FUNDAMENTAL PHASES OF A MUSICAL WORK

1. *Initial conceptions* (intuitions, provisional or definitive data);

2. *Definition of the sonic entities* and of their symbolism communicable with the limits of possible means (sounds of musical instruments, electronic sounds, noises, sets of ordered sonic elements, granular or continuous formations, etc.);

3. *Definition of the transformations* which these sonic entities must undergo in the course of the composition (macrocomposition: general choice of logical framework, i.e., of the elementary algebraic operations and the setting up of relations between entities, sets, and their symbols as defined in 2.); and the arrangement of these operations in lexicographic time with the aid of succession and simultaneity);

4. *Microcomposition* (choice and detailed fixing of the functional or stochastic relations of the elements of 2.), i.e., algebra outside-time, and algebra in-time;

5. *Sequential programming* of 3. and 4. (the schema and *pattern* of the work in its entirety);

6. *Implementation of calculations*, verifications, feedbacks, and definitive modifications of the sequential program;

7. *Final symbolic result* of the programming (setting out the music on paper in traditional notation, numerical expressions, graphs, or other means of solfeggio);

8. *Sonic realization* of the program (direct orchestral performance, manipulations of the type of electromagnetic music, computerized construction of the sonic entities and their transformations).

The order of this list is not really rigid. Permutations are possible in the course of the working out of a composition. Most of the time these phases are unconscious and defective. However, this list does establish ideas and allows speculation about the future. In fact, computers can take in hand phases 6. and 7., and even 8. But as a first approach, it seems that only phases 6. and 7. are immediately accessible. That is to say, that the final symbolic result, at least in France, may be realized only by an orchestra or by manipulations of electroacoustic music on tape recorders, emitted by the existing electroacoustic channels; and not, as would be desirable in the very near future, by an elaborate mechanization which would omit orchestral or tape interpreters, and which would assume the computerized fabrication of the sonic entities and of their transformations.

Here now is an answer to the question put above, an answer that is true for instrumental music, but which can be applied as well to all kinds of

sound production. For this we shall again take up the phases described:

2. Definition of sonic entities. The sonic entities of the classical orchestra can be represented in a first approximation by vectors of four usually independent variables, $E_r(c, h, g, u)$:

c_a = timbre or instrumental family
h_i = pitch of the sound
g_j = intensity of the sound, or dynamic form
u_k = duration of the sound.

The vector E_r defines a point M in the *multidimensional space* provided by a *base* (c, h, g, u). This point M will have as *coordinates* the numbers c_a, h_i, g_j, u_k. For example: c_3 played arco and forte on a violin, one eighth note in length, at one eighth note = 240 MM, can be represented as $c_{\text{viol. arco}}$, h_{39} (= c_3), g_4 (= forte), u_5 (= $\frac{1}{4}$ sec.). Suppose that these points M are plotted on an axis which we shall call E_r, and that through its origin we draw another axis t, at right angles to axis E_r. We shall represent on this axis, called the *axis of lexicographic time*, the lexicographic-temporal succession of the points M. Thus we have defined and conveniently represented a two-dimensional space (E_r, t). This will allow us to pass to phase 3., definition of transformation, and 4., microcomposition, which must contain the answer to the problem posed concerning the minimum of constraints.

To this end, suppose that the points M defined above can appear with no necessary condition other than that of obeying an aleatory law without memory. This hypothesis is equivalent to saying that we admit a stochastic distribution of the events E_r in the space (E_r, t). Admitting a sufficiently weak superficial distribution n, we enter a region where the law of Poisson is applicable:

$$P_k = \frac{n^k}{K!} e^{-n}.$$

Incidentally we can consider this problem as a synthesis of several conveniently chosen linear stochastic processes (law of radiation from radioactive bodies). (The second method is perhaps more favorable for a mechanization of the transformations.)

A sufficiently long fragment of this distribution constitutes the musical work. The basic law defined above generates a whole family of compositions as a function of the superficial density. So we have a formal archetype of composition in which the basic aim is to attain the greatest possible *asymmetry* (in the etymological sense) and the *minimum of constraints, causalities, and rules.* We think that from this archetype, which is perhaps the most general one, we can redescend the ladder of forms by introducing progressively more numerous constraints, i.e., choices, restrictions, and negations. In the analysis in several linear processes we can also introduce other processes: those of Wiener-Lévy, P. Lévy's infinitely divisibles, Markov chains, etc., or mixtures of several. It is this which makes this second method the more fertile.

The exploration of the limits a and b of this archetype $a \leq n \leq b$ is equally interesting, but on another level—that of the mutual comparison of samples. This implies, in effect, a gradation of the increments of n in order that the differences between the families n_i may be recognizable. Analogous remarks are valid in the case of other linear processes.

If we opt for a Poisson process, there are two necessary hypotheses which answer the question of the minimum of constraints: 1. there exists in a given space musical instruments and men; and 2. there exist means of contact between these men and these instruments which permit the emission of rare sonic events.

This is the only hypothesis (cf. the *ekklisis* of Epicurus). From these two constraints and with the aid of stochastics, I built an entire composition without admitting any other restrictions. *Achorripsis* for 21 instruments was composed in 1956–57, and had its first performance in Buenos Aires in 1958 under Prof. Hermann Scherchen. (See Fig. I–8.)

At that time I wrote:*

τὸ γὰρ αὐτὸ νοεῖν ἐστίν τε καὶ εἶναι
τὸ γὰρ αὐτὸ εἶναι ἐστίν τε καὶ οὐκ εἶναι†

ONTOLOGY

In a universe of nothingness. A brief train of waves, so brief that its end and beginning coincide (negative time) disengaging itself endlessly.

Nothingness resorbs, creates.
It engenders being.

Time, Causality.

These rare sonic events can be something more than isolated sounds. They can be melodic figures, cell structures, or agglomerations whose

* The following excerpt (through p. 37) is from "In Search of a Stochastic Music," *Gravesaner Blätter*, no. 11/12.

† "For it is the same to think as to be" (*Poem* by Parmenides); and my paraphrase, "For it is the same to be as not to be."

characteristics are also ruled by the laws of chance, for example, clouds of sound-points or speed-temperatures.[6] In each case they form a sample of a succession of rare sonic events.

This sample may be represented by either a simple table of probabilities or a double-entry table, a matrix, in which the cells are filled by the frequencies of events. The rows represent the particular qualifications of the events, and the columns the dates (see Matrix M, Fig. I–9). The frequencies in this matrix are distributed according to Poisson's formula, which is the law for the appearances of rare random events.

We should further define the sense of such a distribution and the manner in which we realize it. There is an advantage in defining chance as an aesthetic law, as a normal philosophy. Chance is the limit of the notion of evolving symmetry. Symmetry tends to asymmetry, which in this sense is equivalent to the negation of traditionally inherited behavioral frameworks. This negation not only operates on details, but most importantly on the composition of structures, hence tendencies in painting, sculpture, architecture, and other realms of thought. For example, in architecture, plans worked out with the aid of regulating diagrams are rendered more complex and dynamic by exceptional events. Everything happens as if there were one-to-one oscillations between symmetry, order, rationality, and asymmetry, disorder, irrationality in the reactions between the epochs of civilizations.

At the beginning of a transformation towards asymmetry, exceptional events are introduced into symmetry and act as aesthetic stimuli. When these exceptional events multiply and become the general case, a jump to a higher level occurs. The level is one of disorder, which, at least in the arts and in the expressions of artists, proclaims itself as engendered by the complex, vast, and rich vision of the brutal encounters of modern life. Forms such as abstract and decorative art and action painting bear witness to this fact. Consequently chance, by whose side we walk in all our daily occupations, is nothing but an extreme case of this controlled disorder (that which signifies the richness or poverty of the connections between events and which engenders the dependence or independence of transformations); and by virtue of the negation, it conversely enjoys all the benevolent characteristics of an artistic regulator. It is a regulator also of sonic events, their appearance, and their life. But it is here that the iron logic of the laws of chance intervenes; this chance cannot be created without total submission to its own laws. On this condition, chance checked by its own force becomes a hydroelectric torrent.

Fig. I–8. Bars 104–111 of *Achorripsis*

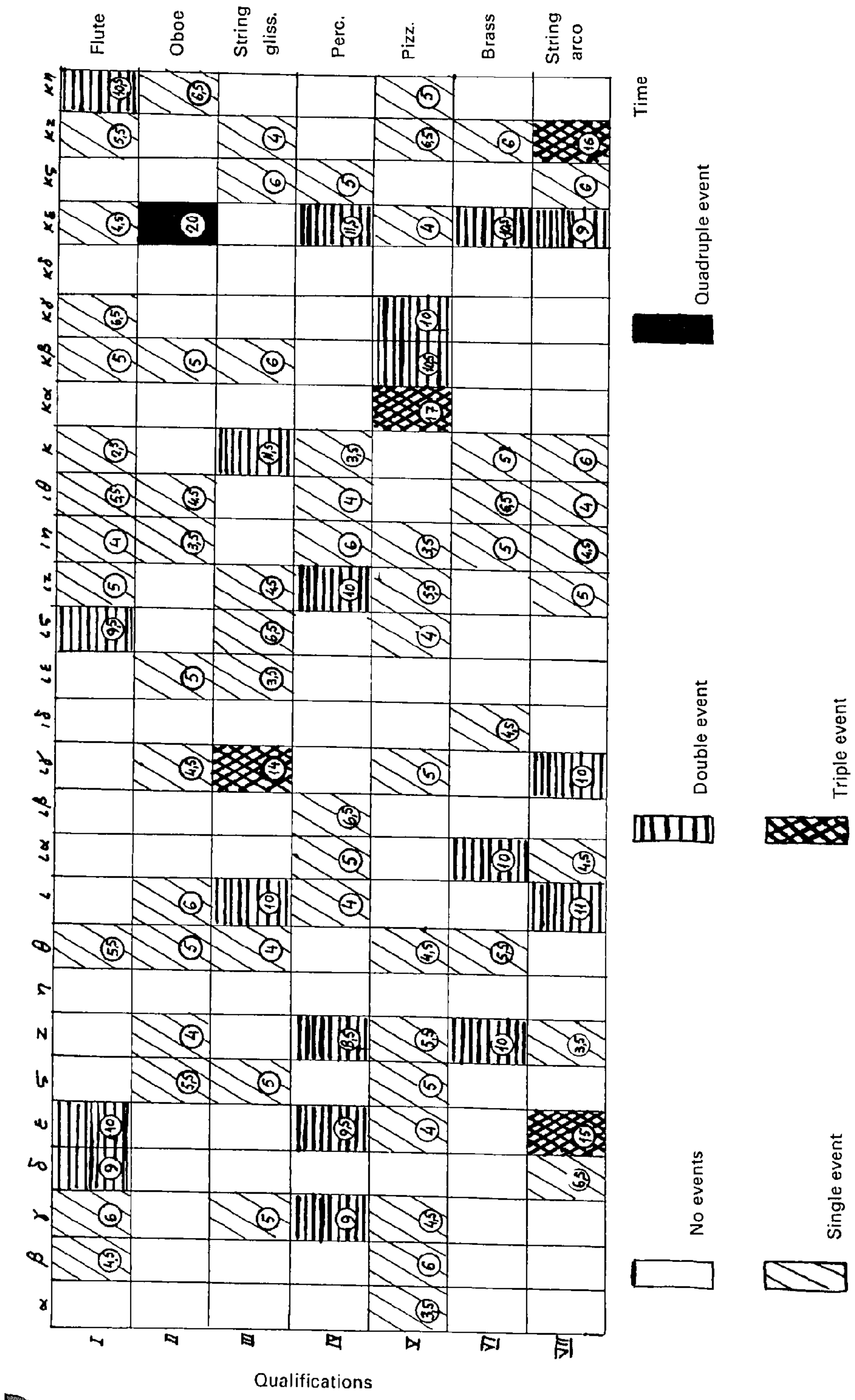


Fig. I–9. Vector Matrix *M*, Matrix of *Achorripsis*

However, we are not speaking here of cases where one merely plays heads and tails in order to choose a particular alternative in some trivial circumstance. The problem is much more serious than that. It is a matter here of a philosophic and aesthetic concept ruled by the laws of probability and by the mathematical functions that formulate that theory, of a coherent concept in a new region of coherence.

The analysis that follows is taken from *Achorripsis.*

For convenience in calculation we shall choose a priori a mean density of events

$$\lambda = 0.6 \text{ events/unit.}$$

Applying Poisson's formula,

$$P_k = \frac{\lambda^k}{K!} e^{-\lambda}$$

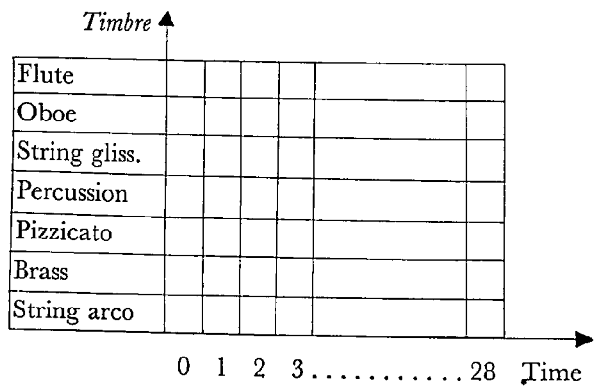
we obtain the table of probabilities:

$$\begin{aligned} P_0 &= 0.5488 \\ P_1 &= 0.3293 \\ P_2 &= 0.0988 \\ P_3 &= 0.0198 \\ P_4 &= 0.0030 \\ P_5 &= 0.0004. \end{aligned} \qquad (1)$$

P_i is the probability that the event will occur i times in the unit of volume, time, etc. In choosing a priori 196 units or cells, the distribution of the frequencies among the cells is obtained by multiplying the values of P_i by 196.

i	Number of cells $196\,P_i$	
0	107	
1	65	
2	19	(2)
3	4	
4	1	

The 196 cells may be arranged in one or several groups of cells, qualified as to timbre and time, so that the number of groups of timbres times the number of groups of durations = 196 cells. Let there be 7 distinct timbres; then 196/7 = 28 units of time. Thus the 196 cells are distributed over a two-dimensional space as shown in (3).



