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Mathematics and Visualization Apps

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HMU Teaching Science

Pre-Calculus Course, iTEM Project

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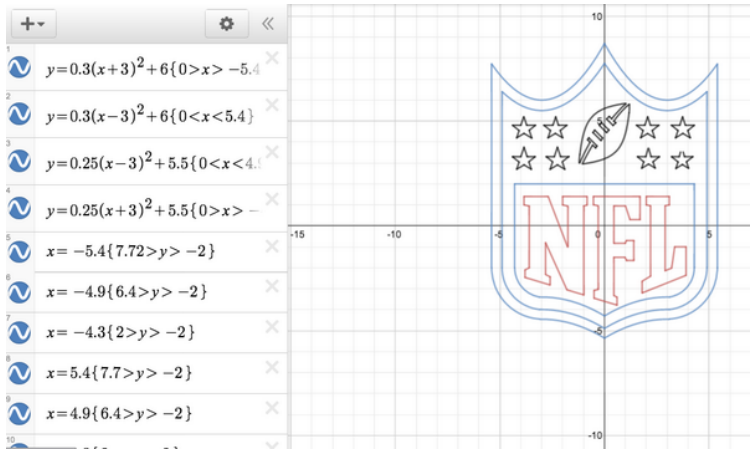

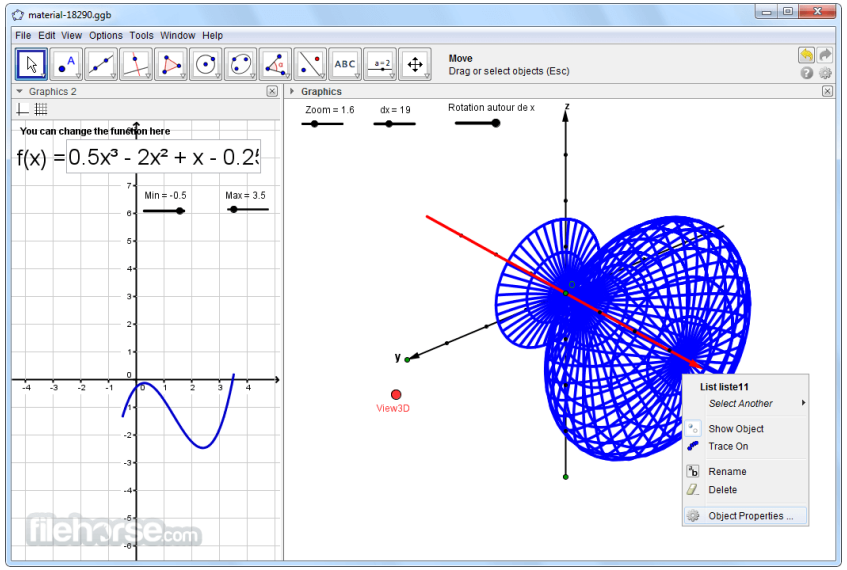

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Math Basics - Addition

Addition means joining things together. It can also be defined as Combining collections of objects together into a larger collection is Addition.It is represented by the '**+**' sign.

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Math Basics - Addition

Example

I have 3 books and my dad gave me 2 more. So now how many books do I have?

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Math Basics - Addition

Answer

I have 5 books in all.

Explanation

3 + 2 = 5

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Math Basics - Substraction

Subtraction means taking things away from a group. We can subtract the integers using the number line. Add a positive integer by moving to the right on number line

Or when we take away a given number of objects from a given collection of objects is Subtraction.

It is represented by the ‘- ’ sign.

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Math Basics - Multiplication

Adding a certain number of objects again and again is

Multiplication. Multiplication means Counting by number. Counting by 2's means 2, 4, 6, 8... and so on Similarly Counting by 5's means 5, 10, 15, 20...and so on

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Math Basics - Division

Subtracting a certain number of objects again and again is division.

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Math Basics – Order of Operations

What is the correct order of operations? What is the correct order of operations?

1st Parenthesis

2nd Exponents

3rd Multiplication and Division

4th Addition and Substruction

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Math Basics – Problem

A student in 5 lessons has an average of 7. If in 6th lesson writes 9 what is the new average?

In the 5 lessons there is a sum of 5x7 = 35 and with the 6th the sum becomes 35 + 9 = 44. So the new average is $n_{aver} = \frac{44}{6} = 7.33, (n_{aver} = new\ average)$

A common mistake

$$n_{aver} = \frac{7+9}{2} = 8$$

Why?

because it doesn't have 2 lessons

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Math Basics – Properties of Exponents

What are the properties of exponents?

