Management of ICT projects

PERT diagrams & Probabilities

Evangelia Maniadi

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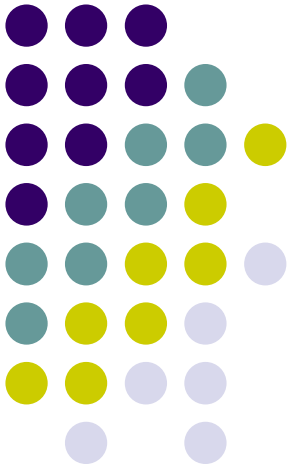


PERT Basics

- At the core, PERT is all about management probabilities. Therefore, PERT involves in many simple statistical methods as well.
- Sometimes, people categorize and put PERT and CPM together. Although CPM (Critical Path Method) shares some characteristics with PERT, PERT has a different focus.

Estimating Probabilities

- There are three estimation times involved in PERT:
  - Optimistic Time Estimation (T_{opt}),
  - Most Likely Time Estimation (T_{prob}, and
  - Pessimistic Time Estimation (T_{pess}).
- In PERT, these three estimated times are derived for each activity.
- This way, a range of time is given for each activity with the most probable value, T_{LIKELY}.

PERT Mathematics

- The expected completion time (E) of each activity is calculated as below:
  - $E = (T_{opt} + 4 \times T_{prob} + T_{pess}) / 6$
- At the same time, the possible variance (V) of the estimated time of each activity is calculated as below:
  - $V = (T_{pess} - T_{opt})^2 / 6^2$

PERT Mathematics

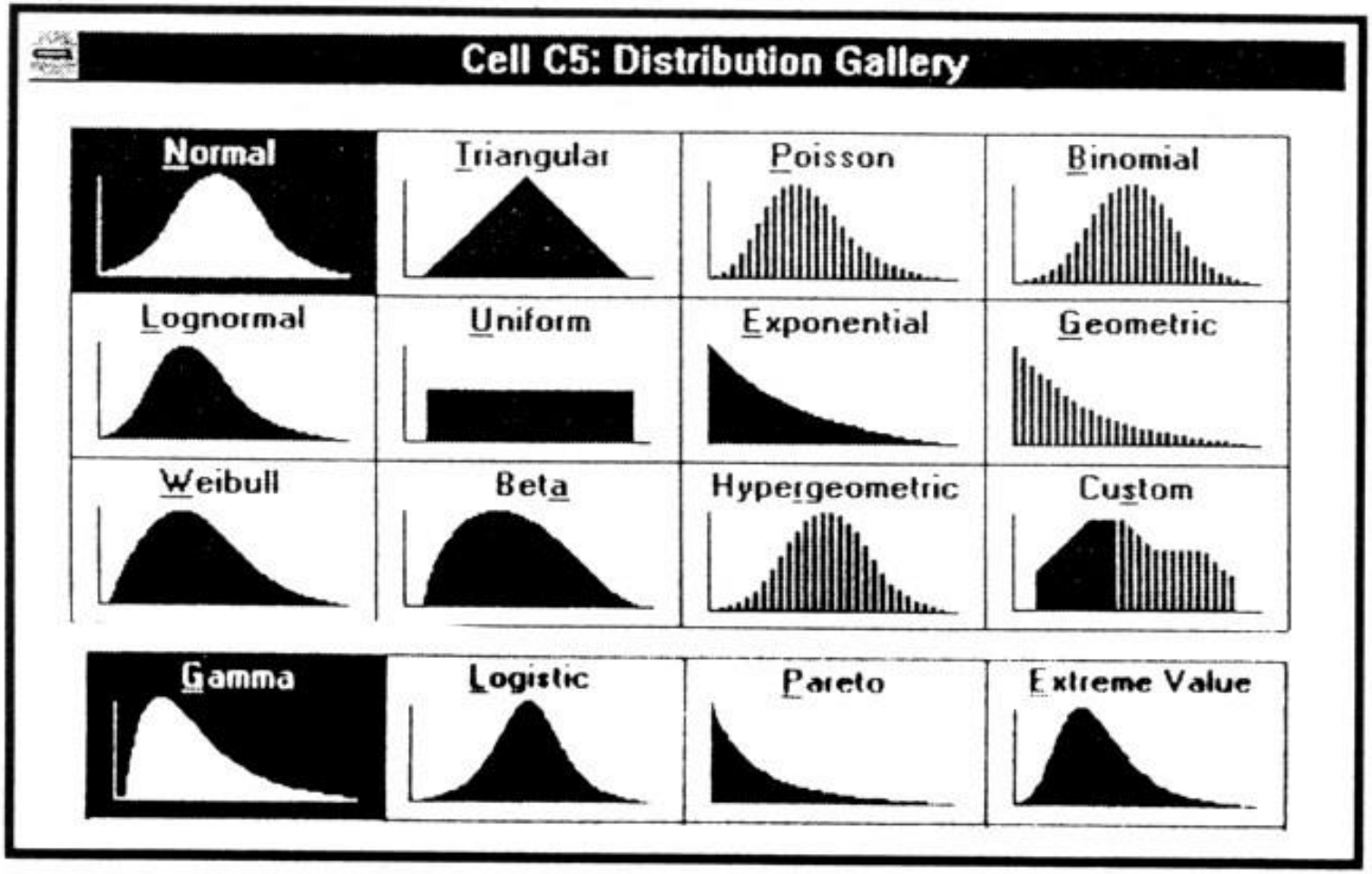
- Now, following is the process we follow with the two values:
  - For every activity in the critical path, E and V are calculated.
  - Then, the total of all Es are taken. This is the overall expected completion time for the project.