(3)

If the musical sample is to last 7 minutes (a subjective choice) the unit of time U_t will equal 15 sec., and each U_t will contain 6.5 measures at MM = 26.

How shall we distribute the frequencies of zero, single, double, triple, and quadruple events per cell in the two-dimensional space of Matrix (3)? Consider the 28 columns as cells and distribute the zero, single, double, triple, and quadruple events from table (2) in these 28 new cells. Take as an example the single event; from table (2) it must occur 65 times. Everything happens as if one were to distribute events in the cells with a mean density $\lambda = 65/28 = 2.32$ single events per cell (here cell = column).

In applying anew Poisson's formula with the mean density $\lambda = 2.32$ ($2.32 \ll 30$) we obtain table (4).

Poisson Distribution

Frequency K	No. of Columns	Product col × K
0	3	0
1	6	6
2	8	16
3	5	15
4	3	12
5	2	10
6	1	6
7	0	0
Totals	28	65

(4)

Arbitrary Distribution

Frequency K	No. of Columns	Product col × K
0	10	0
1	3	3
2	0	0
3	9	27
4	0	0
5	1	5
6	5	30
7	0	0
Totals	28	65

(5)

One could choose any other distribution on condition that the sum of single events equals 65. Table (5) shows such a distribution.

But in this axiomatic research, where chance must bathe all of sonic space, we must reject every distribution which departs from Poisson's law. And the Poisson distribution must be effective not only for the columns but also for the rows of the matrix. The same reasoning holds true for the diagonals, etc.

Contenting ourselves just with rows and columns, we obtain a homogeneous distribution which follows Poisson. It was in this way that the distributions in rows and columns of Matrix (M) (Fig. I–9) were calculated.

So a *unique* law of chance, the law of Poisson (for rare events) through the medium of the arbitrary mean λ is capable of conditioning, on the one hand, a whole sample matrix, and on the other, the partial distributions following the rows and columns. The a priori, arbitrary choice admitted at the beginning therefore concerns the variables of the "vector-matrix."

Variables or entries of the "vector-matrix"

1. Poisson's Law
2. The mean λ
3. The number of cells, rows, and columns

The distributions entered in this matrix are not always rigorously defined. They really depend, for a given λ, on the number of rows or columns. The greater the number of rows or columns, the more rigorous is the definition. This is the law of large numbers. But this indeterminism allows free will if the artistic inspiration wishes it. It is a second door that is open to the subjectivism of the composer, the first being the "state of entry" of the "Vector-Matrix" defined above.

Now we must specify the unit-events, whose frequencies were adjusted in the standard matrix (M). We shall take as a single event a cloud of sounds with linear density δ sounds/sec. Ten sounds/sec is about the limit that a normal orchestra can play. We shall choose $\delta = 5$ sounds/measure at MM 26, so that $\delta = 2.2$ sounds/sec ($\approx 10/4$).

We shall now set out the following correspondence:

Event	Cloud of density δ =		Mean number of sounds/cell (15 sec)
	Sounds/ measure 26MM	Sounds/ sec	
zero	0	0	0
single	5	2.2	32.5
double	10	4.4	65
triple	15	6.6	97.5
quadruple	20	8.8	130

The hatchings in matrix (M) show a Poisson distribution of frequencies, homogeneous and verified in terms of rows and columns. We notice that the rows are interchangeable (= interchangeable timbres). So are the columns. This leads us to admit that the determinism of this matrix is weak in part, and that it serves chiefly as a basis for thought—for thought which manipulates frequencies of events of all kinds. The true work of molding sound consists of distributing the clouds in the two-dimensional space of the matrix, and of anticipating a priori all the sonic encounters before the calculation of details, eliminating prejudicial positions. It is a work of patient research which exploits all the creative faculties instantaneously. This matrix is like a game of chess for a single player who must follow certain rules of the game for a prize for which he himself is the judge. This game matrix has no unique strategy. It is not even possible to disentangle any balanced goals. It is very general and incalculable by pure reason.

Up to this point we have placed the cloud densities in the matrix. Now with the aid of calculation we must proceed to the coordination of the aleatory sonic elements.

HYPOTHESES OF CALCULATION

Let us analyze as an example cell III, ιz of the matrix: third row, sounds of continuous variation (string glissandi), seventeenth unit of time (measures 103–11). The density of the sounds is 4.5 sounds/measure at MM 26 ($\delta = 4.5$); so that 4.5 sounds/measure times 6.5 measures = 29 sounds for this cell. How shall we place the 29 glissando sounds in this cell?

Hypothesis 1. The acoustic characteristic of the glissando sound is assimilated to the speed v = df/dt of a uniformly continuous movement. (See Fig. I–10.)

Hypothesis 2. The quadratic mean α of all the possible values of v is proportional to the sonic density δ. In this case $\alpha = 3.38$ (temperature).

Hypothesis 3. The values of these speeds are distributed according to the most complete asymmetry (chance). This distribution follows the law of

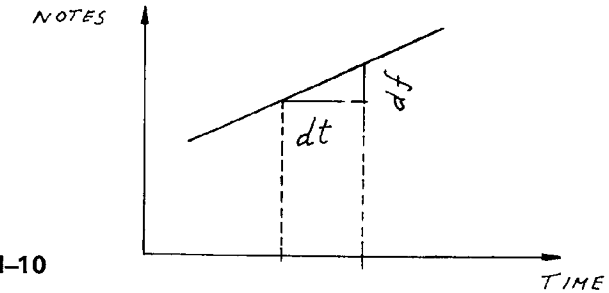


Fig. I–10

Gauss. The probability $f(v)$ for the existence of the speed v is given by the function

$$f(v) = \frac{2}{a\sqrt{\pi}} e^{-v^2/a^2};$$

and the probability $P(\lambda)$ that v will lie between v_1 and v_2, by the function

$$P(\lambda) = \theta(\lambda_2) - \theta(\lambda_1),$$

in which $\lambda_1 = v_1/a$ and

$$\theta(\lambda) = \frac{2}{\sqrt{\pi}} \int_0^\lambda e^{-\lambda^2} d\lambda \quad \text{(normal distribution).}$$

Hypothesis 4. A glissando sound is essentially characterized by *a.* the moment of its departure; *b.* its speed $v_m = df/dt$, $(v_1 < v_m < v_2)$; and *c.* its register.

Hypothesis 5. Assimilate time to a line and make each moment of departure a point on that line. It is as if one were to distribute a number of points on a line with a linear density $\delta = 4.5$ points at MM = 26. This, then, is a problem of continuous probabilities. These points define segments and the probability that the i-th segment will have a length x_i between x and $x + dx$ is

$$P_x = \delta e^{-\delta x} dx.$$

Hypothesis 6. The moment of departure corresponds to a sound. We shall attempt to define its pitch. The strings have a range of about 80 semitones, which may be represented by a line of length a = 80 semitones. Since between two successive or simultaneous glissandi there exists an interval between the pitches at the moments of departure, we can define not only the note of attack for the first glissando, but also the melodic interval which separates the two origins.

Put thus, the problem consists of finding the probability that a segment s within a line segment of length a will have a length between j and $j + dj$ $(0 \leq j \leq a)$. This probability is given by the formula

$$\theta(j)\,dj = \frac{2}{a}\left(1 - \frac{j}{a}\right)dj. \qquad \text{(See Appendix I.)}$$

Hypothesis 7. The three essential characteristics of the glissando sound defined in Hypothesis 4 are independent.

From these hypotheses we can draw up the three tables of probability: a table of durations, a table of speeds, and a table of intervals.

All these tables furnish us with the elements which materialize in cell III, *ιz*. The reader is encouraged to examine the score to see how the results of the calculations have been used. Here also, may we emphasize, a great liberty of choice is given the composer. The restrictions are more of a general canalizing kind, rather than peremptory. The theory and the calculation define the tendencies of the sonic entity, but they do not constitute a slavery. Mathematical formulae are thus tamed and subjugated by musical thought. We have given this example of glissando sounds because it contains all the problems of stochastic music, controlled, up to a certain point, by calculation.

Table of Durations

δ = 4.5 sounds/measure at MM 26
Unit x = 0.10 of the measure at 26 MM
$4.5 \cdot 6.5$ = 29 sounds/cell, i.e., 28 durations

x	δx	$e^{-\delta x}$	$\delta e^{-\delta x}$	$\delta e^{-\delta x}\,dx$	$28P_x$
0.00	0.00	1.000	4.500	0.362	10
0.10	0.45	0.638	2.870	0.231	7
0.20	0.90	0.407	1.830	0.148	4
0.30	1.35	0.259	1.165	0.094	3
0.40	1.80	0.165	0.743	0.060	2
0.50	2.25	0.105	0.473	0.038	1
0.60	2.70	0.067	0.302	0.024	1
0.70	3.15	0.043	0.194	0.016	0
Totals			12.415	0.973	28

An approximation is made by considering dx as a constant factor.

$$\sum_{0}^{\infty} \delta e^{-\delta x}\,dx = 1.$$

Therefore

$$dx = 1 \Big/ \sum_{0}^{\infty} \delta e^{-\delta x}.$$

In this case $dx = 1/12.415 = 0.805$.

Table of Speeds

δ = 4.5 glissando sounds/measure at 26 MM
α = 3.88, quadratic mean of the speeds
v is expressed in semitones/measure at 26 MM
v_m is the mean speed $(v_1 + v_2)/2$
$4.5 \cdot 6.5$ = 29 glissando sounds/cell.

v	$\lambda = v/\alpha$	$\theta(\lambda)$	$P(\lambda) = \theta(\lambda_2) - \theta(\lambda_1)$	$29\,P(\lambda)$	v_m
0	0.000	0.0000			
			0.2869	9	0.5
1	0.258	0.2869			
			0.2510	7	1.5
2	0.516	0.5379			
			0.1859	5	2.5
3	0.773	0.7238			
			0.1310	4	3.5
4	1.032	0.8548			
			0.0771	2	4.5
5	1.228	0.9319			
			0.0397	1	5.5
6	1.545	0.9716			
			0.0179	1	6.5
7	1.805	0.9895			
			0.0071	0	7.5

Table of Intervals

δ = 4.5 glissandi/measure at 26 MM.
a = 80 semitones, or 18 times the arbitrary unit of 4.5 semitones.
j is expressed in multiples of 4.5 semitones.
dj is considered to be constant. Therefore $dj = 1/\sum\theta(j)$ or $dj = a/(m + 1)$, and we obtain a step function. For $j = 0$, $\theta(j)dj = 2/(m + 1) = 0.105$; for $j = 18$, $\theta(j)dj = 0$.
$4.5 \cdot 6.5 = 29$ glissando sounds per cell.
We can construct the table of probabilities by means of a straight line.

j	$\theta(j)\,dj = P(j)$	$29\,P(j)$
0	0.105	3
1		3
2		3
3		3
4		2
5		2
6		2
7		2
8		2
9		2
10		1
11		1
12		1
13		1
14		1
15		0
16		0
17		0
18		0

We shall not speak of the means of verification of liaisons and correlations between the various values used. It would be too long, complex, and tedious. For the moment let us affirm that the basic matrix was verified by the two formulae:

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2}\sqrt{\sum (y - \bar{y})^2}},$$

and

$$z = \tfrac{1}{2}\log\frac{1 + r}{1 - r}.$$

Let us now imagine music composed with the aid of matrix (M). An observer who perceived the frequencies of events of the musical sample would deduce a distribution due to chance and following the laws of probability. Now the question is, when heard a number of times, will this music keep its surprise effect? Will it not change into a set of *foreseeable* phenomena through the existence of memory, despite the fact that the law of frequencies has been derived from the laws of chance?

In fact, the data will appear aleatory only at the first hearing. Then, during successive rehearings the relations between the events of the sample ordained by "chance" will form a network, which will take on a definite meaning in the mind of the listener, and will initiate a special "logic," a new cohesion capable of satisfying his intellect as well as his aesthetic sense; that is, if the artist has a certain flair.

If, on the other hand, we wish the sample to be unforeseeable at all times, it is possible to conceive that at each repetition certain data might be transformed in such a way that their deviations from theoretical frequencies would not be significant. Perhaps a programming useful for a first, second, third, etc., performance will give aleatory samples that are not identical in an absolute sense, whose deviations will also be distributed by chance.

Or again a system with electronic computers might permit variations of the parameters of entrance to the matrix and of the clouds, under certain conditions. There would thus arise a music which can be distorted in the course of time, giving the same observer n results apparently due to chance for n performances. In the long run the music will follow the laws of probability and the performances will be *statistically* identical with each other, the identity being defined once for all by the "vector-matrix."

The sonic scheme defined under this form of vector-matrix is consequently capable of establishing a more or less self-determined regulation of the rare sonic events contained in a musical composition sample. It represents a compositional attitude, a fundamentally stochastic behavior, a unity of superior order. [1956–57].

If the first steps may be summarized by the process vision → rules → works of art, the question concerning the minimum has produced an inverse path: rules $\rightarrow$ vision. In fact stochastics permits a philosophic vision, as the example of *Achorripsis* bears witness.

