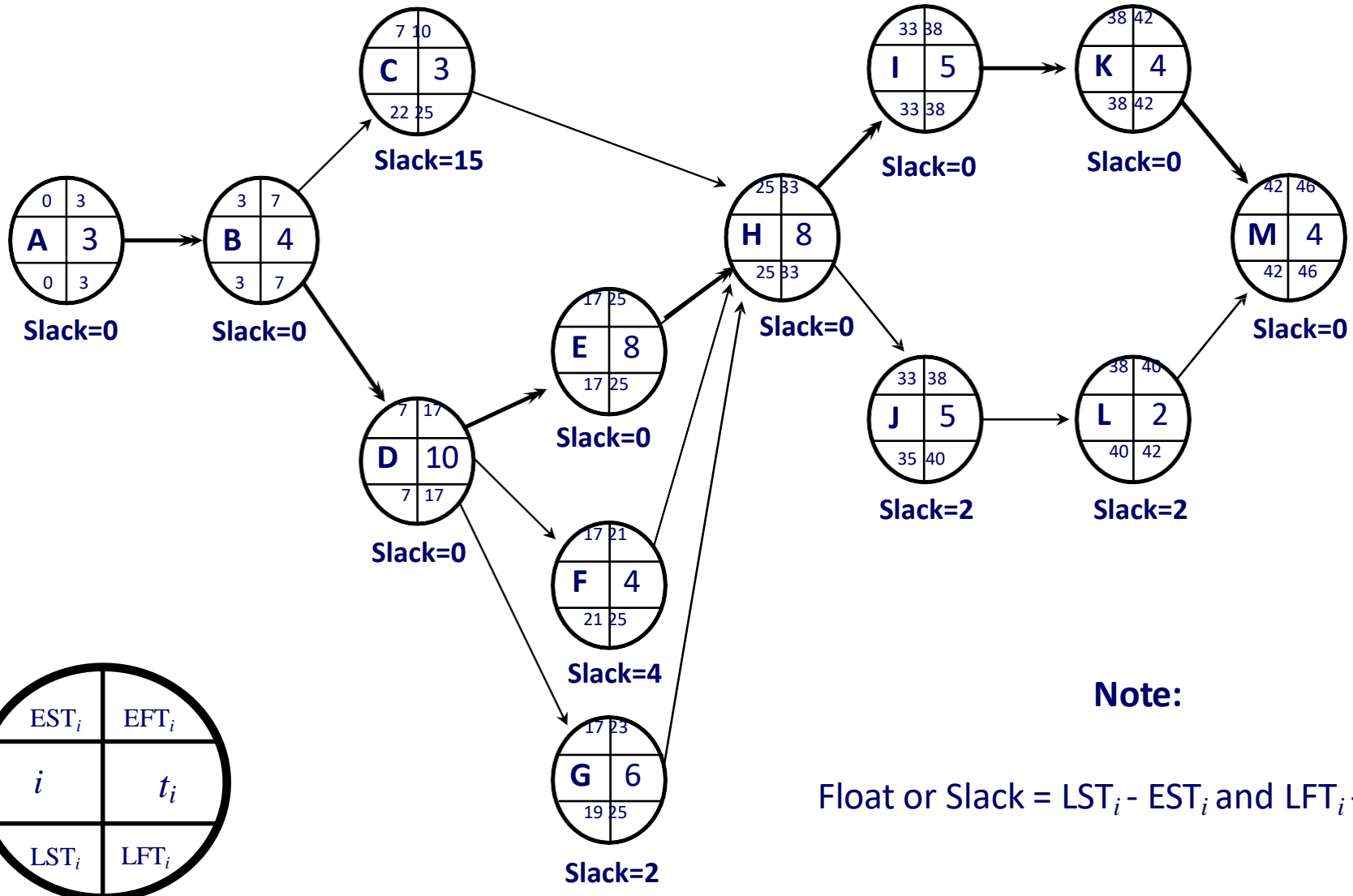
Float or Slack

❖ The amount of *slack or float* time F_i available is given by

$$F_i = LST_i - EST_i$$

which is the amount by which we can increase the time taken to complete *activity i* without *changing* (increasing) the overall *project completion time*.

