

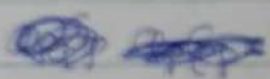
$t = 0 \Rightarrow T = 200^\circ \text{C}$ ομοιογενώς

$k = 10 \text{ W/mK}$, $\rho c = 10 \cdot 10^6 \text{ J/m}^3 \text{K}$

Κυβερνήτης εφάρμογης: $\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(\frac{k}{\rho c} \frac{\partial T}{\partial x} \right) \Leftrightarrow$

$\Leftrightarrow \rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right)$

Διακριτική με πρόοχημα



Ενδομέσος υποβλη 2-4:

$a_p T_p = a_w T_w + a_e T_e + (a_p - (a_w + a_e)) T_p^0 + S_{ut} + S_{pbc} \quad (1)$

όπου: $a_w = a_e = \frac{k}{\Delta x} = \frac{10}{0.001} = 2500$ $S_u = S_p = 0$ για την.

$a_p = a_p^0 = \frac{\rho c \Delta x}{\Delta t} = 10 \cdot 10^6 \cdot \frac{0.001}{\Delta t} = \frac{40000}{\Delta t}$ $S_{up} = S_{pbc} = 0$

σημασία έχει όταν: $\Delta t < \frac{\rho c \Delta x^2}{2\gamma} = 10 \cdot 10^6 \cdot \frac{16 \cdot 10^{-6}}{20} = 8 \text{ s}$

εδώ έχουμε $\Delta t = 2 \text{ s} \Rightarrow a_p = a_p^0 = 20000$

$(1) \Rightarrow 20000 T_p = 2500 T_w + 2500 T_e + (20000 - 5000) T_p^0 \Leftrightarrow$

$\Leftrightarrow 20000 T_p = 2500 T_w + 2500 T_e + 15000 T_p^0$

Καθώς 1 δυνάμει ορρο $\frac{\partial T}{\partial x} = 0$

$$a_e = \frac{V}{\Delta x} = \frac{10}{0.004} = 2500, a_w = 0$$

$$a_p = a_p^o = 20000$$

$$s_u = s_p = 0$$

$$s_{p,BC} = 0, s_{u,BC} = 0$$

$$(1) \Rightarrow 20000 T_p = \cancel{25000} 2500 T_e^o + (20000 - (2500 - 0)) T_p^o \Leftrightarrow$$

$$\Leftrightarrow 20000 T_p = 2500 T_e^o + 17500 T_p^o \Leftrightarrow$$

$$\Leftrightarrow 200 T_p = 25 T_e^o + 175 T_p^o$$

Καθώς 5 ανατολική ορρο $T_B = 0$

$$a_e = 0, a_w = 2500, s_u = s_p = 0,$$

$$s_{p,BC} = -\frac{1}{R} = -\frac{1}{\cancel{\frac{1}{2K}} \frac{2K}{\Delta x}} = -\frac{2K}{\Delta x} = -\frac{20}{0.004} = -5000$$

$$s_{u,BC} = T_B/R = 0$$

$$(1) \Rightarrow 20000 T_p = 2500 T_w^o + (20000 - (2500 + 5000)) T_p^o \Leftrightarrow$$

$$\Leftrightarrow 20000 T_p = 2500 T_w^o + 12500 T_p^o \Leftrightarrow$$

$$\Leftrightarrow 200 T_p = 25 T_w^o + 125 T_p^o$$

Αλγεβρικό σύστημα

$$\begin{aligned} 200T_1 + 0T_2 + 0T_3 + 0T_4 + 0T_5 &= 25T_2^0 + 175T_1^0 \\ 0T_1 + 200T_2 + 0T_3 + 0T_4 + 0T_5 &= 25T_1^0 + 25T_3^0 + 150T_2^0 \\ 0T_1 + 0T_2 + 200T_3 + 0T_4 + 0T_5 &= 25T_2^0 + 25T_4^0 + 150T_3^0 \\ 0T_1 + 0T_2 + 0T_3 + 200T_4 + 0T_5 &= 25T_2^0 + 25T_5^0 + 150T_4^0 \\ 0T_1 + 0T_2 + 0T_3 + 0T_4 + 200T_5 &= 25T_4^0 + 125T_5^0 \end{aligned}$$