CHANCE—IMPROVISATION

Before generalizing further on the essence of musical composition, we must speak of the principle of improvisation which caused a furore among the neo-serialists, and which gives them the right, or so they think, to speak of chance, of the aleatory, which they thus introduce into music. They write scores in which certain combinations of sounds may be freely chosen by the interpreter. It is evident that these composers consider the various possible circuits as equivalent. Two logical infirmities are apparent which deny them the right to speak of chance on the one hand and "composition" on the other (composition in the broad sense, that is):

1. The interpreter is a highly conditioned being, so that it is not possible to accept the thesis of unconditioned choice, of an interpreter acting like a roulette game. The martingale betting at Monte Carlo and the procession of suicides should convince anyone of this. We shall return to this.

2. The composer commits an act of resignation when he admits several possible and equivalent circuits. In the name of a "scheme" the problem of choice is betrayed, and it is the interpreter who is promoted to the rank of composer by the composer himself. There is thus a substitution of authors.

The extremist extension of this attitude is one which uses graphical signs on a piece of paper which the interpreter reads while improvising the whole. The two infirmities mentioned above are terribly aggravated here. I would like to pose a question: If this sheet of paper is put before an interpreter who is an incomparable expert on Chopin, will the result not be modulated by the style and writing of Chopin in the same way that a performer who is immersed in this style might improvise a Chopin-like cadenza to another composer's concerto? From the point of view of the composer there is no interest.

On the contrary, two conclusions may be drawn: first, that serial composition has become so banal that it can be improvised like Chopin's, which confirms the general impression; and second, that the composer resigns his function altogether, that he has nothing to say, and that his function can be taken over by paintings or by cuneiform glyphs.

Chance needs to be calculated

To finish with the thesis of the roulette-musician, I shall add this: Chance is a rare thing and a snare. It can be constructed up to a certain

point with great difficulty, by means of complex reasoning which is summarized in mathematical formulae; it can be constructed a little, but never improvised or intellectually imitated. I refer to the demonstration of the impossibility of imitating chance which was made by the great mathematician Emile Borel, who was one of the specialists in the calculus of probabilities. In any case—to play with sounds like dice—what a truly simplistic activity! But once one has emerged from this primary field of chance worthless to a musician, the calculation of the aleatory, that is to say stochastics, guarantees first that in a region of precise definition slips will not be made, and then furnishes a powerful method of reasoning and enrichment of sonic processes.

STOCHASTIC PAINTING?

In line with these ideas, Michel Philippot introduced the calculus of probabilities into his painting several years ago, thus opening new directions for investigation in this artistic realm. In music he recently endeavored to analyze the act of composition in the form of a *flow chart* for an *imaginary machine*. It is a fundamental analysis of voluntary choice, which leads to a chain of aleatory or deterministic events, and is based on the work *Composition pour double orchestre* (1960). The term imaginary machine means that the composer may rigorously define the entities and operating methods, just as on an electronic computer. In 1960 Philippot commented on his *Composition pour double orchestre*:

> If, in connection with this work, I happened to use the term "experimental music," I should specify in what sense it was meant in this particular case. It has nothing to do with concrete or electronic music, but with a very banal score written on the usual ruled paper and requiring none but the most traditional orchestral instruments. However, the experiment of which this composition was in some sense a by-product does exist (and one can think of many industries that survive only through the exploitation of their by-products).
>
> The end sought was merely to effect, in the context of a work which I would have written independent of all experimental ambitions, an exploration of the processes followed by my own cerebral mechanism as it arranged the sonic elements. I therefore devised the following steps:
>
> 1. Make the most complete inventory possible of the set of my gestures, ideas, mannerisms, decisions, and choices, etc., which were mine when I wrote the music.

2. Reduce this set to a succession of simple decisions, binary, if possible; i.e., accept or refuse a particular note, duration, or silence in a situation determined and defined by the context on one hand, and by the conditioning to which I had been subjected and my personal tastes on the other.

3. Establish, if possible, from this sequence of simple decisions, a scheme ordered according to the following two considerations (which were sometimes contradictory): the manner in which these decisions emerged from my imagination in the course of the work, and the manner in which they would have to emerge in order to be most useful.

4. Present this scheme in the form of a flow chart containing the logical chain of these decisions, the operation of which could easily be controlled.

5. Set in motion a mechanism of simulation respecting the rules of the game in the flow chart and note the result.

6. Compare this result with my *musical* intentions.

7. Check the differences between result and intentions, detect their causes, and correct the operating rules.

8. Refer these corrections back to the sequence of experimental phases, i.e., start again at 1. until a satisfactory result has been obtained.

If we confine ourselves to the most general considerations, it would simply be a matter of proceeding to an analysis of the complexity, considered as an accumulation, in a certain order, of single events, and then of reconstructing this complexity, at the same time verifying the nature of the elements and their rules of combination. A cursory look at the flow chart of the first movement specifies quite well by a mere glance the method I used. But to confine oneself to this first movement would be to misunderstand the essentials of musical composition.

In fact the "preludial" character which emerges from this combination of notes (elementary constituents of the orchestra) should remind us of the fact that composition in its ultimate stage is also an assembly of groups of notes, motifs, or themes and their transformations. Consequently the task revealed by the flow charts of the following movements ought to make conspicuous a grouping of a higher order, in which the data of the first movement were used as a sort of "prefabricated" material. Thus appeared the phenomenon, a rather banal one, of autogeneration of complexity by juxtaposition and combination of a large number of single events and operations.

At the end of this experiment I possessed at most some insight into my own musical tastes, but to me, the obviously interesting aspect of

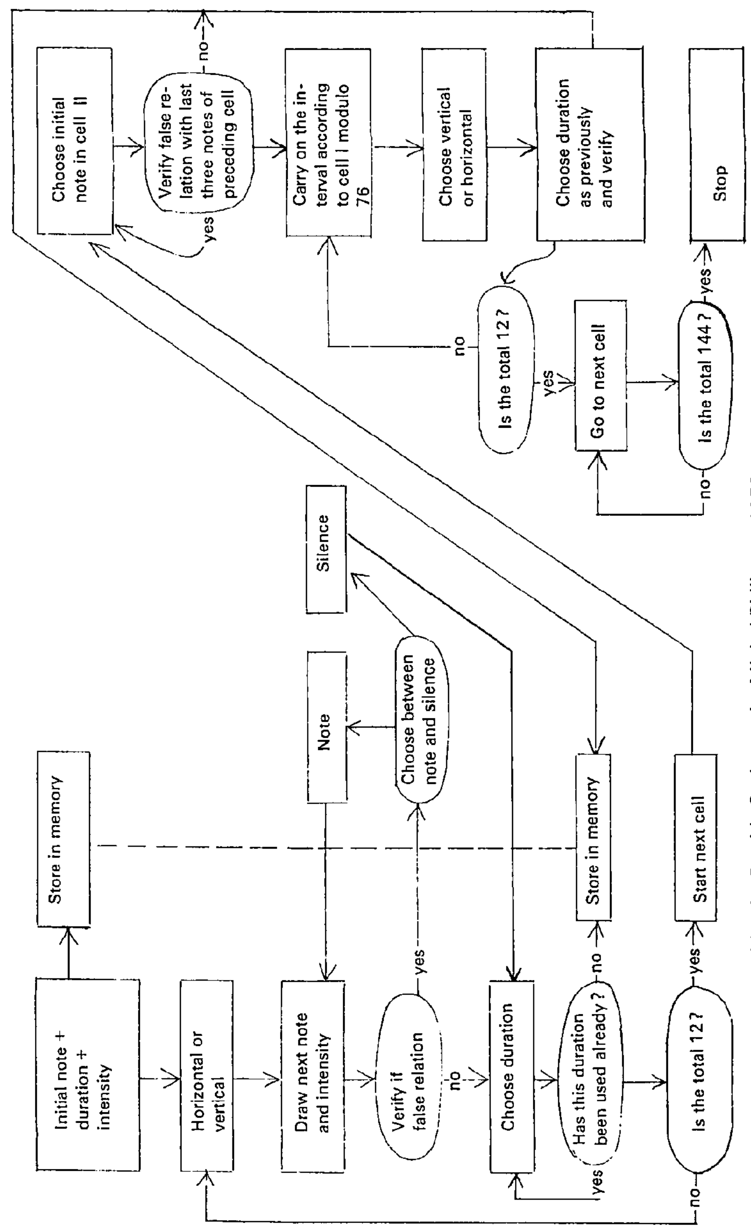


Fig. I–11. *Composition for Double Orchestra*, by Michel Philippot, 1959
Flow Chart of the First Movement

it (as long as there is no error of omission!) was the analysis of the composer, his mental processes, and a certain liberation of the imagination.

The biggest difficulty encountered was that of a conscious and voluntary split in personality. On one hand, was the composer who already had a clear idea and a precise audition of the work he wished to obtain; and on the other was the experimenter who had to maintain a lucidity which rapidly became burdensome in these conditions—a lucidity with respect to his own gestures and decisions. We must not ignore the fact that such experiments must be examined with the greatest prudence, for everyone knows that no observation of a phenomenon exists which does not disturb that phenomenon, and I fear that the resulting disturbance might be particularly strong when it concerns such an ill-defined domain and such a delicate activity. Moreover, in this particular case, I fear that observation might provoke its own disturbance. If I accepted this risk, I did not underestimate its extent. At most, my ambition confined itself to the attempt to project on a marvelous unknown, that of aesthetic creation, the timid light of a dark lantern. (The dark lantern had the reputation of being used especially by housebreakers. On several occasions I have been able to verify how much my thirst for investigation has made me appear in the eyes of the majority as a dangerous housebreaker of inspiration.)

Chapter II

Markovian Stochastic Music—Theory

Now we can rapidly generalize the study of musical composition with the aid of stochastics.

The first thesis is that stochastics is valuable not only in instrumental music, but also in electromagnetic music. We have demonstrated this with several works: *Diamorphoses* 1957–58 (B.A.M. Paris), *Concret PH* (in the Philips Pavilion at the Brussels Exhibition, 1958); and *Orient-Occident*, music for the film of the same name by E. Fulchignoni, produced by UNESCO in 1960.

The second thesis is that stochastics can lead to the creation of new sonic materials and to new forms. For this purpose we must as a preamble put forward a temporary hypothesis which concerns the nature of sound, of all sound [19].

BASIC TEMPORARY HYPOTHESIS (lemma) AND DEFINITIONS

All sound is an integration of grains, of elementary sonic particles, of sonic quanta. Each of these elementary grains has a threefold nature: duration, frequency, and intensity.[1] All sound, even all continuous sonic variation, is conceived as an assemblage of a large number of elementary grains adequately disposed in time. So every sonic complex can be analyzed as a series of pure sinusoidal sounds even if the variations of these sinusoidal sounds are infinitely close, short, and complex. In the attack, body, and decline of a complex sound, thousands of pure sounds appear in a more or less short interval of time, Δt. Hecatombs of pure sounds are necessary for the creation of a complex sound. A complex sound may be imagined as a multi-colored firework in which each point of light appears and instantaneously disappears against a black sky. But in this firework there would be such a quantity of points of light organized in such a way that their rapid and teeming succession would create forms and spirals, slowly unfolding, or conversely, brief explosions setting the whole sky aflame. A line of light would be created by a sufficiently large multitude of points appearing and disappearing instantaneously.

If we consider the duration Δt of the grain as quite small but invariable, we can ignore it in what follows and consider frequency and intensity only. The two physical substances of a sound are frequency and intensity in association. They constitute two sets, F and G, independent by their nature. They have a set product $F \times G$, which is the elementary grain of sound. Set F can be put in any kind of correspondence with G: many-valued, single-valued, one-to-one mapping, ... The correspondence can be given by an extensive representation, a matrix representation, or a canonical representation.

EXAMPLES OF REPRESENTATIONS

Extensive (term by term):

Frequencies	f_1	f_2	f_3	f_4	$\cdots$
Intensities ↓	g_3	g_n	g_3	g_h	$\cdots$

Matrix (in the form of a table):

↓	f_1	f_2	f_3	f_4	f_5	f_6	f_7	$\cdots$
g_1	+	0	+	0	0	0	+	
g_2	0	+	0	0	0	+	0	
g_3	0	0	0	+	+	0	0	
$\vdots$								

Canonical (in the form of a function):

$$\sqrt{f} = Kg$$

f = frequency
g = intensity
K = coefficient.

The correspondence may also be indeterminate (stochastic), and here the most convenient representation is the matrical one, which gives the transition probabilities.

Example:

↓	f_1	f_2	f_3	f_4	$\cdots$
g_1	0.5	0	0.2	0	$\cdots$
g_2	0	0.3	0.3	1	$\cdots$
g_3	0.5	0.7	0.5	0	$\cdots$

The table should be interpreted as follows: for each value f_i of f there are one or several corresponding intensity values g_i, defined by a probability. For example, the two intensities g_2 and g_3 correspond to the frequency f_2, with 30% and 70% chance of occurrence, respectively. On the other hand, each of the two sets F and G can be furnished with a structure—that is to say, internal relations and laws of composition.

Time t is considered as a totally ordered set mapped onto F or G in a lexicographic form.

Examples:

a. $f_1 \quad f_2 \quad f_3 \quad \cdots$
$t = 1, 2, \cdots$

b. $f_{0.5} \quad f_3 \quad f_{\sqrt{11}} \quad f_x \quad \cdots$
$t = 0.5, 3, \sqrt{11}, x, \cdots$

c.

$$t = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c} f_1 & f_1 & f_2 & f_1 & f_2 & f_2 & f_n & f_3 & \cdots & \cdots & \cdots \\ A & B & C & D & E & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \Delta t & \Delta t & \Delta t & \Delta t & \Delta t & \Delta t & \Delta t & \cdots & \cdots & \cdots & \cdots \end{array}$$

$$\Delta t = \Delta t$$

Example *c.* is the most general since continuous evolution is sectioned into slices of a single thickness Δt, which transforms it in discontinuity; this makes it much easier to isolate and examine under the magnifying glass.