PERT Mathematics

- Now, the corresponding V is added to each activity of the critical path. This is the variance for the entire project. This is done only for the activities in the critical path as only the critical path activities can accelerate or delay the project duration.
- Then, standard deviation of the project is calculated. This equals to the square root of the variance (V).
- Now, the normal probability distribution is used for calculating the project completion time with the desired probability.

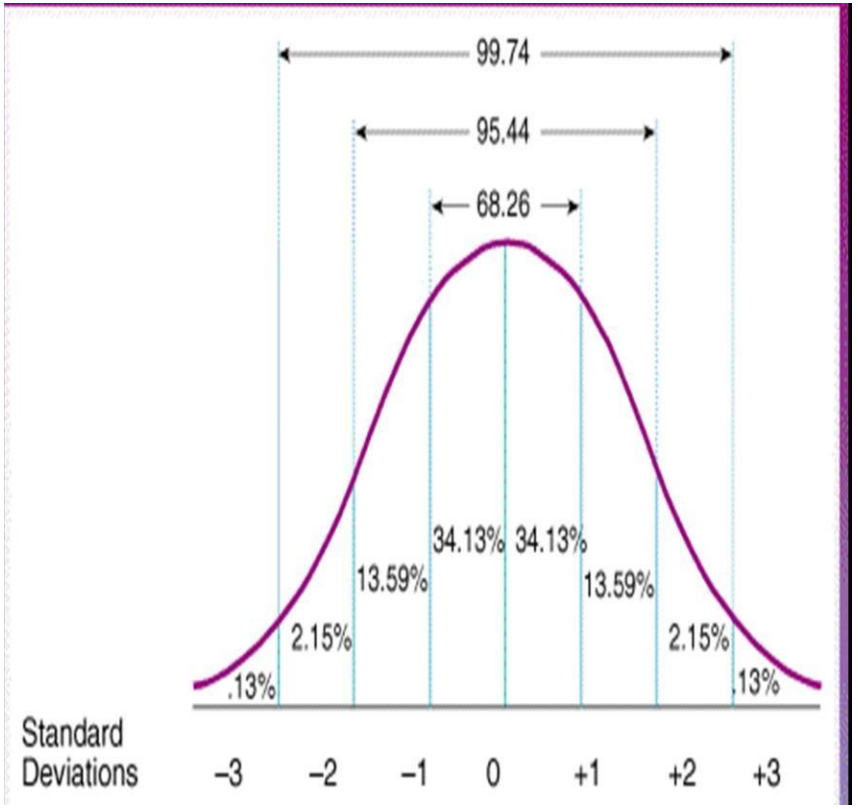
Probability and Distributions

Figure 1a



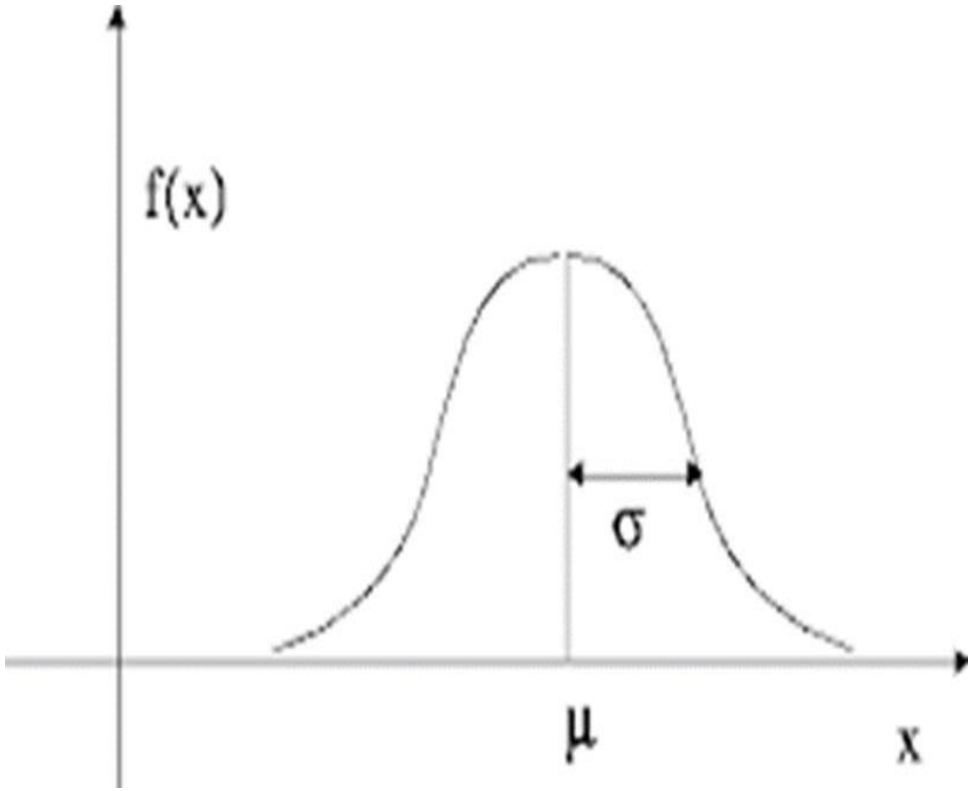
Normal distribution

- In probability theory, the normal distribution is a very common continuous probability distribution. Normal distributions are important in statistics and are often used in the natural and social sciences to represent real-valued random variables whose distributions are not known.