Διαφάνισμα με χωροχρόνο Gxipca

~~Ενδογενής χωροχρόνος~~

$$q_{pp} = a_w q_w + a_e q_e + a_p q_p^0 + S_u + S_{u,BC} \quad (2)$$

από $a_w = \frac{K}{\Delta x} = a_e = 2500$

$$a_p = a_p^0 + a_e + a_w - S_p - S_{p,BC}$$

$$a_p^0 = \rho c \frac{\Delta x}{\Delta t}$$

Εισαγωγή νέου οπώ. Εξωτερικοί κοίλι $\Delta t = 2s$.

Ενδογενής χωροχρόνος 2-4

$$a_w = a_e = 2500$$

$$S_u = S_p = S_{u,BC} = S_{p,BC} = 0$$

$$a_p^0 = \rho c \frac{\Delta x}{\Delta t} = 20000$$

$$a_p = 20000 + 5000 = 25000$$

$$(2) \Rightarrow 25000 T_p = 2500 T_e + 2500 T_w + 20000 T_p^0 \Leftrightarrow$$

$$\Leftrightarrow 250 T_p = 25 T_w + 25 T_e + 200 T_p^0$$

4ο οπώ - Κόρυφος 1

$$a_w = 0, S_{p,BC} = 0 = S_{u,BC}, a_p = 20000 + 2500 = 22500$$

$$(2) \Rightarrow 225 T_p = 25 T_e + 200 T_p^0$$

Ανταρτικό οπώ - Κόρυφος 5

$$a_e = 0, a_w = 2500, S_{p,BC} = -5000, S_{u,BC} = 0, a_p = 20000 + 2500 + 5000 = 27500$$

$$(2) \Rightarrow 275 T_p = 25 T_w + 200 T_p^0$$

Αλγεβρικές εξισώσεις

1 ερώτηση

$$\begin{aligned} 225T_1 - 25T_2 + 0T_3 + 0T_4 + 0T_5 &= 200T_1^o \\ -25T_1 + 250T_2 - 25T_3 + 0T_4 + 0T_5 &= 200T_2^o \\ 0T_1 - 25T_2 + 250T_3 - 25T_4 + 0T_5 &= 200T_3^o \\ 0T_1 + 0T_2 - 25T_3 + 250T_4 - 25T_5 &= 200T_4^o \\ 0T_1 + 0T_2 + 0T_3 - 25T_4 + 275T_5 &= 200T_5^o \end{aligned}$$

Αναμετρώμεν με το σχήμα Crank-Nicolson

$$a_p \phi_p = a_w \frac{\phi_w + \phi_w^o}{2} + a_e \frac{\phi_e + \phi_e^o}{2} + \left[a_p^o - \frac{1}{2}(a_e + a_w - S_p - S_{p,BC}) \right] \phi_p^o + S_u + S_{u,BC}$$

$$\text{όπου } a_w = \Gamma / \Delta x_{wp}, \quad a_e = \Gamma / \Delta x_{ep}$$

$$a_p = a_p^o + \frac{1}{2}(a_e + a_w - S_p - S_{p,BC})$$

$$a_p^o = \rho \frac{\Delta x}{\Delta t} \Rightarrow a_p^o = \rho C \frac{\Delta x}{\Delta t}$$

$$\text{Σχίσμα Σωθών Ευαίσθητο: } \rho C \frac{\Delta x}{\Delta t} > \frac{k}{\Delta x} \Leftrightarrow$$

$$\Leftrightarrow \Delta t < \frac{\rho C \Delta x^2}{k} = \frac{10 \cdot 10^6 \cdot 16 \cdot 10^{-6}}{10} = 16s$$