GRAPHICAL REPRESENTATIONS

We can plot the values of pure frequencies in units of octaves or semitones on the abscissa axis, and the intensity values in decibels on the ordinate axis, using logarithmic scales (see Fig. II–1). This cloud of points is the cylindrical projection on the plane (FG) of the grains contained in a thin slice Δt (see Fig. II–2). The graphical representations Figs. II–2 and II–3 make more tangible the abstract possibilities raised up to this point.

Psychophysiology

We are confronted with a cloud of evolving points. This cloud is the product of the two sets F and G in the slice of time Δt. What are the possible

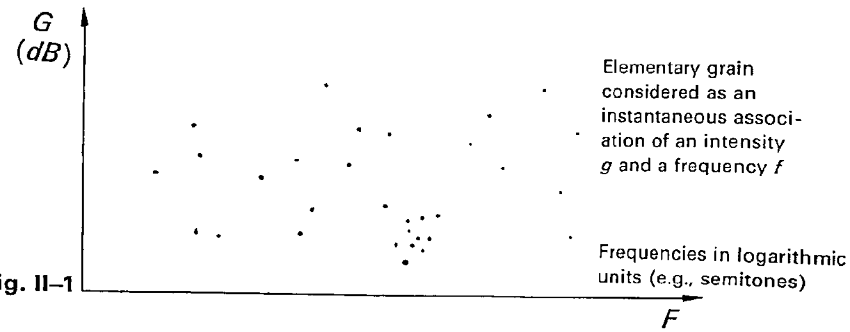


Fig. II–1

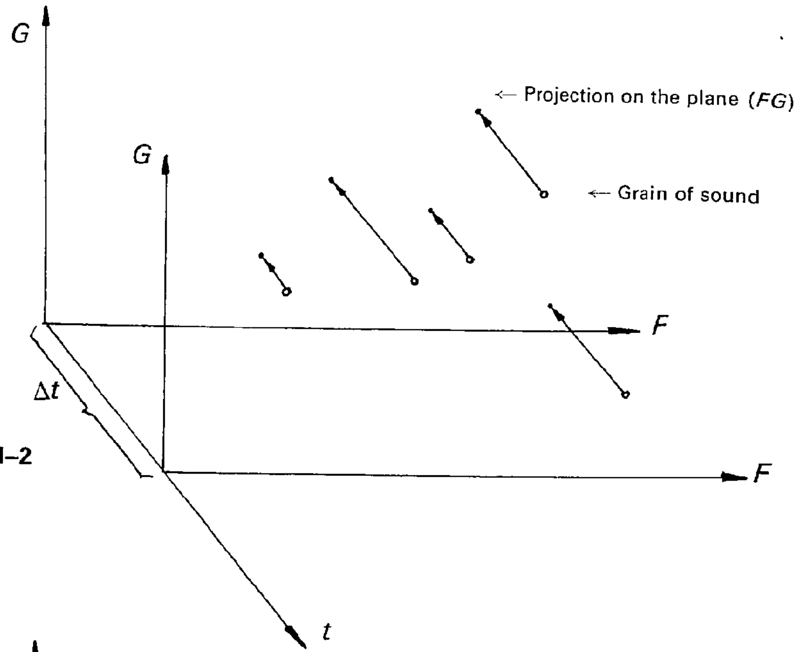


Fig. II–2

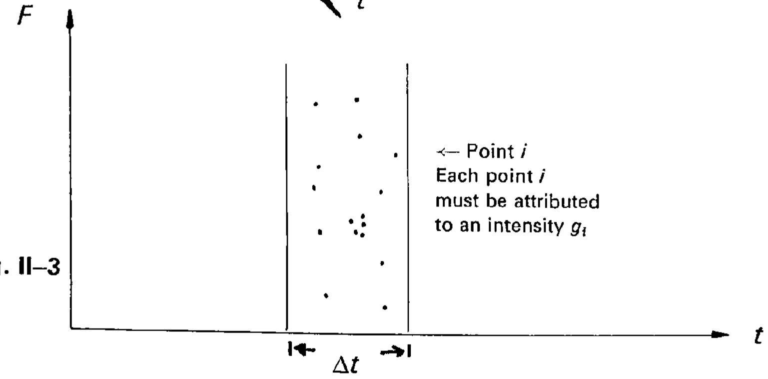


Fig. II–3

restrictive limits of human psychophysiology? What are the most general manipulations which may be imposed on the clouds and their transformations within psychophysiological limits?

The basic abstract hypothesis, which is the granular construction of all possible sounds, gives a very profound meaning to these two questions. In fact within human limits, using all sorts of manipulations with these grain clusters, we can hope to produce not only the sounds of classical instruments and elastic bodies, and those sounds generally preferred in concrete music, but also sonic perturbations with evolutions, unparalleled and unimaginable until now. The basis of the timbre structures and transformations will have nothing in common with what has been known until now.

We can even express a more general supposition. Suppose that each point of these clusters represents not only a pure frequency and its satellite intensity, but an already present structure of elementary grains, ordered a priori. We believe that in this way a sonority of a second, third, or higher order can be produced.

Recent work on hearing has given satisfactory answers to certain problems of perception. The basic problems which concern us and which we shall suppose to be resolved, even if some of the solutions are in part lacking, are [2, 3]: 1. What is the minimum perceptible duration (in comfort) of a sinusoidal sound, as a function of its frequency and its intensity? 2. What are the minimum values of intensities in decibels compatible with minimum frequencies and durations of sinusoidal sounds? 3. What are the minimum melodic interval thresholds, as a function of register, intensity, and duration? A good approximation is the Fletcher-Munson diagram of equal loudness contours (see Fig. II–4).

The total number of elementary audible grains is about 340,000. The ear is more sensitive at the center of the audible area. At the extremities it perceives less amplitude and fewer melodic intervals, so that if one wished to represent the audible area in a homogeneous manner using the coordinates F and G, i.e., with each surface element $\Delta F \Delta G$ containing the same density of grains of perceptible sounds, one would obtain a sort of mappa mundi (Fig. II–5).

In order to simplify the reasoning which will follow without altering it, we shall base our argument on Fletcher's diagram and suppose that an appropriate one-to-one transformation applied to this group of coordinates will change this curved space into an ordinary rectangle (Fig. II–6).

All the above experimental results were established in ideal conditions and without reference to the actual complexity of the natural sounds of the orchestra and of elastic bodies in general, not to mention the more complex

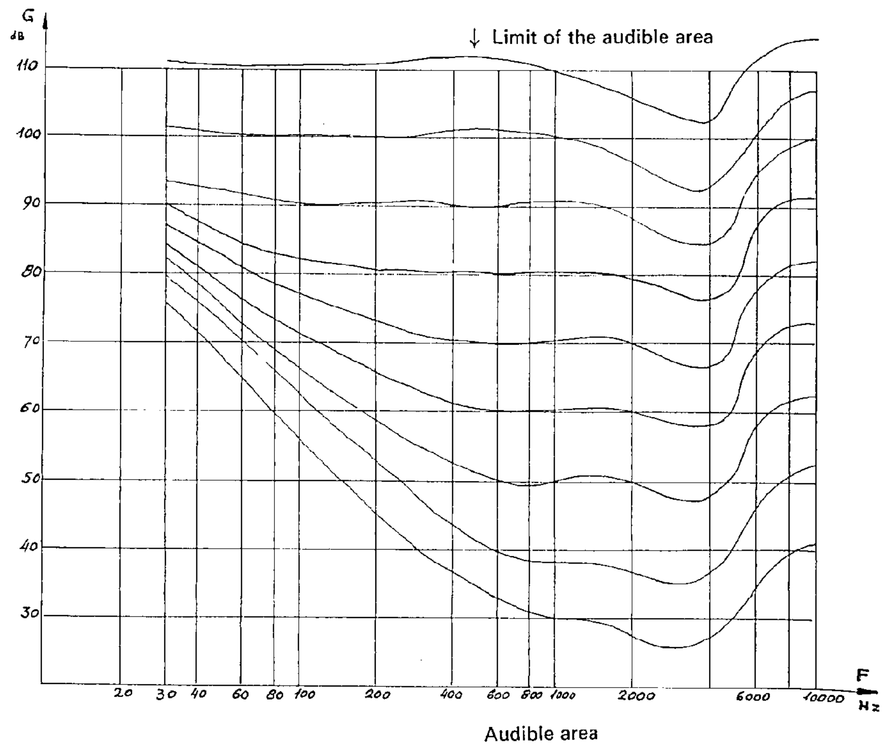


Fig. II–4. Fletcher-Munson Diagram
Equal Loudness Contours

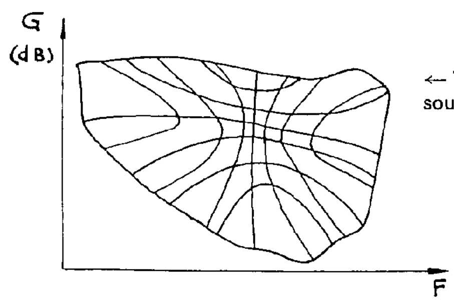
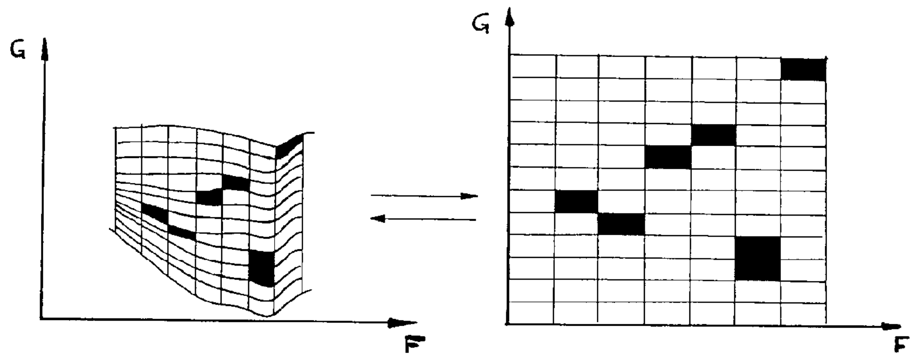


Fig. II–5



sounds of industry or of chaotic nature [4]. Theoretically [5] a complex sound can only be exhaustively represented on a three-dimensional diagram *F*, *G*, *t*, giving the instantaneous frequency and intensity as a function of time. But in practice this boils down to saying that in order to represent a momentary sound, such as a simple noise made by a car, months of calculations and graphs are necessary. This impasse is strikingly reminiscent of classical mechanics, which claimed that, given sufficient time, it could account for all physical and even biological phenomena using only a few formulae. But just to describe the state of a gaseous mass of greatly reduced volume at one instant *t*, even if simplifications are allowed at the beginning of the calculation, would require several centuries of human work!

This was a false problem because it is useless; and as far as gaseous masses are concerned, the Maxwell-Boltzmann kinetic theory of gases, with its statistical method, has been very fruitful [6]. This method reestablished the value of scales of observation. For a macroscopic phenomenon it is the massed total result which counts, and each time a phenomenon is to be observed the scale relationship between observer and phenomenon must first be established. Thus if we observe galactic masses, we must decide whether it is the movement of the whole mass, the movement of a single star, or the molecular constitution of a minute region on a star that interests us.

The same thing holds true for complex as well as quite simple sounds. It would be a waste of effort to attempt to account analytically or graphically for the characteristics of complex sounds when they are to be used in an electromagnetic composition. For the manipulation of these sounds macroscopic methods are necessary.

Inversely, and this is what particularly interests us here, to work like architects on the sonic material in order to construct complex sounds and evolutions of these entities means that we must use macroscopic methods of analysis and construction. Microsounds and elementary grains have no importance on the scale which we have chosen. Only groups of grains and the characteristics of these groups have any meaning. Naturally in very particular cases, the single grain will be reestablished in all its glory. In a Wilson chamber it is the elementary particle which carries theoretical and experimental physics on its shoulders, while in the sun it is the mass of particles and their compact interactions which constitute the solar object.

Our field of evolution is therefore the curved space described above, but simplified to a rectilinear space by means of complete one-to-one transformation, which safeguards the validity of the reasoning which we shall pursue.

SCREENS

The graphical representation of a cloud of grains in a slice of time Δt examined earlier brings a new concept, that of the density of grains per unit of volume, $\Delta F \Delta G \Delta t$ (Fig. II–7). Every possible sound may therefore be cut up into a precise quantity of elements $\Delta F \Delta G \Delta t \Delta D$ in four dimensions, distributed in this space and following certain rules defining this sound, which are summarized by a function with four variables: $s(F, G, D, t)$.

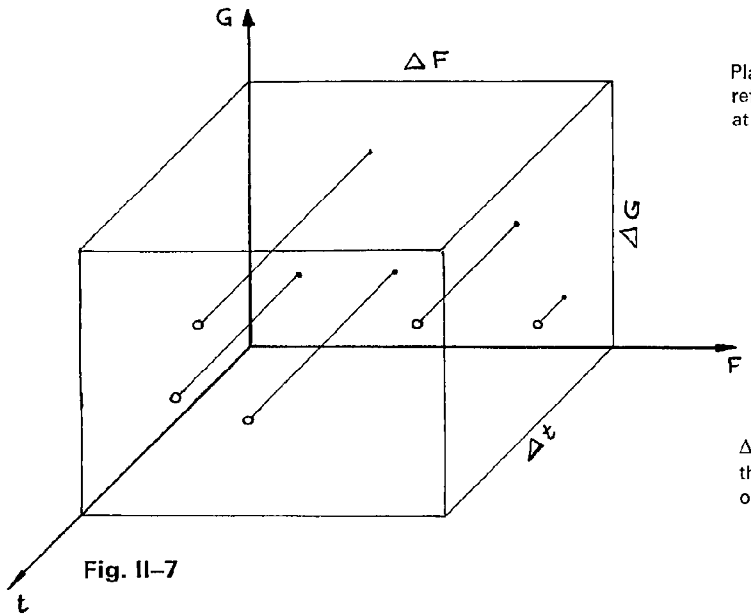


Fig. II–7

The scale of the density will also be logarithmic with its base between 2 and 3.[2] To simplify the explanation we will make an abstraction of this new coordinate of density. It will always be present in our mind but as an entity associated with the three-dimensional element $\Delta F \Delta G \Delta t$.

