



Msc in Informatics Engineering
Advances in Digital Imaging and Computer
Vision
Histogram Matching on MRI scans

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1. Abstract

Magnetic resonance images (MRIs) with different imaging acquisitions have various image intensities. In order to conduct accurate image analysis, such as registration and segmentation, it is essential to perform intensity normalization of MRI. Histogram matching and histogram equalization methods are described and compared to each other.

2. Introduction

Histogram Matching (HM)

In image processing, histogram matching or histogram specification is the transformation of an image so that its histogram matches a specified histogram. It is possible to use histogram matching to balance detector responses as a relative detector calibration technique. It can be used to normalize two images, when the images were acquired at the same local illumination (such as shadows) over the same location, but by different sensors, atmospheric conditions or global illumination.

Raw Image & Reference Image -----> Output image

Histogram Equalization (HE)

Histogram equalization is a method in image processing of contrast adjustment using the image's histogram. Histogram equalization is the best method for image enhancement. It provides better quality of images without loss of any information.

This method usually increases the global contrast of many images, especially when the usable data of the image is represented by close contrast values. Through this adjustment, the intensities can be better distributed on the histogram. This allows for areas of lower local contrast to gain a higher contrast. Histogram equalization accomplishes this by effectively spreading out the most frequent intensity values.

Adaptive Histogram Equalization (AHE)

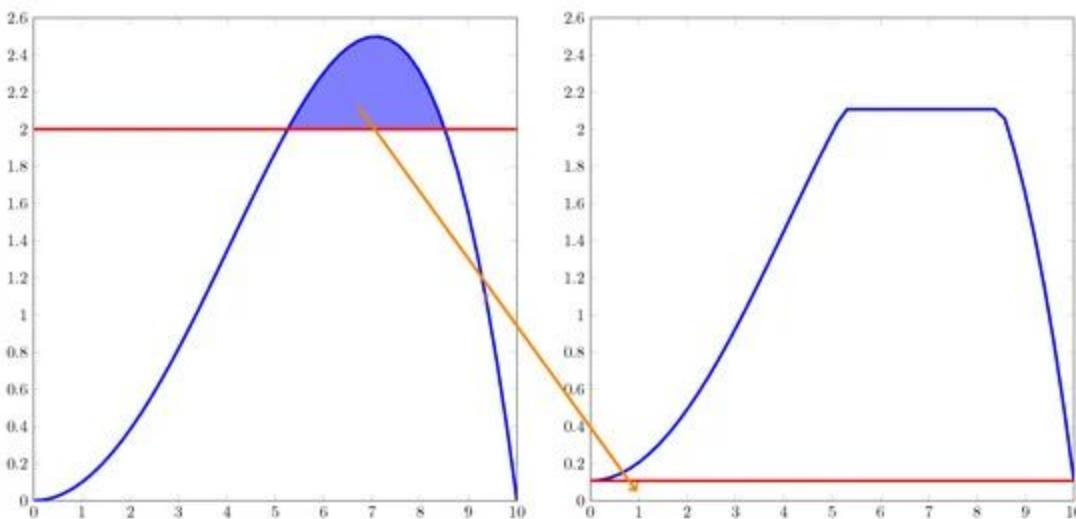
Adaptive histogram equalization (AHE) is a computer image processing technique used to improve contrast in images. It differs from ordinary histogram equalization in the respect that the adaptive method computes several histograms, each corresponding to a distinct section of the image, and uses them to redistribute the lightness values of the image. It is therefore suitable for improving the local contrast and enhancing the definitions of edges in each region of an image.

AHE has a tendency to over-amplify noise in relatively homogeneous regions of an image. A variant of adaptive histogram equalization called contrast limited adaptive histogram equalization (CLAHE) prevents this by limiting the amplification.

Contrast Limited Adaptive Histogram Equalization

In CLAHE, the contrast amplification in the vicinity of a given pixel value is given by the slope of the transformation function. This is proportional to the slope of the neighbourhood cumulative distribution function (CDF) and therefore to the value of the histogram at that pixel value. CLAHE limits the amplification by clipping the histogram at a predefined value before computing the CDF. This limits the slope of the CDF and therefore of the transformation function. The value at which the histogram is clipped, the so-called clip limit, depends on the normalization of the histogram and thereby on the size of the neighbourhood region. Common values limit the resulting amplification to between 3 and 4.