Επιλέγουμε:

$$\Delta t = 2s$$

Ενδιάμεσοι κόμβοι 2-4:

$$a_p T_p = \frac{a_w}{2} T_w + \frac{a_w}{2} T_w^o + \frac{a_e}{2} T_e + \frac{a_e}{2} T_e^o +$$

$$+ \left[a_p^o - \frac{1}{2}(a_e + a_w - S_p - S_{p,BC}) \right] T_p^o + S_u + S_{u,BC} \quad (1)$$

$$\text{όπου: } a_w = a_e = k / \Delta x = 2500$$

$$S_u = S_p = S_{u,BC} = S_{p,BC} = 0$$

$$a_p^o = \rho C \frac{\Delta x}{\Delta t} = 10 \cdot 10^6 \frac{0.004}{2} = 20000$$

$$a_p = 20000 + \frac{1}{2}(2500 + 2500) = 20000 + 2500 = 22500$$

$$(1) \Rightarrow 22500 T_p = 1250 T_W + 1250 T_W^0 + 1250 T_e + 1250 T_e^0 + [20000 - 2500] T_p^0 \Leftrightarrow$$

$$\Leftrightarrow 225 T_p = 12.5 T_W + 12.5 T_W^0 + 12.5 T_e + 12.5 T_e^0 + 175 T_p^0 \Leftrightarrow$$

$$\Leftrightarrow 225 T_p - 12.5 T_W - 12.5 T_e = 12.5 T_W^0 + 12.5 T_e^0 + 175 T_p^0$$

Αντικείμενο κομμάτι 1

$$\alpha_W = 0, \alpha_e = k/\Delta x = 2500, S_{PBC} = 0, S_{URC} = 0, S_p = S_u = 0.$$

$$Q_p^0 = 20000$$

$$Q_p = 20000 + \frac{1}{2} 2500 = 21250 \quad 18750$$

$$(1) \Rightarrow 21250 T_p = 1250 T_e + 1250 T_e^0 + (20000 - 1250) T_p^0 \Leftrightarrow$$

$$\Leftrightarrow 212.5 T_p - 12.5 T_e = 12.5 T_e^0 + 187.5 T_p^0$$

Αντικείμενο κομμάτι 5

$$\alpha_e = 0, \alpha_W = 2500, Q_p^0 = 20000$$

$$S_u = S_p = 0$$

$$S_{URC} = 0$$

$$S_{PBC} = -\frac{1}{R} = -5000$$

$$\alpha_p = 20000 + \frac{1}{2} (2500 + 5000) = 20000 + 3750 = 23750$$

$$(1) \Rightarrow 23750 T_p = 1250 T_W + 1250 T_W^0 + [20000 - \frac{1}{2} (2500 + 5000)] T_p^0 \Leftrightarrow$$

$$\Leftrightarrow 237.5 T_p - 12.5 T_W = 12.5 T_W^0 + 162.5 T_p^0$$

Αντικείμενο αβάνι 4

$$212.5 T_1 - 12.5 T_2 + 0 T_3 + 0 T_4 + 0 T_5 = 12.5 T_1^0 + 187.5 T_1^0$$

$$-12.5 T_1 + 225 T_2 - 12.5 T_3 + 0 T_4 + 0 T_5 = 12.5 T_1^0 + 12.5 T_3^0 + 175 T_2^0$$

$$0 T_1 - 12.5 T_2 + 225 T_3 - 12.5 T_4 + 0 T_5 = 12.5 T_2^0 + 12.5 T_4^0 + 175 T_3^0$$

$$0 T_1 + 0 T_2 - 12.5 T_3 + 225 T_4 - 12.5 T_5 = 12.5 T_3^0 + 12.5 T_5^0 + 175 T_4^0$$

$$0 T_1 + 0 T_2 + 0 T_3 - 12.5 T_4 + 237.5 T_5 = 12.5 T_4^0 + 162.5 T_5^0$$