Normal distribution

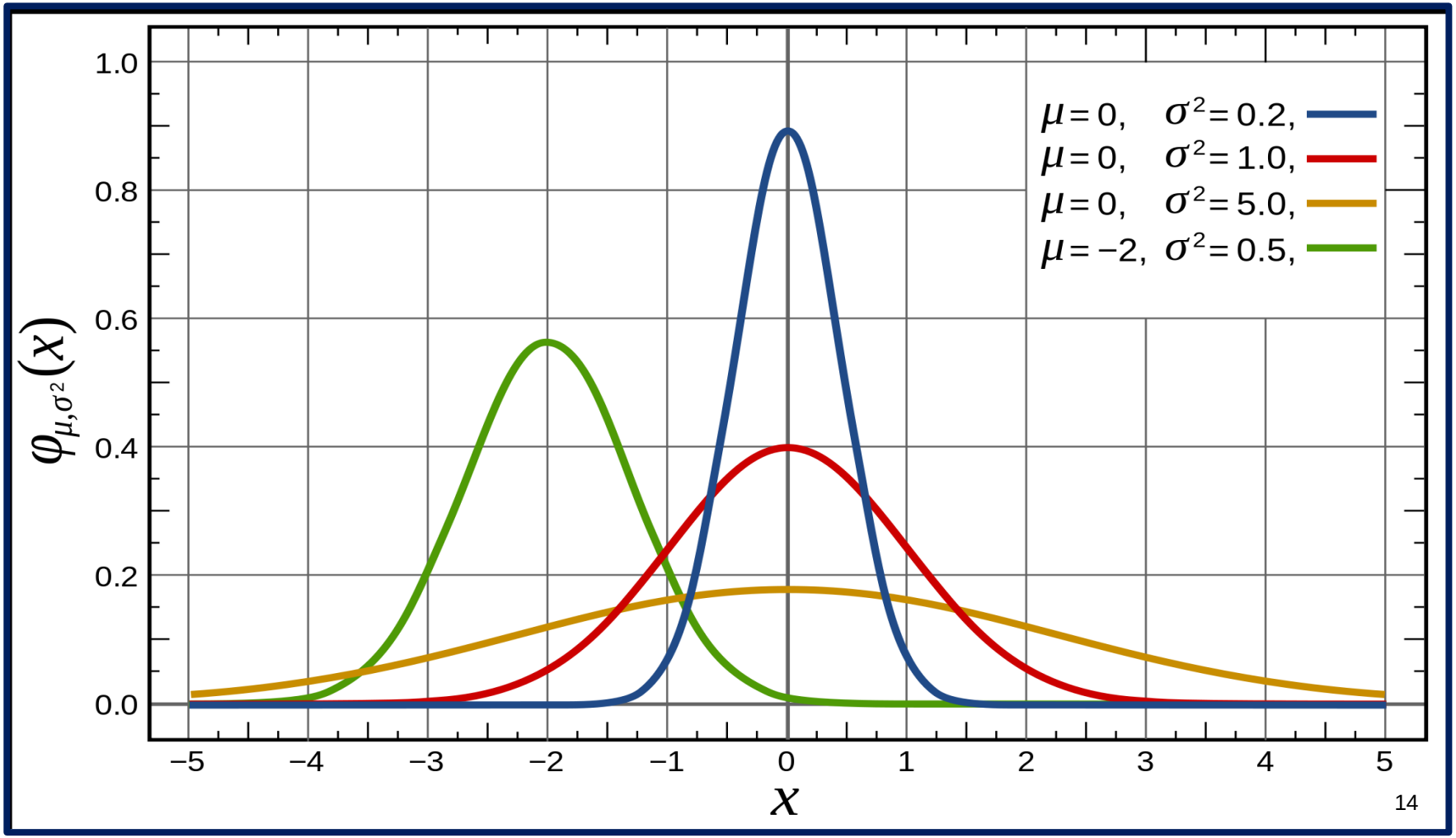
- A normal distribution is determined by two things:
  - the mean, **μ**, and
  - the standard deviation, **σ**, which is a measure that is used to quantify the amount of variation or dispersion of a set of data values



(Standard Deviation)

σ = √ (variance)

Normal distribution - Examples



PERT - Example

(Activity)	(Immediate Predecessor)	(Optimistic) (T_{opt})	(Most Probable) (T_{prob})	(Pessimistic) (T_{pess})
A	-	4	5	12
B	A	3	4.5	15
C	A	2	3	4
D	C	6	8	22
E	B	4	6	8
F	C	3	4	5
G	D,E	1.5	3	4.5
H	B	5	7	15
I	H	3	4	5
J	G,I	2	4	6

What is the probability that the project will be completed within 29 weeks;

Step 1: Calculating the estimated time

- To proceed on solving the problem we must calculate the "Expected time" of the completion for each activity.

- The calculation of the expected Time is given by the equation:

$$E_i = (T_{opt} + 4 \times T_{prob} + T_{pess}) / 6$$

- Where “i” is the activity

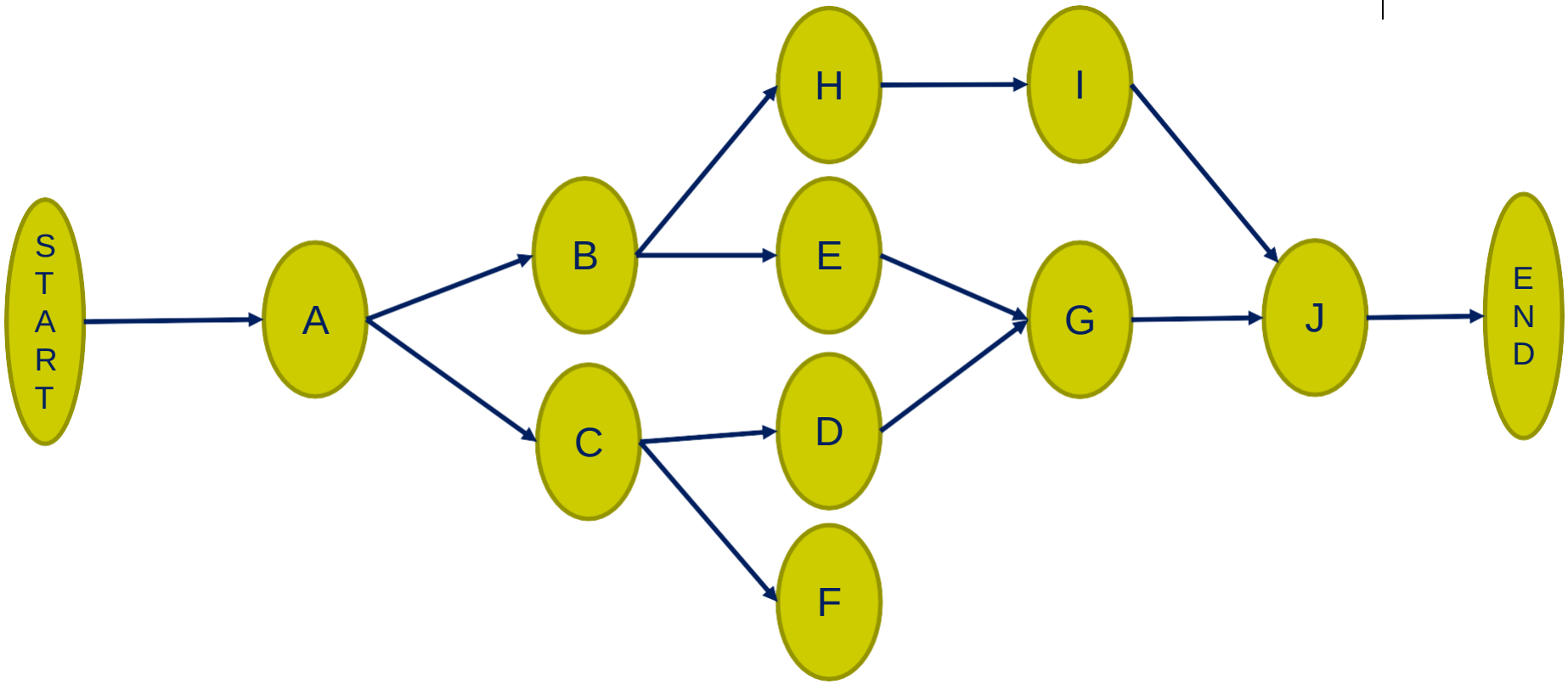
Calculating the estimated time

(Activity)	(Immediate Predecessor)	(T_{opt})	(T_{prob})	(T_{pess})	
A	-	4	5	12	$t_A = [T_{opt} + 4* T_{prob} + T_{pess}] /6$ $= [4 +(4*5) + 12/6]$ $= [4 + 20 + 12] / 6$ $= 36/6$ $= 6$
B	A	3	4.5	15	
C	A	2	3	4	
D	C	6	8	22	
E	B	4	6	8	
F	C	3	4	5	
G	D,E	1.5	3	4.5	
H	B	5	7	15	
I	H	3	4	5	
J	G,I	2	4	6	