If time is considered as a procedure of lexicographic ordering, we can, without loss, assume that the Δt are equal constants and quite small. We can thus reason on a two-dimensional space defined by the axes F and G, on condition that we do not lose sight of the fact that the cloud of grains of sound exists in the thickness of time Δt and that the grains of sound are only artificially flattened on the plane (FG).

Definition of the screen. The screen is the audible area (FG) fixed by a sufficiently close and homogeneous grid as defined above, the cells of which may or may not be occupied by grains. In this way any sound and its history may be described by means of a sufficiently large number of sheets of paper carrying a given screen S. These sheets are placed in a fixed lexicographic order (see Fig. II–8).

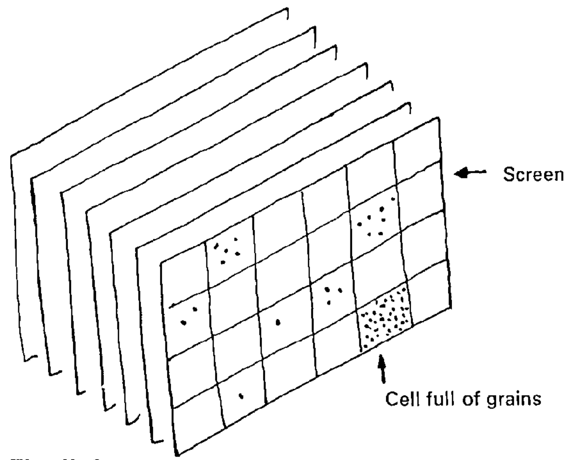


Fig. II–8

A book of screens equals the life of a complex sound

The clouds of grains drawn on the screens will differ from one screen to another by their geographical or topographical position and by their surface density (see Fig. II–9). Screen A contains a small elemental rectangle with a small cluster of density d of mean frequency f and mean amplitude g. It is almost a pure sound. Screen B represents a more complex sound with strong high and low areas but with a weak center. Screen C represents a "white" sound of weak density which may therefore be perceived as a sonic sheen occupying the whole audible area.

What is important in all the statements made up to now is that nothing has been said about the topographic fixity of the grains on the screens. All natural or instrumental sounds are composed of small surface elements filled with grains which fluctuate around a mean frequency and intensity. The same holds for the density. This statement is fundamental, and it is very likely that the failure of electronic music to create new timbres, aside from the inadequacy of the serial method, is largely due to the fixity of the grains, which form structures like packets of spaghetti (Fig. II–10).

Topographic fixity of the grains is a very particular case, the most general case being mobility and the statistical distribution of grains around positions of equilibrium. Consequently in the majority of cases real sounds can be analyzed as quite small rectangles, $\Delta F \Delta G$, in which the topographic positions and the densities vary from one screen to another following more or less well-defined laws.

Thus the sound of example D at this precise instant is formed by the collection of rectangles (f_2g_4), (f_2g_5), (f_4g_2), (f_4g_3), (f_5g_1), (f_5g_2), (f_6g_1), (f_6g_2), (f_6g_5), (f_7g_2), (f_7g_3), (f_7g_4), (f_7g_5), (f_8g_3), (f_8g_4), (f_8g_5), and in each of the rectangles the grains are disposed in an asymmetric and homogeneous manner (see Fig. II–11).

CONSTRUCTION OF THE ELEMENTS $\Delta F \Delta G$ OF THE SCREENS

1. *By calculation.* We shall examine the means of calculating the elements $\Delta F \Delta G \Delta t \Delta D$.

How should the grains be distributed in an elemental volume? If we fix the mean density of the grains (= number of grains per unit of volume) we have to resolve a problem of probability in a four-dimensional space. A simpler method would be to consider and then calculate the four coordinates independently.

For the coordinate t the law of distribution of grains on the axis of time is:

$$P_x = c\,e^{-cx}\,dx \quad \text{or} \quad P_{x_i} = e^{-civ}\,c\Delta x_i. \qquad \text{(r)} \quad \text{(See Appendix I.)}$$

For the coordinates G, F, D the stochastic law will be:

$$f(j)\,dj = \frac{2}{a}\left(1 - \frac{j}{a}\right) dj \qquad \text{(r')}$$

or

$$P_i = \frac{2}{2\cdot 10^n}\left(1 - \frac{i}{2\cdot 10^n - 1}\right). \quad \text{(See Appendix I.)}$$

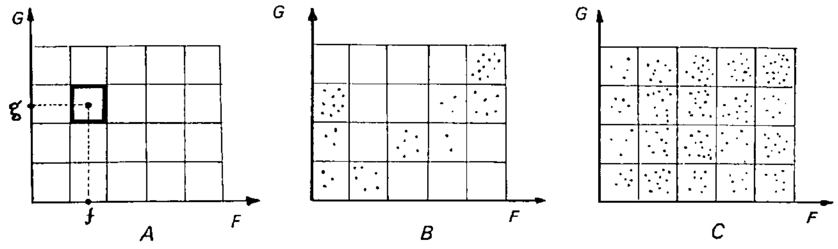


Fig. II–9

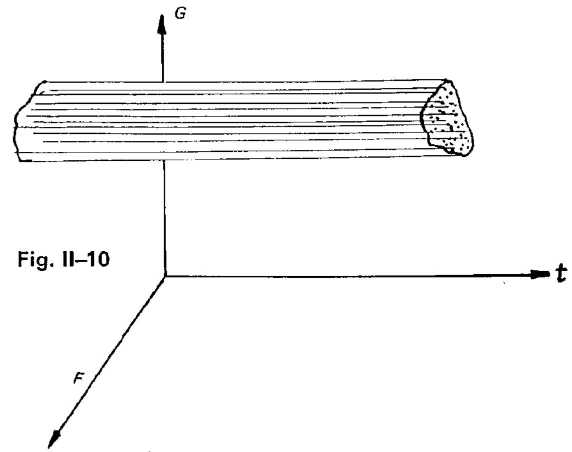


Fig. II–10

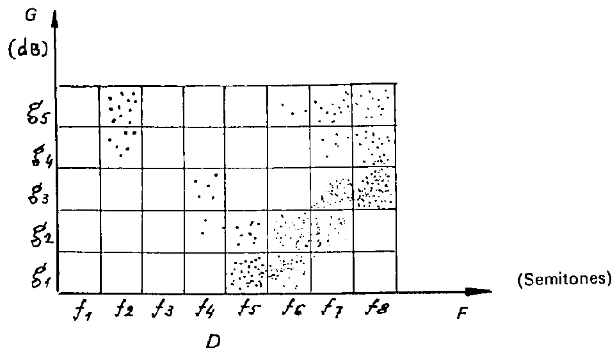


Fig. II–11

From these formulae we can draw up tables of the frequencies of the values t, G, F, D (see the analogous problem in Chapter I). These formulae are in our opinion privileged, for they arise from very simple reasoning, probably the very simplest; and it is essential to start out with a minimum of terms and constraints if we wish to keep to the principle of the tabula rasa (1st and 3rd rules of Descartes's Discourse on Method).

Let there be one of these elemental volumes $\Delta t \Delta D \Delta F \Delta G$ of the screen at the moment t. This volume has a density D taken from the table derived from formula (r'). Points on Δt are defined with a linear density $D = c$ according to the table defined by formula (r). To each point is attributed a sonic grain of frequency f and intensity g, taken from within the rectangle $\Delta F \Delta G$ by means of the table of frequencies derived from formula (r'). The correspondences are made graphically or by random successive drawings from urns composed according to the above tables.

2. *Mechanically*. *a*. On the tape recorder: The grains are realized from sinusoidal sounds whose durations are constant, about 0.04 sec. These grains must cover the selected elemental area $\Delta F \Delta G$. Unfolding in time is accomplished by using the table of durations for a minimum density $c = D$. By mixing sections of this tape with itself, we can obtain densities varying geometrically with ratio 1:2:3 . . . according to the number of tracks that we use. *b*. On computers: The grains are realized from wave forms duly programmed according to Gabor's signals, for a computer to which an analogue converter has been coupled. A second program would provide for the construction of the elemental volumes $\Delta t \Delta D \Delta F \Delta G$ from formulae (r) and (r').

First General Comment

Take the cell $\Delta F \Delta G \Delta t$. Although occupied in a homogeneous manner by grains of sound, it varies in time by fluctuating around a mean density d_m. We can apply another argument which is more synthetic, and admit that these fluctuations will exist in the most general case anyhow (if the sound is long enough), and will therefore obey the laws of chance. In this case, the problem is put in the following manner:

Given a prismatic cloud of grains of density d_m, of cross section $\Delta F \Delta G$ and length $\sum \Delta t$, what is the probability that d grains will be found in an elemental volume $\Delta F \Delta G \Delta t$? If the number d_m is small enough, the probability is given by Poisson's formula:

$$P_k = \frac{(d_m)^k}{K!} e^{-d_m}.$$

For the definition of each grain we shall again use the methods described above.

Second General Comment (Vector Space) [8]

We can construct elemental cells $\Delta F \Delta G$ of the screens not only with points, but with elemental vectors associated with the grains (vector space). The mean density of 0.04 sec/grain really implies a small vector. The particular case of the grain occurs when the vector is parallel to the axis of time, when its projection on the plane (FG) is a point, and when the frequency of the grain is constant. In general, the frequencies and intensities of the grains can be variable and the grain a very short glissando (see Fig. II–12).

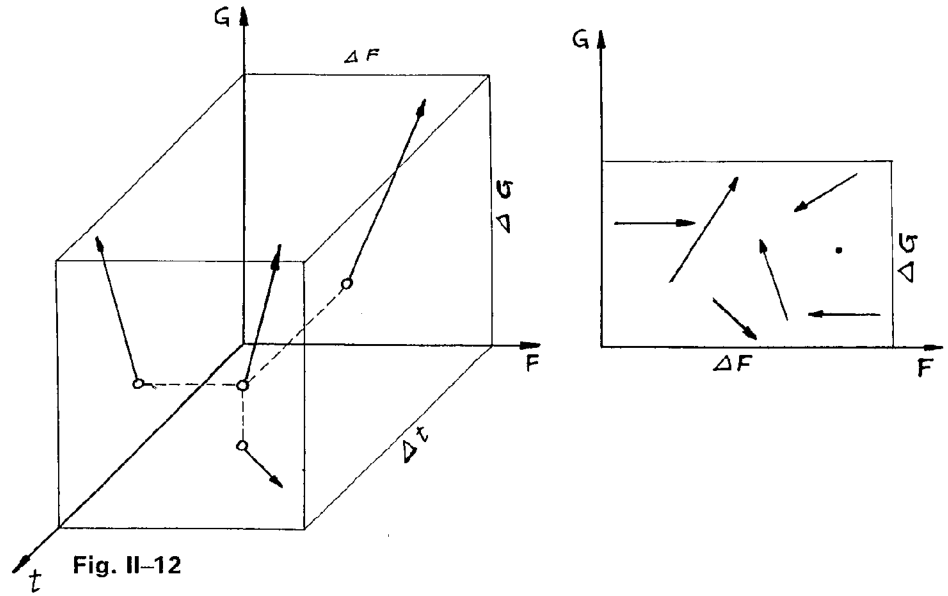


Fig. II–12

In a vector space (FG) thus defined, the construction of screens would perhaps be cumbersome, for it would be necessary to introduce the idea of speed and the statistical distribution of its values, but the interest in the undertaking would be enormous. We could imagine screens as the basis of granular fields which are magnetized or completely neutral (disordered).

In the case of total disorder, we can calculate the probability $f(v)$ of the existence of a vector v on the plane (FG) using Maxwell's formula as applied to two dimensions [11]:

$$f(v) = \frac{2v}{a^2} e^{-v^2/a^2}.$$

For the mean value $v_1 \leq v_m \leq v_2$,

$$P(v_m) = \frac{2\sqrt{\pi}}{a}\left\{\theta(\lambda_1) - \theta(\lambda_2)\right\}$$

in which $\lambda_i = v_i/a$ and

$$\theta(\lambda_i) = \frac{1}{\sqrt{\pi}}\int_{-\lambda_i}^{+\lambda_i} e^{-\lambda^2}\, d\lambda,$$

for $\lambda_1 \leq \lambda \leq \lambda_2$ (normal Gaussian law) [12]. In any case, whether it is a matter of a vector space or a scalar space does not modify the arguments [13].

Summary of the Screens

1. A screen is described by a set of clouds that are themselves a set of elemental rectangles $\Delta F \Delta G$, and which may or may not contain grains of sound. These conditions exist at the moment t in a slice of time Δt, as small as desired.

2. The grains of sound create a density peculiar to each elemental rectangle $\Delta F \Delta G$ and are generally distributed in the rectangles in an ergodic manner. (The ergodic principle states that the capricious effect of an operation that depends on chance is regularized more and more as the operation is repeated. Here it is understood that a very large succession of screens is being considered [14].)