As an alternative to using [histeq](#), you can perform contrast-limited adaptive histogram equalization (CLAHE) and also (AHE) using the [adaphisteq](#)

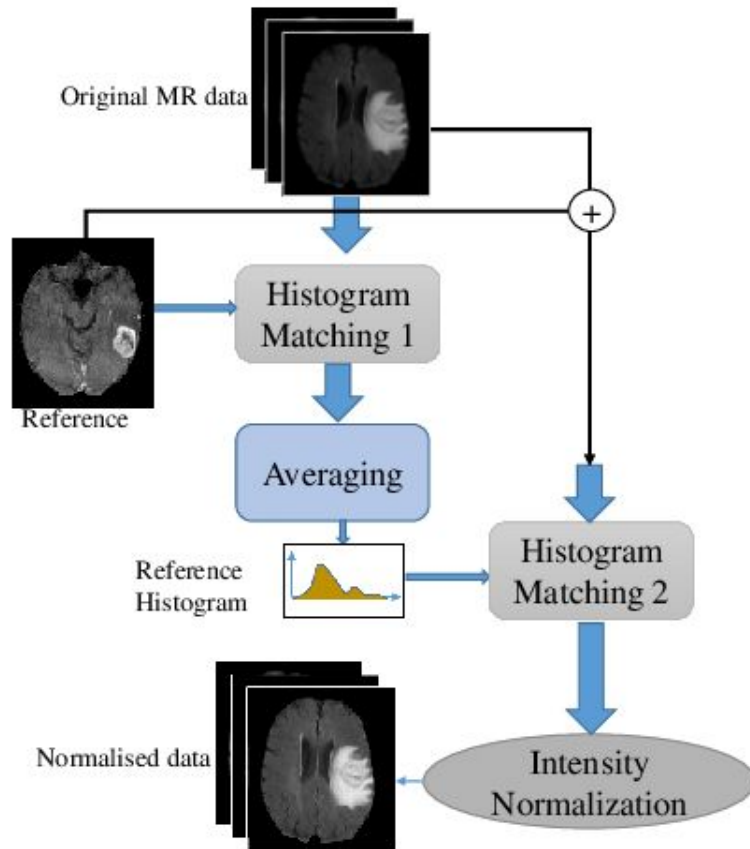


3. State of the art

Image processing is a vast and challenging domain with its applications in fields like medical, aerial and satellite images, industrial applications, law enforcement, and science. Image enhancement is used in the following cases: Removal of noise from image, enhancement of the dark image and highlight the edges of the objects in an image. The goal of image enhancement is to improve the image quality so that the processed image is better than the original image for a specific application or set of objectives.

MRI medical images are recently considered one of the most widely utilized for disease diagnostic in the field of medicinal. With this signification the MRI image suffers from contrast degradations. These contrast degradations in MRI images can be solved with image contrast enhancement techniques to make it more suitable for medical applications. The image contrast enhancement techniques are used to enhance the visibility quality of internal human body texture in magnetic resonance imaging (MRI) images. In this paper the Histogram Equalization (HE) and Adaptive Histogram Equalization (AHE) are applied to improve the contrast of MRI medical images. Experimental results have achieved good efficiency to improve the contrast of MRI medical images with very high quality. Various measures of quality like MSE, PSNR and SNR are been taken into account for evaluation of the quality of enhanced MRI medical images.

[14]



3.1. Techniques

3.1.1. Histogram matching/specification technique [12]

In image processing, histogram matching or histogram specification is the transformation of an image so that its histogram matches a specified histogram. The well-known histogram equalization method is a special case in which the specified histogram is uniformly distributed.

It is possible to use histogram matching to balance detector responses as a relative detector calibration technique. It can be used to normalize two images, when the images were acquired at the same local illumination (such as shadows) over the same location, but by different sensors, atmospheric conditions or global illumination.

3.1.2. Brightness Preserving Bi-Histogram Equalization (BPBHE) [10]

In Bi-histogram equalization the histogram of the original image is separated into two sub-histograms based on the mean of the histogram of the original image, the sub-histograms are equalized independently using refined histogram equalization, which produces flatter histogram.

BPBHE preserves the brightness compared to HE where output MEAN is always the middle gray level.

Algorithm

Step 1: Start the program

Step 2: Read the original image.

Step 3: Make the histogram of the image.

Step 4: Calculate the mean of the histogram.

Step 5: Divide the Histogram into two parts Based on the mean

Step 6: Equalize each Partition independently using Probability Density Function and Cumulative Density Function

Step 7: Stop

Disadvantage

Higher degree of brightness preservation is not possible to avoid annoying artifacts. In some images, this level of brightness preservation is not sufficient to avoid unpleasant artefacts. They clearly show that higher degree of brightness preservation is required for these images to avoid unpleasant artefacts. In this case RMSHE produces better results.