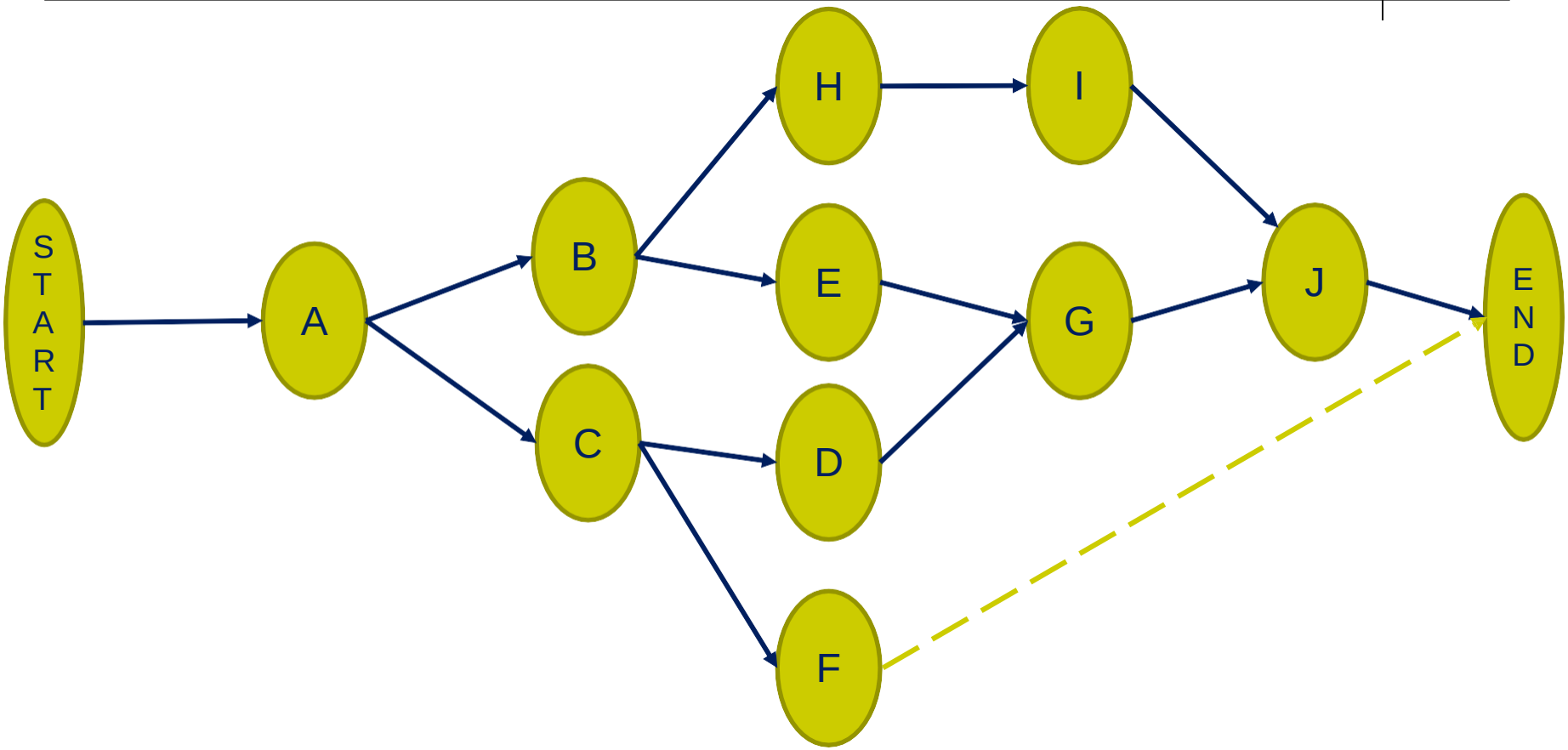
Calculating the estimated time

(Activity)	(Immediate Predecessors)	(T_{opt})	(T_{prob})	(T_{pess})	
A	-	4	5	12	$t_A = [T_{opt} + 4 * T_{prob} + T_{pess}] /6$ $= [4 +(4*5) + 12/6]$ $= [4 + 20 + 12] / 6$ $= 36/6$ $= 6$
B	A	3	4.5	15	$t_B = 6$
C	A	2	3	4	$t_C = 3$
D	C	6	8	22	$t_D = 10$
E	B	4	6	8	$t_E = 6$
F	C	3	4	5	$t_F = 4$
G	D,E	1.5	3	4.5	$t_G = 3$
H	B	5	7	15	$t_H = 8$
I	H	3	4	5	$t_I = 4$
J	G,I	2	4	6	$t_J = 4$