3. The conception of the elemental volume $\Delta F \Delta G \Delta T \Delta D$ is such that no simultaneity of grains is generally admitted. Simultaneity occurs when the density is high enough. Its frequency is bound up with the size of the density. It is all a question of scale and this paragraph refers above all to realization. The temporal dimension of the grain (vector) being of the order of 0.04 sec., no systematic overlapping of two grains (vectors) will be accepted when the elementary density is, for example, $D_0 = 1.5$ grains/sec. And as the surface distribution of the grains is homogeneous, only chance can create this overlapping.

4. The limit for a screen may be only one pure sound (sinusoidal), or even no sound at all (empty screen).

ELEMENTARY OPERATIONS ON SCREENS

Let there be a complex sound. At an instant t of its life during a thickness Δt it can be represented by one or several clouds of grains or vectors on the plane (FG). This is the definition which we gave for the

screen. The junction of several of these screens in a given order describes or prescribes the life of this sonic complex. It would be interesting to envisage in all its generality the manner of combining and juxtaposing screens to describe, and above all to construct, sonic evolutions, which may be continuous or discontinuous, with a view to playing with them in a composition. To this end we shall borrow the terminology and symbolism of modern algebra, but in an elementary manner and as a form of introduction to a further development which we shall not undertake at the moment.

Comment: It does not matter whether we place ourselves on the plane of physical phenomena or of perception. In general, on the plane of perception we consider arithmetically that which is geometrical on the physical plane. This can be expressed in a more rigorous manner. Perception constitutes an additive group which is *almost* isomorphic with a physical excitation constituting a multiplicative group. The "almost" is necessary to exorcise approximations.

Grains or vectors on the plane (FG) constitute a cloud. A screen can be composed of no grain at all or of several clouds of grains or vectors (see Fig. II–13).

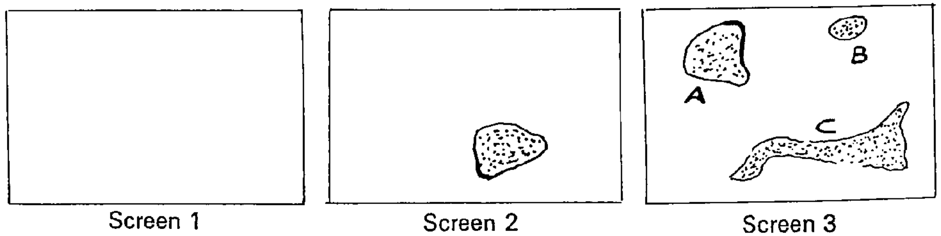


Fig. II–13

To notate that a grain or vector a belongs to a cloud E, we write $a \in E$ the contrary is written $a \notin E$. If all the grains of a cloud X are grains of another cloud Y, it is said that X is included in Y or that X is a part or sub-cloud of Y. This relation is notated $X \subset Y$ (inclusion).

Consequently we have the following properties:

$$X \subset X \quad \text{for any } X.$$
$$X \subset Y \quad \text{and} \quad Y \subset Z \quad \text{imply} \quad X \subset Z.$$

When $X \subset Y$ and $Y \subset X$, the clouds X and Y consist of the same grains; they are indistinguishable and the relation is written: $X = Y$ (equation).

A cloud may contain as little as a single grain. A cloud X is said to be empty when it contains no grain a, such that $a \in X$. The empty cloud is notated $\varnothing$.

ELEMENTARY OPERATIONS

These operations apply equally well to clouds and to screens. We can therefore use the terms "screen" and "cloud" indiscriminately, with cloud and grain as "constitutive elements."

The *intersection* of two screens A and B is the screen of clouds which belong to both A and B. This is notated as $A \cap B$ and read as "A inter B" (Fig. II–14). When $A \cap B = \varnothing$, A and B are said to be *disjoint* (Fig. II–15). The *union* of two screens A and B is the set of clouds which belong to both A and/or B (Fig. II–16). The *complement* of a screen A in relation to a screen E containing A is the set of clouds in E which do *not* belong to A. This is notated C_EA when there is no possible uncertainty about E (Fig. II–17). The *difference* $(A - B)$ of A and B is the set of clouds of A which do not belong to B. The immediate consequence is $A - B = A - (A \cap B) = C_A(A \cap B)$ (Fig. II–18).

We shall stop this borrowing here; however, it will afford a stronger, more precise conception on the whole, better adapted for the manipulations and arguments which follow.

DISTINCTIVE CHARACTERISTICS OF THE SCREENS

In our desire to create sonic complexes from the temporary accepted primary matter of sound, sine waves (or their replacements of the Gabor sort), and to create sonic complexes as rich as but more extraordinary than natural sounds (using scientifically controlled evolutions on very general abstract planes), we have implicitly recognized the importance of three basic factors which seem to be able to dominate both the theoretical construction of a sonic process and its sensory effectiveness: 1. the density of the elementary elements, 2. the topographic situation of events on the screens, and 3. the order or disorder of events.

At first sight then the density of grains or vectors, their topography, and their degree of order are the indirect entities and aspects perceived by our macroscopic ears. It is wonderful that the ear and the mind follow objective reality and react directly in spite of gross inherent or cultural imperfections. Measurement has been the foundation of the experimental sciences. Man voluntarily treats himself as a sensory invalid, and it is for this reason that he has armed himself, justifiably, with machines that measure other machines. His ears and eyes do measure entities or physical phenomena, but they are transformed as if a distorting filter came between immediate perception and consciousness. About a century ago the logarithmic

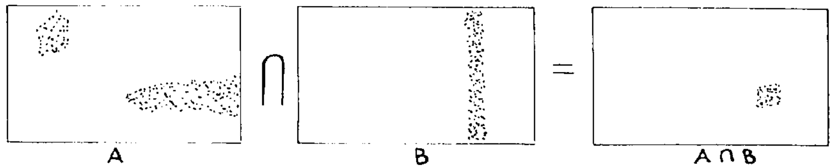


Fig. II–14

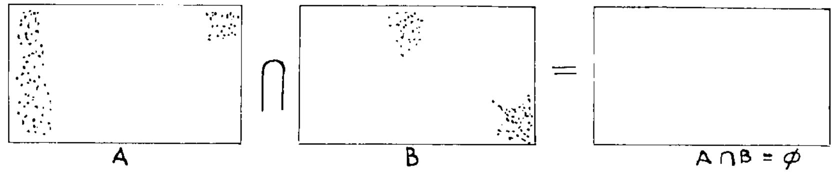


Fig. II–15

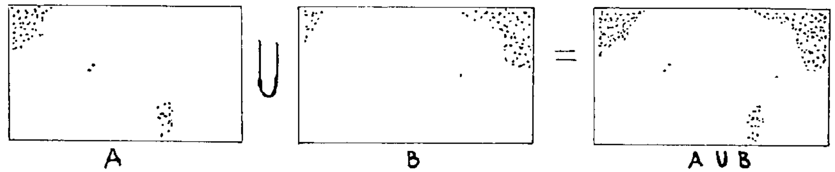


Fig. II–16

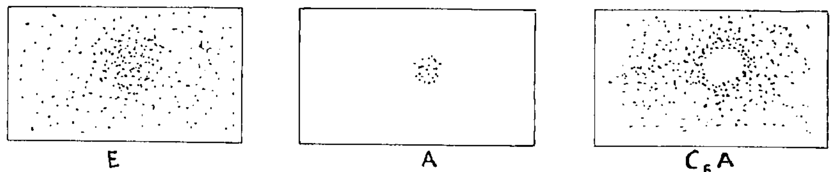


Fig. II–17

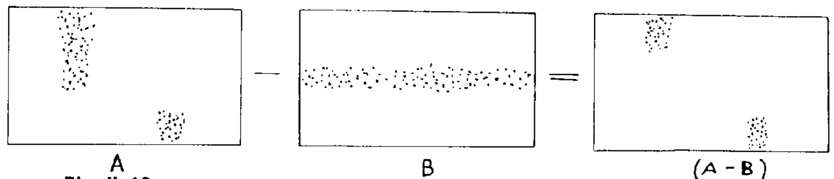


Fig. II–18

law of sensation was discovered; until now it has not been contradicted. But as knowledge never stops in its advance, tomorrow's science will without doubt find not only a greater flexibility and exactitude for this law, but also the beginnings of an explanation of this distorting filter, which is so astonishing.

This statistical, but none the less quasi-one-to-one transformation of excitation into perception has up to now allowed us to argue about physical entities, such as screens, all the while thinking "perceived events." A reciprocity of the same kind between perception and its comprehension permits us to pass from the screens to the consequent distinctive characteristics. Thus the arguments which we shall pursue apply equally well to pure concepts and to those resulting from perception or sensory events, and we may take the attitude of the craftsman or the listener.

We have already remarked on the density and the topography of grains and cells and we have acknowledged the concepts of order and disorder in the homogeneous superficial distribution or grains.

We shall examine closely the concept of order, for it is probably hidden behind the other two. That is to say, density and topography are rather palpably simplified embodiments of this fleeting and many-sided concept of disorder.

When we speak of order or disorder we imply first of all "objects" or "elements." Then, and this is already more complex, we define the very "elements" which we wish to study and from which we wish to construct order or disorder, and their scale in relation to ours. Finally we qualify and endeavor to measure this order or disorder. We can even draw up a list of all the orders and disorders of these entities on all scales, from all aspects, for all measurements, even the characteristics of order or disorder of this very list, and establish anew aspects and measurements.

Take the example of the gases mentioned above. On the molecular scale (and we could have descended to the atomic level), the absolute values of the speeds, directions, and distributions in space are of all sorts. We can distinguish the "elements" which carry order or disorder. Thus if we could theoretically isolate the element "directions" and assume that there is an obligation to follow certain privileged directions and not all directions, we could impose a certain degree of order which would be independent of the other elements constituting the concept "gas." In the same way, given enough time, the values of the speeds of a single molecule will be distributed around a mean value and the size of the deviations will follow Gauss's law. There we will have a certain order since these values are vastly more

numerous in the neighborhood of their mean than anywhere else, from infinitely small to infinitely large.

Let us take another example, more obvious and equally true. A crowd of 500,000 persons is assembled in a town square. If we examine the group displacement of this crowd we can prove that it does not budge. However, each individual moves his limbs, his head, his eyes, and displaces his center of gravity by a few centimeters in every direction. If the displacements of the centers of gravity were very large the crowd would break up with yells of terror because of the multiple collisions between individuals. The statistical values of these displacements normally lie between very narrow limits which vary with the density of the crowd, From the point of view of these values as they affect immobility, the disorder is weak.

Another characteristic of the crowd is the orientation of the faces. If an orator on a balcony were to speak with a calming effect, 499,000 faces would look at the balcony and 998,000 ears would listen to the honeyed words. A thousand or so faces and 2000 ears would be distracted for various reasons: fatigue, annoyance, imagination, sexuality, contempt, theft, etc. We could confirm, along with the mass media, without any possible dispute, that crowd and speaker were in complete accord, that 500,001 people, in fact, were unanimous. The degree of order that the speaker was after would attain a maximum for a few minutes at least, and if unanimity were expressed equally strongly at the conclusion of the meeting, the orator could be persuaded that the ideas were as well ordered in the heads of the crowd as in his own.

We can establish from these two extreme examples that the concept of order and disorder is basic to a very large number of phenomena, and that even the definition of a phenomenon or an object is very often attributable to this concept. On the other hand, we can establish that this concept is founded on precise and distinct groups of elements; that the scale is important in the choice of elements; and finally, that the concept of order or disorder implies the relationship between effective values over all possible values that the elements of a group can possess. This introduces the concept of probability in the quantitative estimate of order or disorder.

We shall call the number of distinct elements in a group its *variety*. We shall call the degree of order or disorder definable in a group of elements its *entropy*. Entropy is linked with the concept of variety, and for that very reason, it is linked to the probability of an element in the group. These concepts are those of the theory of communications, which itself borrows from the second law of thermodynamics (Boltzmann's theorem H) [15].

Variety is expressed as a pure number or as its logarithm to the base 2. Thus human sex has two elements, male and female, and its variety is 2, or 1 bit: 1 bit = $\log_2 2$.

Let there be a group of probabilities (a group of real numbers p, positive or zero, whose sum is 1). The entropy H of this group is defined as

$$H = -K \sum p_i \log p_i.$$

If the logarithmic base is 2, the entropy is expressed in bits. Thus if we have a sequence of heads and tails, the probability of each is $\frac{1}{2}$, and the entropy of this sequence, i.e., its uncertainty at each throw, will be 1 bit. If both sides of the coin were heads, the uncertainty would be removed and the entropy H would be zero.

Let us suppose that the advent of a head or a tail is not controlled by tossing the coin, but by a fixed, univocal law, e.g., heads at each even toss and tails at each odd toss. Uncertainty or disorder is always absent and the entropy is zero. If the law becomes very complex the appearance of heads or tails will seem to a human observer to be ruled by the law of chance, and disorder and uncertainty will be reestablished. What the observer could do would be to count the appearances of heads and tails, add up their respective frequencies, deduce their probabilities, and then calculate the entropy in bits. If the frequency of heads is equal to that of tails the uncertainty will be maximum and equal to 1 bit.

This typical example shows roughly the passage from order to disorder and the means of calibrating this disorder so that it may be compared with other states of disorder. It also shows the importance of scale. The intelligence of the observer would assimilate a deterministic complexity up to a certain limit. Beyond that, in his eyes, the complexity would swing over into unforeseeability and would become chance or disorder; and the visible (or macroscopic would slide into the invisible (or microscopic). Other methods and points of view would be necessary to observe and control the phenomena.

At the beginning of this chapter we admitted that the mind and especially the ear were very sensitive to the order or disorder of phenomena. The laws of perception and judgment are probably in a geometrical or logarithmic relation to the laws of excitation. We do not know much about this, and we shall again confine ourselves to examining general entities and to tracing an overall orientation of the poetic processes of a very general kind of music, without giving figures, moduli, or determinisms. We are still optimistic enough to think that the interdependent experiment and action of abstract

hypotheses can cut biologically into the living conflict between ignorance and reality(if there is any reality).