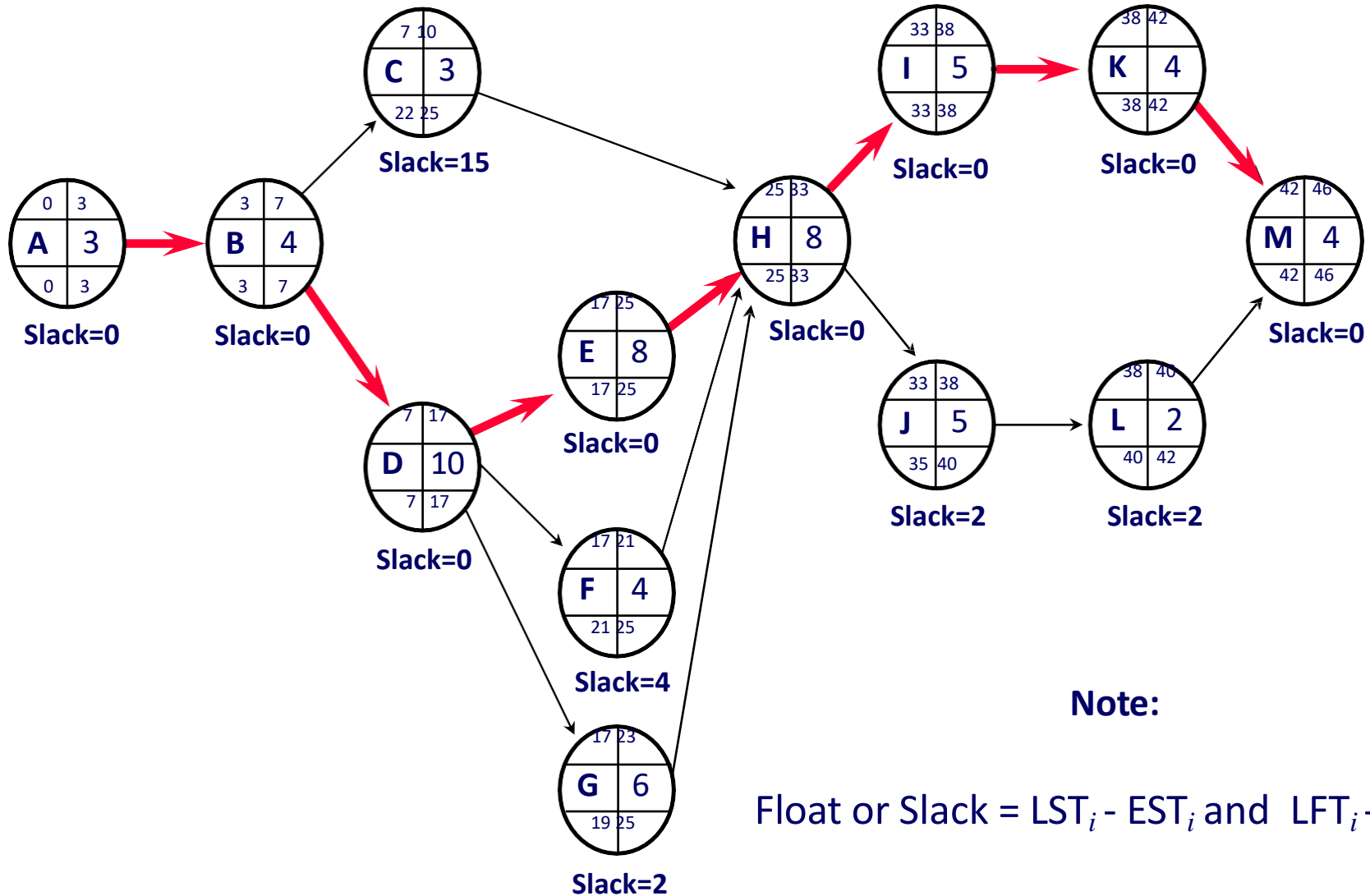
Determining slack



Determining The Critical Path

- ❖ Critical activities have **zero slack** and **cannot be delayed** without delaying the completion of the project;
- ❖ The **slack** for non-critical activities **represents the amount of time** by which the **start** of these activities **can be delayed** without delaying the completion of the entire project (assuming that all predecessor activities start at their earliest start times);
- ❖ The longest path on the network;
- ❖ Could also be those activities with the least slack.

The Critical Path



Q & A