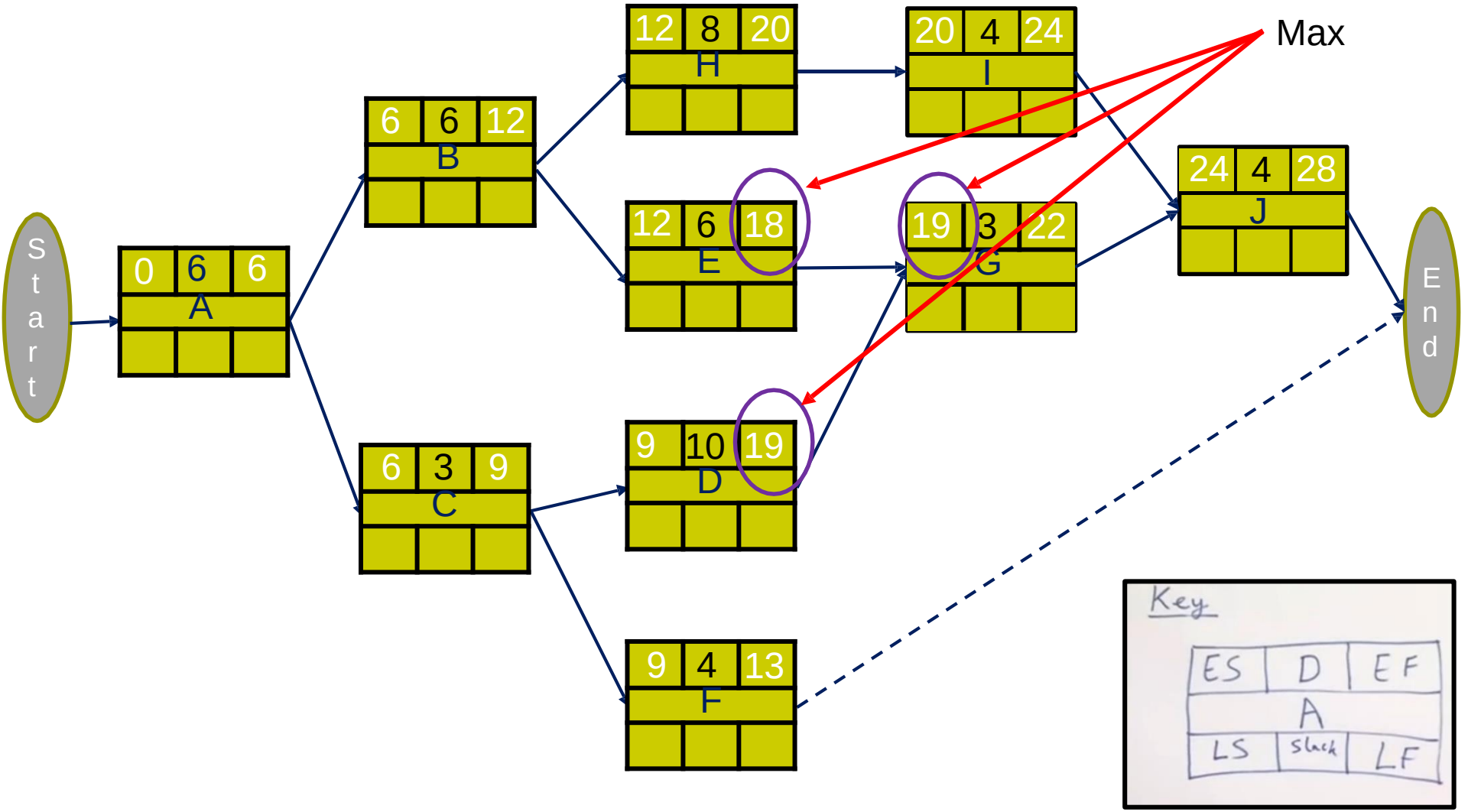
Step 2: Creating the network diagram



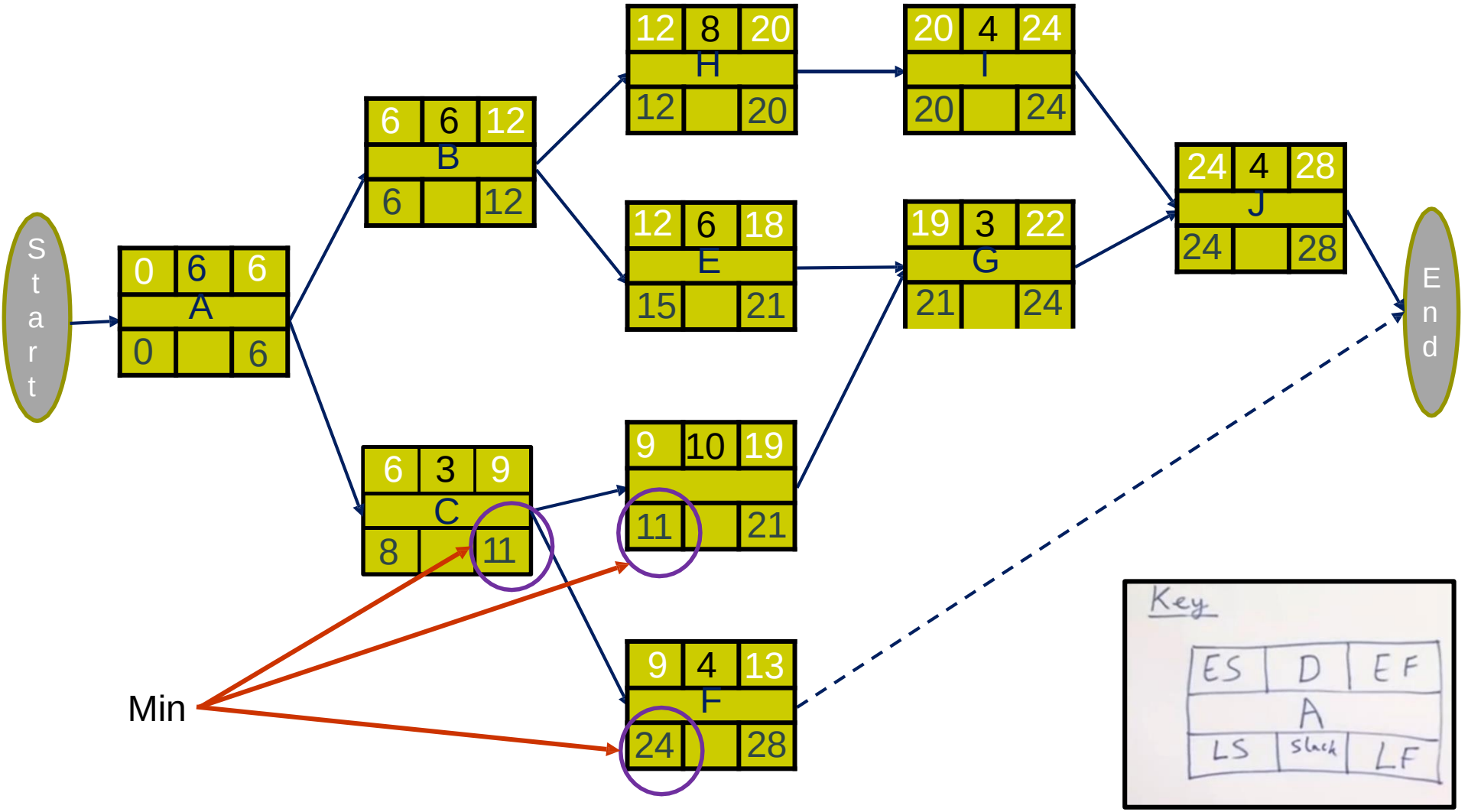
Creating the network diagram