Study of Ataxy (order or disorder) on the Plane of a Cloud of Grains or Vectors

Axis of time: The degree of ataxy, or the entropy, is a function of the simultaneity of the grains and of the distinct intervals of time between the emission of each grain. If the *variety* of the durations of the emissions is weak, the entropy is also weak. If, for example, in a given Δt each grain is emitted at regular intervals of time, the temporal variety will be 1 and the entropy zero. The cloud will have zero ataxy and will be completely ordered. Conversely, if in a fairly long succession of Δt the grains are emitted according to the law $P_x = \delta e^{-\delta x}\, dx$, the degree of ataxy will be much larger. The limit of entropy is infinity, for we can imagine all possible values of time intervals with an equal probability. Thus, if the variety is $n \rightarrow \infty$, the probability for each time interval is $p_i = 1/n$, and the entropy is

$$H = -K \sum_{i=0}^{n} p_i \log p_i$$

$$H = -K \sum_{i=0}^{n} \frac{1}{n} \log \frac{1}{n} = -K\, n \frac{1}{n} \log \frac{1}{n} = -K \log \frac{1}{n} = K \log n$$

for $n \rightarrow \infty$, $H \rightarrow \infty$.

This is less true in practice, for a Δt will never offer a very great variety of durations and its entropy will be weak. Furthermore a sonic composition will rarely have more than 100,000 Δt's, so that $H \leq \log 100{,}000$ and $H \leq 16.6$ bits.

Axis of frequencies (*melodic*): The same arguments are valid here but with greater restriction on the variety of melodic intervals and on the absolute frequencies because of the limits of the audible area.

Entropy is zero when the variety of frequencies of grains is 1, i.e., when the cloud contains only one pure sound.

Axis of intensity and density: The above observations are valid. Therefore, if at the limit, the entropies following the three axes of an element $\Delta F \Delta G \Delta t \Delta D$ are zero, this element will only contain one pure sound of constant intensity emitted at regular intervals.

In conclusion, a cloud may contain just one single pure sound emitted at regular intervals of time (see Fig. II–19), in which case its mean entropy (arithmetic mean of the three entropies) would be zero. It may contain chaotically distributed grains, with maximum ataxy and maximum mean entropy (theoretically ∞). Between these two limits the grains may be distributed in an infinite number of ways with mean entropies between 0 and the maximum and able to produce both the Marseillaise and a raw, dodecaphonic series.

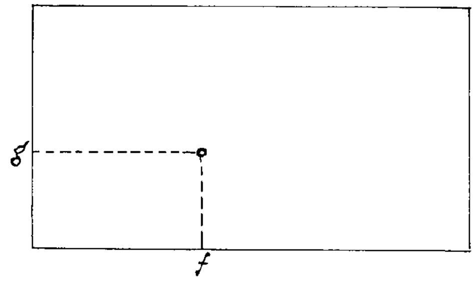


Fig. II–19

A single grain emitted at regular intervals of time

Parentheses

GENERAL OBSERVATIONS ON ATAXY

Taking this last possibility as a basis, we shall examine the very general formal processes in all realms of thought, in all physical and psychic realities.

To this end we shall imagine a "Primary Thing," malleable at will; capable of deforming instantaneously, progressively, or step-by-step; extendible or retractable; unique or plural; as simple as an electron (!) or as complex as the universe (as compared to man, that is).

It will have a given mean entropy. At a defined time we will cause it to undergo a transformation. From the point of view of ataxy this transformation can have one of three effects:

1. The degree of complexity (variety) does not change; the transformation is neutral; and the overall entropy does not change.
2. The degree of complexity increases and so does the entropy.
3. The transformation is a simplifying one, and the entropy diminishes.

Thus the neutral transformation may act on and transform: perfect disorder into perfect disorder (fluctuations), partial disorder into partial disorder, and perfect order into perfect order.

Multiplicative transformation transforms: perfect disorder into perfect disorder, partial disorder into greater disorder, and perfect order into partial disorder.

And simplifying transformation transforms: perfect disorder into partial disorder, partial order into greater order, and partial order into perfect order. Fig. II–20 shows these transformations in the form of a kinematic diagram.

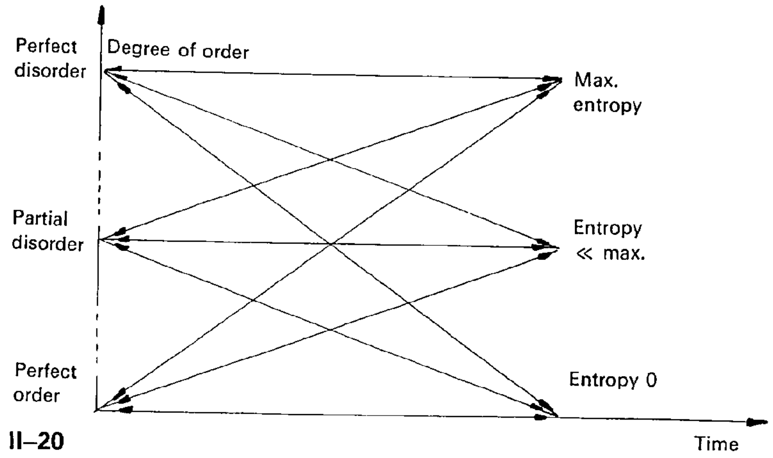


Fig. II–20

STUDY OF ATAXY AT THE LEVEL OF SCREENS (SET OF CLOUDS)

From the above discussion, a screen which is composed of a set of cells $\Delta F \Delta G$ associated with densities during a slice of time Δt, may be dissociated according to the two characters of the grains, frequency and amplitude, and affected by a mean entropy. Thus we can classify screens according to the criterion of ataxy by means of two parameters of disorder: the variety of the frequencies and the variety of the intensities. We shall make an abstraction of the temporal distribution of the grains in Δt and of the density, which is implicitly bound up with the varieties of the two fundamental sizes of the grain. In symbolic form:

Perfect disorder $= \infty$
Partial disorder $= n$ or m
Partial order $= m$ or n
Perfect order $= 0$.

From the point of view of ataxy a screen is formulated by a pair of entropy values ascribed to a pair of frequencies and intensities of its grains. Thus the pair (n, ∞) means a screen whose frequencies have quite a small entropy (partial order or disorder) and whose intensities have maximum entropy (more or less perfect disorder).

CONSTRUCTION OF THE SCREENS

We shall quickly survey some of the screens in the entropy table in Fig. II–21.

Perfect disorder F	Perfect disorder G	Partial disorder F	Partial disorder G	Perfect order F	Perfect order G	Symbol	Description	Diagram	Diagram
F	G					∞, ∞	Unique screen		
F			G			∞, *n*	Infinite number of screens		
F					G	∞, 0	Unique screen		
	G	F				*n*, ∞	Infinite number of screens		
	G			F		0, ∞	Unique screen, pure sounds		
		F	G			*n*, *m*	Infinite number of screens		
		F			G	*n*, 0	Infinite number of screens		
			G	F		0, *n*	Infinite number of screens		
				F	G	0, 0	Unique screen, pure sound		

Fig. II–21. Screen Entropy Table

SCREEN (∞, ∞)

Let there be a very large number of grains distributed at random over the whole range of the audible area and lasting an interval of time equal to Δt. Let there also be a grid fine enough so that the average density will not be more than 30 grains per cell. The distribution law is then given by Poisson's formula

$$P_k = \frac{(d_m)^k}{K!} e^{-d_m},$$

where d_m is the mean density and P_k the probability that there will be k grains in a cell. If d_m becomes greater than about 30, the distribution law will become *normal.*

Fig. II–22 is an example of a Poisson distribution for a mean density $d_m = 0.6$ grains/cell in a grid of 196 cells for a screen (∞, ∞).

Thus we may construct the (∞, ∞) screens by hand, according to the distributions for the rows and columns, or with suitable computer programs.

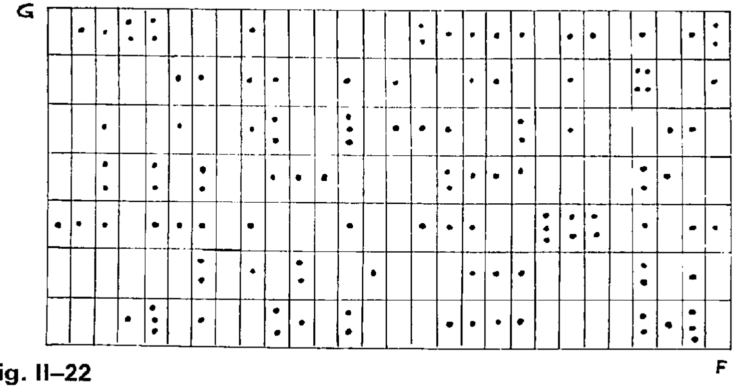


Fig. II–22

For a very high mean density the screens in which disorder is perfect (maximum) will give a very rich sound, almost a white sound, which will never be identical throughout time. If the calculation is done by hand we can construct a large number of (∞, ∞) screens from the first (∞, ∞) screen in order to avoid work and numerical calculation for each separate screen. To this end we permute the cells by column and row (see Fig. II–23).

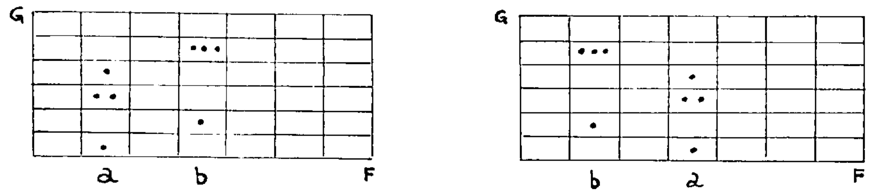


Fig. II–23. Example of Permutation by Columns

Discussion. It is obvious that for a high mean density, the greater the number of cells, the more the distribution of grains in one region of the screen tends to regularize (ergodism) and the weaker are the fluctuations from one cell or cloud to another. But the absolute limits of the density in the cells in the audible area will be a function of the technical means available: slide rules, tables, calculating machines, computers, ruled paper, orchestral instruments, tape recorders, scissors, programmed impulses of pure sounds, automatic splicing devices, programmed recordings, analogue converters, etc.

If each cell is considered as a symbol defined by the number of grains k, the entropy of the screen (for a given fineness of grid) will naturally be affected by the mean density of the grains per cell and will grow at the same time. It is here that a whole series of statistical experiments will have to circumscribe the perceptible limits of ataxy for these screens (∞, ∞) and even express the color nuances of white sound. It is very possible that the ear classifies in the same file a great number of screens whose entropies vary tremendously. There would result from this an impoverishment and a simplification of the communication: physical information $\rightarrow$ perception, but at least there will be the advantage that the work involved in constructing screens will be considerably reduced.

ALL SCREENS

Starting from a few screens and applying the *elementary operations* we can construct all the screens of the entropy table. See Fig. II–24 for a few examples. In practice, frequency and intensity filters imitate these elementary operations perfectly.

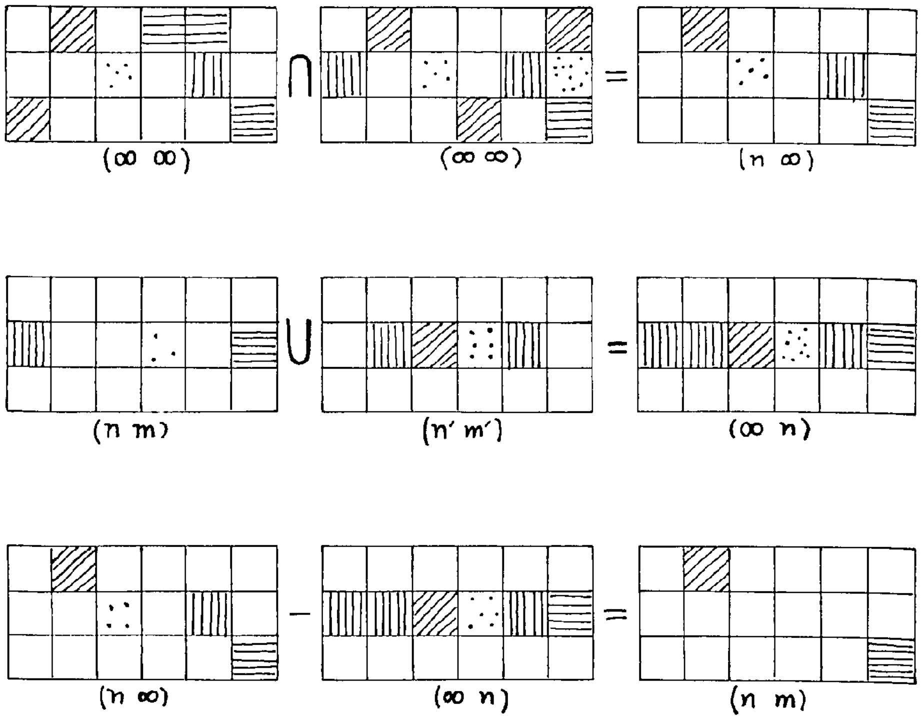


Fig. II–24

LINKING THE SCREENS

Up to now we have admitted that any sound or music could be described by a number of screens arranged in the lexicographic order of the pages of a book. If we represent each screen by a specific symbol (one-to-one coding), the sound or the music can be translated by a succession of symbols called a protocol:

$$a\ b\ g\ k\ a\ b \cdots b\ g \cdots$$

each letter identifying screens and moments t for isochronous Δt's.