Product of a Power: When you multiply exponentials with the same base, you add their **exponents** (or powers). Power to a Power: When you have a power to a power, you multiply the **exponents** (or powers). Quotient of Powers: When you divide exponentials with the same base, you subtract the **exponents** (or powers).

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Math Basics – Properties of Exponents

What is an exponent?

The base a raised to the power of n is equal to the multiplication of a, n times:

$$a^n = a \times a \times \ldots \times a$$

Examples

$3^1 = 3$ | $3^2 = 3 \times 3 = 9$ | $3^3 = 3 \times 3 \times 3 = 27$ | $3^4 = 3 \times 3 \times 3 \times 3 = 81$ | $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

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Math Basics – Properties of Exponents

Rule name	Rule	Example
Product rules	$a^n \cdot a^m = a^{n+m}$	$2^3 \cdot 2^4 = 2^{3+4} = 128$
	$a^n \cdot b^n = (a \cdot b)^n$	$3^2 \cdot 4^2 = (3 \cdot 4)^2 = 144$
Quotient rules	$a^n / a^m = a^{n-m}$	$2^5 / 2^3 = 2^{5-3} = 4$
	$a^n / b^n = (a / b)^n$	$4^3 / 2^3 = (4/2)^3 = 8$
Power rules	$(b^n)^m = b^{n \cdot m}$	$(2^3)^2 = 2^{3 \cdot 2} = 64$
	${}_b n^m = {}_b(n^m)$	${}_2 3^2 = {}_2(3^2) = 512$
Negative exponents	$b^{-n} = 1 / b^n$	$2^{-3} = 1/2^3 = 0.125$
Zero rules	$b^0 = 1$	$5^0 = 1$
	$0^n = 0$, for $n>0$	$0^5 = 0$
One rules	$b^1 = b$	$5^1 = 5$
	$1^n = 1$	$1^5 = 1$

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Math Basics – Properties of Exponents

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Math Basics – Properties of Exponents

Solve this

$$\frac{2\times 3^2+4\times(5^2-17)^2}{2^3-4\times 5} = ???$$

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Math Basics – Properties of Exponents

Solution

$$\frac{2\times3^2+4\times(5^2-17)^2}{2^3-4\times5}=\frac{2\times9+4\times(25-17)^2}{8-4\times5}$$

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Math Basics – Properties of Exponents

Solution

$$\frac{2\times3^2+4\times(5^2-17)^2}{2^3-4\times5}=\frac{2\times9+4\times(25-17)^2}{8-4\times5}=\frac{2\times9+4\times8^2}{8-4\times5}$$

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Math Basics – Properties of Exponents

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$$-19.6$$

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Math Basics – Properties of Exponents

Find the values of the following numerical operations

a. $2^3 \times (3^2 - 3^1) - 4 \times [(2^3 + 3^2) \times 2 - 3^3]$

b. $8^2 - [4 \times (3^2 + 5) + 6 \times 5 - 2^3 \times 3^2]$

c. $10^3 - 10 \times [2 \times (7^2 - 1^8) - 3 \times 2^4 + 2^5]$

d. $\dfrac{3\times(2^3-1)+9+2\times(4\times3^2-11)}{(5-2^2)\times(3\times2^3-2\times3^2)+7\times(3^2-3)}$

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Math Basics – Multiplication of Polynomials

Multiplication of a polynomial by a polynomial

to calculate the result of polynomials apply the property

$$(a+b)\times(c+d)=(a+b)\times c+(a+b)\times d=a\times c+b\times c+a\times d+b\times d$$

$$=a\times c+a\times d+b\times c+b\times d$$

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Math Basics – Multiplication of Polynomials

Example1

$$(x-2y)(x^2-3xy+4y^2)$$

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Math Basics – Multiplication of Polynomials

Example2

$$x^2(3x^2 - 5x + 6) - (x^2 - 4x)(x^2 + 2)$$

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Math Basics – Addition and removal of homonymous fractions

Homonyms called fractions with the same denominator

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b} \text{ and } \frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$$

Find the solution

$$\frac{x^2-5x}{2x^2-8}+\frac{x+4}{2x^2-8}$$

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Visualization Apps - GeoGebra

GeoGebra is dynamic mathematics software for all levels of education that brings together geometry, algebra, spreadsheets, graphing, statistics and calculus in one easy-to-use package. GeoGebra is a rapidly expanding community of millions of users located in just about every country.

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Visualization Apps - GeoGebra

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- 3D Calculator
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- GeoGebra Classic 5

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Visualization Apps - Desmos

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Visualization Apps - GeoGebra

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