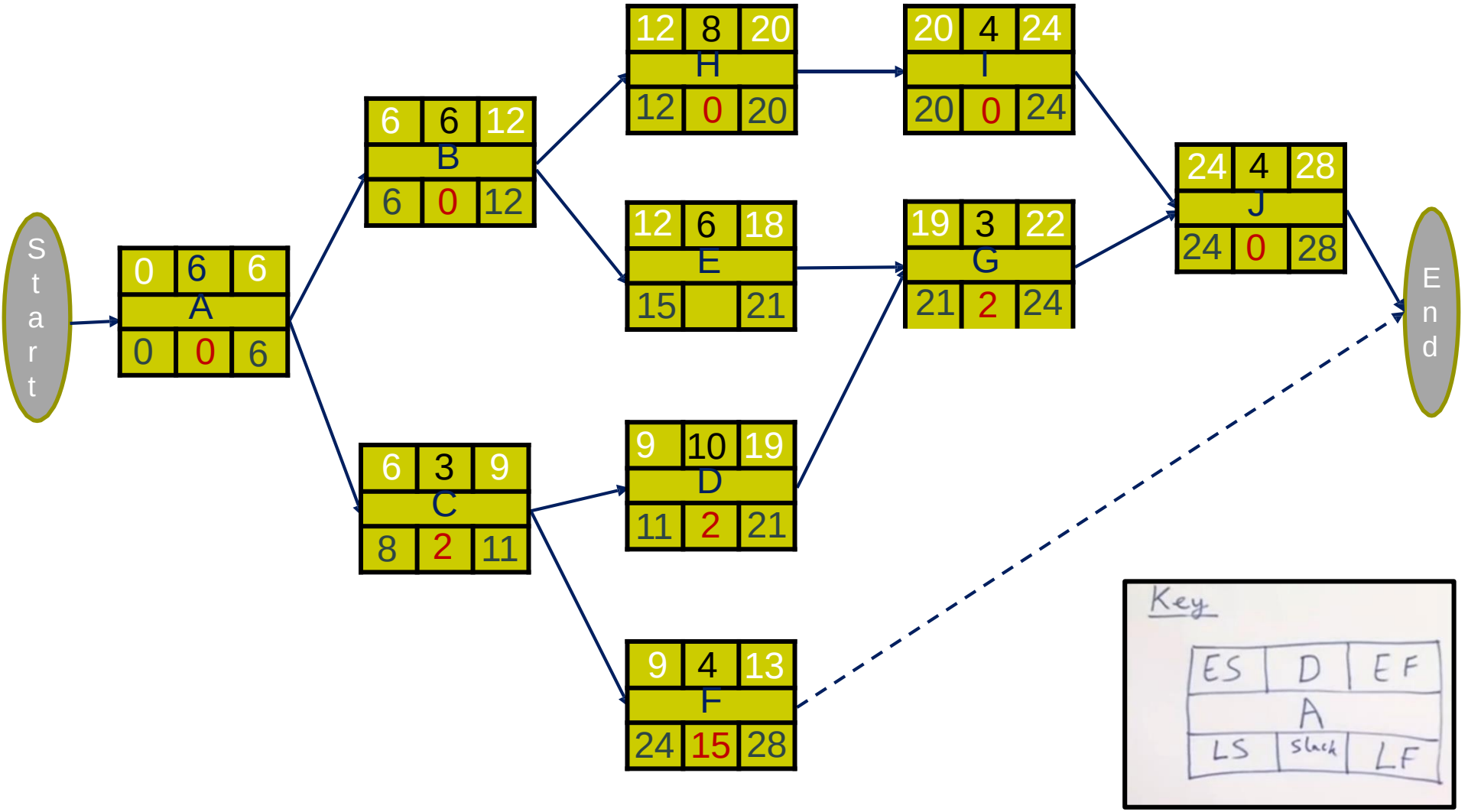
Step 3: Identifying the critical path (forward pass)



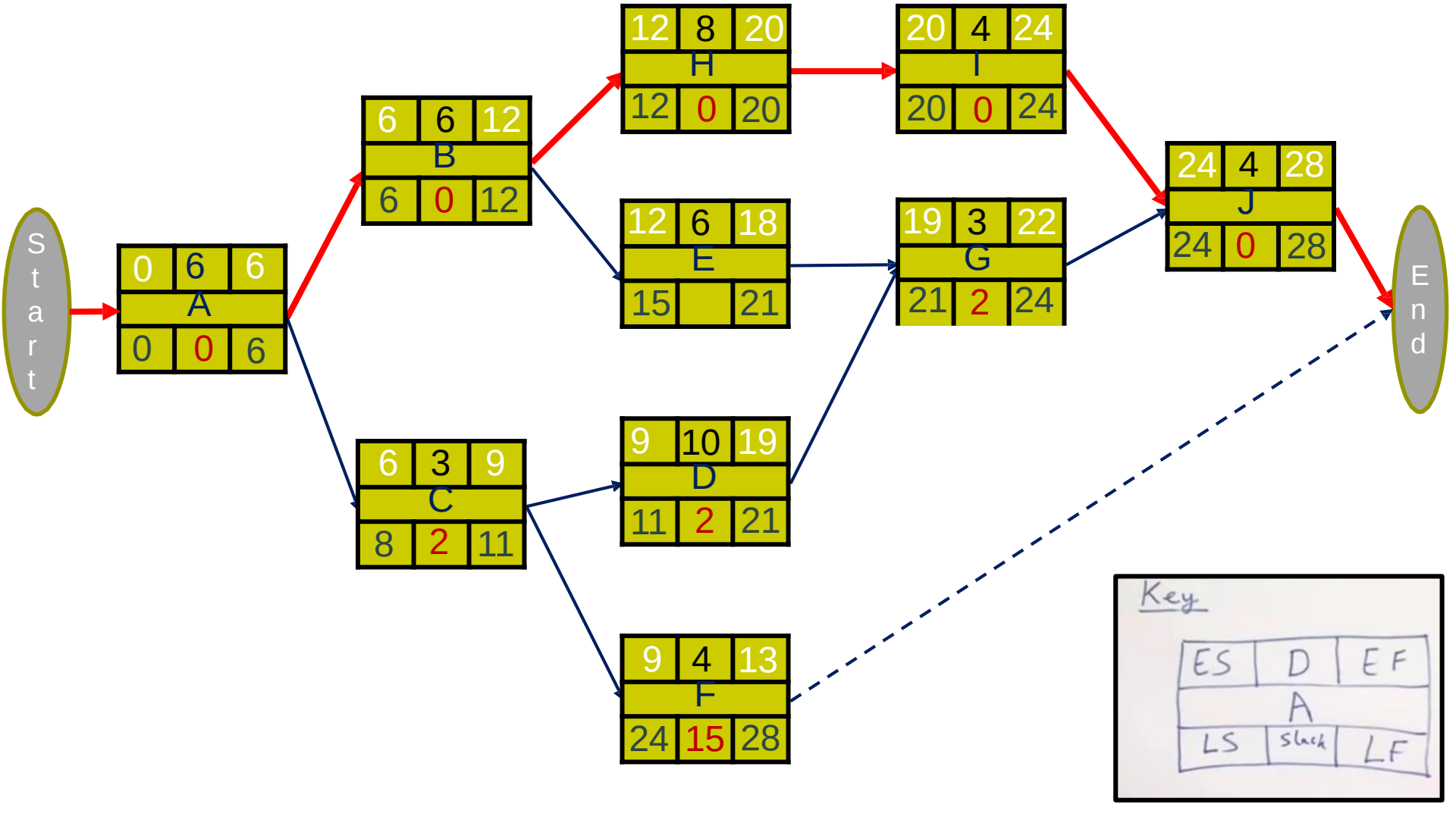
Identifying the critical path (backward pass)