Without seeking the causes of a particular succession of screens, i.e., without entering into either the physical structure of the sound or the logical structure of the composition, we can disengage certain modes of succession and species of protocols [16]. We shall quickly review the elementary definitions.

Any matter or its unique symbol is called a *term*. Two successive terms cause a *transition* to materialize. The second term is called the *transform* and the change effected is represented by term $A \rightarrow$ term B, or $A \rightarrow B$.

A *transformation* is a collection of transitions. The following example is drawn from the above protocol:

$$\downarrow \begin{array}{lllll} a & b & g & k & \cdots \\ b & g & k & a & \cdots, \end{array}$$

another transformation with musical notes:

$$\downarrow \begin{array}{llll} C & D & E & \cdots \\ B & G & A & \cdots \end{array}$$

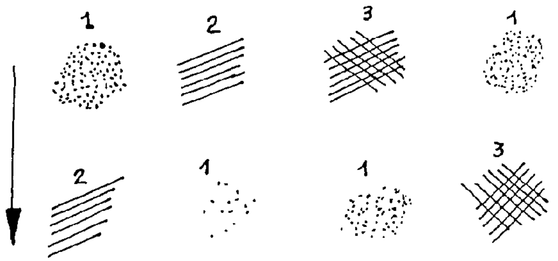
A transformation is said to be *closed* when the collection of transforms contains only elements belonging to the collection of terms, for example: the alphabet,

$$\downarrow \begin{array}{lllll} a & b & c & \cdots & z \\ b & c & d & \cdots & a \end{array}$$

musical notes,

↓	C	D♭	D	E♭	E	F	G♭	G	A	B♭	B
	D	G♭	G	C	F	B	A	D♭	E♭	E	B♭

musical sounds,



1. Cloud of sound-points, e.g., pizzicati
2. Network of parallel glissandi in one direction
3. Network of parallel glissandi in two directions.

an infinity of terms,

↓	1	2	3	4	5	6	...
	6	7	4	100	1	2	...

A transformation is *univocal* or single-valued (mapping) when each term has a single transform, for example:

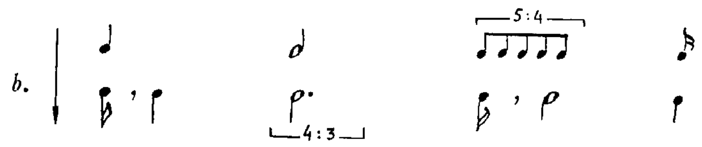
↓	b	a	c	e	...
	a	b	c	d	...

The following are examples of transformations that are not univocal:

a.

↓	a	b	c
	b, c	d	m, n, p

b.



c. timbre change of a group of values

Timbres ↓	clarinets	oboes	strings	timpani	brass
	timpani, strings	timpani, bassoon	brass	oboes	strings, oboes

and *d.* concrete music characteriology [4, 5]

"Manner" ↓	nil	vibrated	trembled	cyclical	irregular
	cyclical or trembled	irregular	nil or irregular	trembled	nil or vibrated or cyclical

A transformation is a one-to-one mapping when each term has a single transform and when each transform is derived from a single term, for example:

↓	a	b	c	d	...
	b	a	d	c	...

MATRICAL REPRESENTATION

A transformation:

↓	a	b	c
	a	c	c

can be represented by a table as follows:

↓	a	b	c
a	+	0	0
b	0	0	0
c	0	+	+

or

↓	a	b	c
a	1	0	0
b	0	0	0
c	0	1	1

This table is a matrix of the transitions of the collection of terms to a collection of transforms.

PRODUCT

Let there be two transformations T and U:

T:

↓	a	b	c	d
	b	d	a	b

and U:

↓	a	b	c	d
	d	c	d	b

In certain cases we can apply to a term n of T a transformation T, then a transformation U. This is written: $U[T(n)]$, and is the product of the two transformations T and U, on condition that the transforms of T are terms of U. Thus, first T: $a \rightarrow b$, then U: $b \rightarrow c$, which is summarized as $V = UT: a \rightarrow c$.

To calculate the product applied to all the terms of T we shall use the following matrical representation:

$$T: \begin{array}{c|cccc} \downarrow & a & b & c & d \\ \hline a & 0 & 0 & 1 & 0 \\ b & 1 & 0 & 0 & 1 \\ c & 0 & 0 & 0 & 0 \\ d & 0 & 1 & 0 & 0 \end{array} \qquad U: \begin{array}{c|cccc} \downarrow & a & b & c & d \\ \hline a & 0 & 0 & 0 & 0 \\ b & 0 & 0 & 0 & 1 \\ c & 0 & 1 & 0 & 0 \\ d & 1 & 0 & 1 & 0 \end{array}$$

the total transformation V equals the product of the two matrices T and U in the order U, T.

$$\overset{U}{\begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{vmatrix}} \times \overset{T}{\begin{vmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}} = \overset{V}{\begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}}$$

KINEMATIC DIAGRAM

The kinematic or transition diagram is a graphical expression of transformation. To draw it each term is connected to its transform by an arrow pointed at the transform. The *representative point* of a kinematic diagram is an imaginary point which moves in jumps from term to term following the arrows of the diagram; for an example see Fig. II–25.

$$T: \begin{array}{|cccccccc} A & C & D & I & L & N & P & A \\ \downarrow D & D & I & A & N & A & N & N \end{array}$$

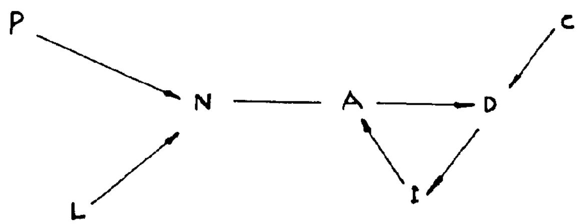


Fig. II–25

A transformation is really a mechanism and theoretically all the mechanisms of the physical or biological universes can be represented by

transformations under five conditions of correspondence:

1. Each state of the mechanism (continuity is broken down into discrete states as close together as is desired) is in a one-to-one correspondence with a term of the transformation.

2. Each sequence of states crossed by the mechanism by reason of its internal structure corresponds to an uninterrupted sequence of the terms of the transformation.

3. If the mechanism reaches a state and remains there (absorbing or stationary state), the term which corresponds to this state has no transform.

4. If the states of a mechanism reproduce themselves in the same manner without end, the transformation has a kinematic diagram in closed circuit.

5. A halt of the mechanism and its start from another state is represented in the diagram by a displacement of the representative point, which is not due to an arrow but to an arbitrary action on the paper.

The mechanism is determined when the corresponding transformation is univocal and closed. The mechanism is not determined when the corresponding transformation is many-valued. In this case the transformation is said to be *stochastic*. In a stochastic mechanism the numbers 0 and 1 in the transformation matrix must be replaced by relative frequencies. These are the alternative probabilities of various transformations. The determined mechanism is a particular case of the stochastic mechanism, in which the probabilities of transition are 0 and 1.

Example: All the harmonic or polyphonic rules of classical music could be represented by mechanisms. The fugue is one of the most accomplished and determined mechanisms. One could even generalize and say that the avant-garde composer is not content with following the mechanisms of his age but proposes new ones, for both detail and general form.

If these probabilities are constant over a long period of time, and if they are independent of the states of origin, the stochastic sequence is called, more particularly, a Markov chain.

Let there be two screens A and B and a protocol of 50 transitions:

ABABBBABAABABABABBBBABAABABBAABABBABAAABABBA ABBABBA.

The real frequencies of the transitions are:

$A \to B$	17 times	$B \to A$	17 times
$A \to A$	6 times	$B \to B$	10 times
	23 times		27 times

a diminution of the entropy. If melodic or harmonic liaisons are effected and perceived in the same distribution, unpredictability and entropy are both diminished.

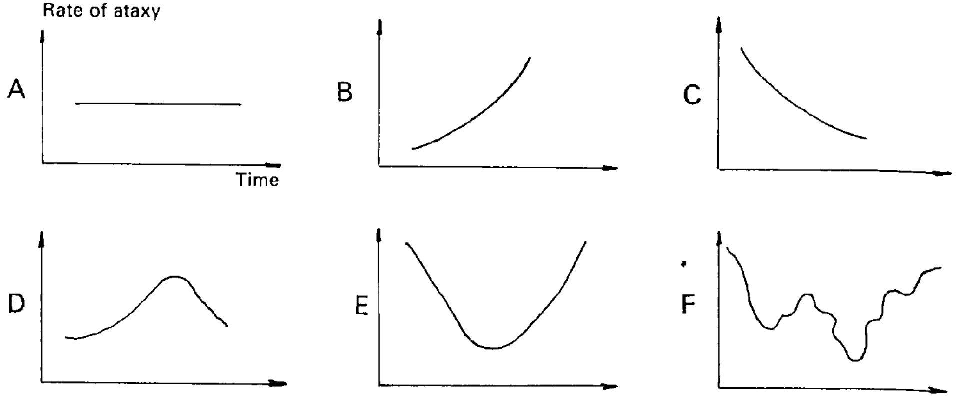


Fig. II–26

A. The evolution is nil. B. The rate of disorder and the richness increase. C. Ataxy decreases. D. Ataxy increases and then decreases. E. Ataxy decreases and then increases. F. The evolution of the ataxy is very complex, but it may be analyzed from the first three diagrams.

Thus after the first unfolding of a series of twelve sounds of the tempered scale, the unpredictability has fallen to zero, the constraint is maximum, the choice is nil, and the entropy is zero. Richness and hence interest are displaced to other fields, such as harmonies, timbres, and durations, and many other compositional wiles are aimed at reviving entropy. In fact sonic discourse is nothing but a perpetual fluctuation of entropy in all its forms [17].

However, human sensitivity does not necessarily follow the variation in entropy even if it is logarithmic to an appropriate base. It is rather a succession or a protocol of strains and relaxations of every degree that often excites the listener in a direction contrary to that of entropy. Thus Ravel's *Bolero*, in which the only variation is in the dynamics, has a virtually zero entropy after the third or fourth repetition of the fundamental idea. However, the interest, or rather the psychological agitation, grows with time through the very fact of this immobility and banality.

All incantatory manifestations aim at an effect of maximum tension with minimum entropy. The inverse is equally true, and seen from a certain

angle, white noise with its maximum entropy is soon tiresome. It would seem that there is no correspondence aesthetics ↔ entropy. These two entities are linked in quite an independent manner at each occasion. This statement still leaves some respite for the free will of the composer even if this free will is buried under the rubbish of culture and civilization and is only a shadow, at the least a tendency, a simple stochasm.

The great obstacle to a too hasty generalization is chiefly one of logical order; for an object is only an object as a function of its definition, and there is, especially in art, a near-infinity of definitions and hence a near-infinity of entropies, for the notion of entropy is an epiphenomenon of the definition. Which of these is valid? The ear, the eye, and the brain unravel sometimes inextricable situations with what is called intuition, taste, and intelligence. Two definitions with two different entropies can be perceived as identical, but it is also true that the set of definitions of an object has its own degree of disorder. We are not concerned here with investigating such a difficult, complex, and unexplored situation, but simply with looking over the possibilities that connected realms of contemporary thought promise, with a view to action.

To conclude briefly, since the applications which follow are more eloquent than explanatory texts, we shall accept that a collection or book of screens can be expressed by matrices of transition probabilities having parameters. They are affected by a degree of ataxy or entropy which is calculable under certain conditions. However, in order to render the analysis and then the synthesis of a sonic work within reach of understanding and the slide rule, we shall establish three criteria for a screen:

1. TOPOGRAPHIC CRITERION

The position of the cells $\Delta F \Delta G$ on the audible area is qualitatively important, and an enumeration of their possible combinations is capable of creating a group of well defined terms to which we can apply the concept of entropy and its calculation.

2. DENSITY CRITERION

The superficial density of the grains of a cell $\Delta F \Delta G$ also constitutes a quality which is immediately perceptible, and we could equally well define terms to which the concept and calculation of entropy would be applicable.

3. CRITERION OF PURE ATAXY (defined in relation to the grains of a screen)

A cell has three variables: mean frequency, mean amplitude, and mean density of the grains. For a screen we can therefore establish three independent or connected protocols, then three matrices of transition probabilities which may or may not be coupled. Each of the matrices will have its entropy and the three coupled matrices will have a mean entropy. In the procession of sound we can establish several series of three matrices and hence several series of mean entropies, their variations constituting the criterion of ataxy.

The first two criteria, which are general and on the scale of screens or cells, will not concern us in what follows. But the third, more conventional criterion will be taken up in detail in the next chapter.

Chapter III

Markovian Stochastic Music—Applications

In this chapter we will discuss two musical applications: *Analogique A*, for string orchestra, and *Analogique B*, for sinusoidal sounds, both composed in 1958–59.

We shall confine ourselves to a simple case in which each of the components G, F, D of the screen take only two values, following matrices of transition probability which will be coupled by means of parameters. In addition, the choice of probabilities in the matrices will be made in such a way that we shall have only the regular case, conforming to the chain of events theory as it has been defined in the work of Maurice Fréchet [14].

It is obvious that richer and more complex stochastic mechanisms are highly interesting to construct and to put in work, but in view of the considerable volume of calculations which they necessitate it would be useless to undertake them by hand, but very desirable to program them for the computer.

Nevertheless, despite the structural simplicity of what follows, the stochastic mechanism which will emerge will be a model, a standard subjacent to any others that are far more complex, and will serve to catalyze further studies of greater elaboration. For although we confine ourselves here to the study of screens as they have been defined in this study (sets of elementary grains), it goes without saying that nothing prevents the generalization of this method of structuralization (composition) for definitions of sonic entities of more than three dimensions. Thus, let us no longer suppose screens, but *criteria* of definitions of a sonic entity, such that for the timbre, degree of order, density, variation, and even the *criteria* of more or less