Identifying the critical path (slack)



Critical Path



Step 4: Calculating the expected completion time of the project

- To calculate the total expected completion time of the project we have to calculate the expected completion time of each critical activity add the times of these activities.

Project's critical path is: A, B, H, I, J.

Total expected completion time:
= 6 + 6 + 8 + 4 + 4 = 28 weeks

Step 5: Calculating the project variance

To calculate the project variance we need to calculate the variance of each critical activity

$$V = (T_{pess} - T_{opt})^2 / 6^2$$

Then the project variance is given by the sum of the variances along the critical path

$V_A = (T_{pess} - T_{opt})^2 / 6^2 = 1.78$

$V_B = (T_{pess} - T_{opt})^2 / 6^2 = 4$

$V_H = (T_{pess} - T_{opt})^2 / 6^2 = 2.78$

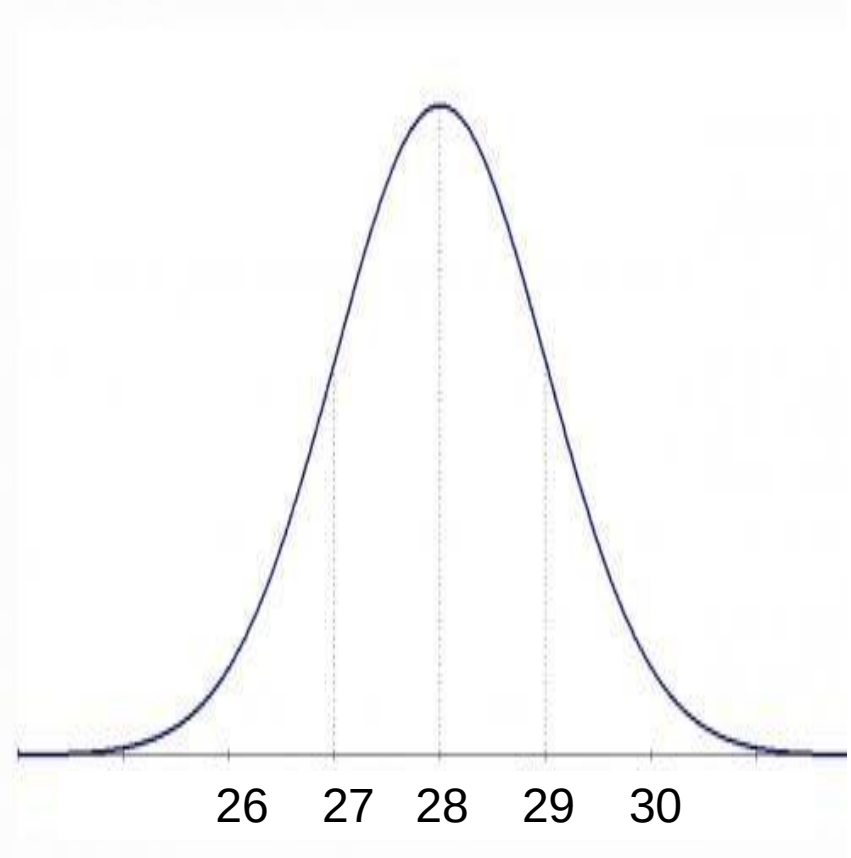
$V_I = (T_{pess} - T_{opt})^2 / 6^2 = 0.11$

$V_I = (T_{pess} - T_{opt})^2 / 6^2 = 0.44$

Project variance $\sigma_\rho^2 = 1.78 + 4 + 2.78 + 0.11 + 0.44 = 9.11$

Question

- What is the probability that the project will be completed within 29 weeks;



Step 6: Calculating the Z - score

- To answer that we need to calculate the z-score.

$$Z = (\text{Due Date} - \text{Expected Date}) / \sigma_p$$

thus, we must first calculate the standard deviation

$\sigma_p = \sqrt{9,1} = 3.01$

- $Z = (\text{Due Date} - \text{Expected Date}) / \sigma_p$
  $Z = (29-28) / 3.01 = 0.33$

- Using the normal distribution table, we are sure that the project will be finished till the 29^{th} day with 62.93% probability.

Normal distribution table

Tables of the Normal Distribution

Probability Content from -oo to Z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990