3.1.3. Background Brightness Preserving Histogram Equalization (BBPHE) [10]

This method is able to enhance the image contrast while preserving the background brightness for images with well-defined background brightness. In this method the histogram is divided according to the foreground and the background levels

The steps for performing this method are as follows:.

- Input the image
- Find the histogram of the image
- Separate the input image into sub-images based on background levels and non-background levels range
- Each sub-image is then equalized independently, and
- Combine the sub-images and then we get the final output image.

3.1.4. Recursive Mean Separated Histogram Equalization (RMSHE) [10]

Recursive Mean-Separate Histogram Equalization (RMSHE) is another technique to provide better and scalable brightness preservation for gray scale and colour images. While the separation is done only once in BHE, RMSHE performs the separation recursively based on their respective mean. It is analyzed mathematically that the output images mean brightness will converge to the input images mean brightness as the number of recursive mean separation increases.

3.1.5. Brightness Preserving Dynamic Histogram Equalization (BPDHE) [9]

This method can produce the output image with the mean intensity almost equal to the mean intensity of the input, thus fulfilling the requirement of maintaining the mean brightness of the image. First, the method smoothes the input histogram with one dimensional Gaussian filter, and then partitions the smoothed histogram based on its local maximums. Next, each partition will be assigned to a new dynamic range. After that, the histogram equalization process is applied independently to these partitions, based on this new dynamic range. For sure, the changes in dynamic range, and also histogram equalization process will alter the mean brightness of the image. Therefore, the last step in this method is to normalize the output image to the input mean brightness.

3.1.6. Brightness Preserving Dynamic Fuzzy Histogram equalization (BPDFHE)

The modified technique, called Brightness Preserving Dynamic Fuzzy Histogram Equalization (BPDFHE), uses fuzzy statistics of digital images for their representation and processing. Representation and processing of images in the fuzzy domain enables the technique to handle the inexactness of gray level values in a better way, resulting in improved performance. Execution time is dependent on image size and nature of the histogram, however experimental results show it to be faster as compared to the techniques compared here.

3.2. Techniques of Edge detection techniques [11]

3.2.1. Sobel

The Sobel edge detection method was introduced by Sobel in 1970 (Rafael C.Gonzalez (2004)). The Sobel method of edge detection for image segmentation finds edges using the Sobel approximation to the derivative. It precedes the edges at those points where the gradient is highest. The Sobel technique performs a 2-D spatial gradient quantity on an image and so highlights regions of high spatial frequency that correspond to edges. In general it is used to find the estimated absolute gradient magnitude at each point in n input grayscale image. In conjecture at least the operator consists of a pair of 3x3 convolution kernels as given away in the table. One kernel is simply the other rotated by 90°. This is very alike to the Roberts Cross operator.

3.2.2. Prewitt

The Prewitt edge detection was proposed by Prewitt in 1970 (Rafael C.Gonzalez [1]). To estimate the magnitude and orientation of an edge Prewitt is a correct way. Even though different gradient edge detection wants a quite time consuming calculation to estimate the direction from the magnitudes in the x and y-directions, the compass edge detection obtains the direction directly from the kernel with the highest response. It is limited to 8 possible directions; however knowledge shows that most direct direction estimates are not much more perfect. This gradient based edge detector is estimated in the 3x3 neighborhood for eight directions. All the eight convolution masks are calculated. One convolution mask is then selected, namely with the purpose of the largest module.

Prewitt detection is slightly simpler to implement computationally than the Sobel detection, but it tends to produce somewhat noisier results.

3.2.3. Canny

In industry, the Canny edge detection technique is one of the standard edge detection techniques. It was first created by John Canny for his Master's thesis at MIT in 1983, and still outperforms many of the newer algorithms that have been developed. To find edges by separating noise from the image before finding edges of the image the Canny is a very important method. Canny method is a better method without disturbing the features of the edges in the image; afterwards it applies the tendency to find the edges and the serious value for threshold. The algorithmic steps are as follows:

- Convolve image $f(r, c)$ with a Gaussian function to get a smooth image $f^{\wedge}(r, c)$. $f^{\wedge}(r, c) = f(r, c) * G(r, c, 6)$
- Apply first difference gradient operator to compute edge strength then edge magnitude and direction are obtained as before.
- Apply non-maximal or critical suppression to the gradient magnitude.
- Apply threshold to the non-maximal suppression image.

Unlike Roberts and Sobel, the Canny operation is not very susceptible to noise. If the Canny detector worked well it would be superior.

3.2.4. Roberts

The Roberts edge detection is introduced by Lawrence Roberts (1965). It performs a simple, quick to compute, 2-D spatial gradient measurement on an image. This method emphasizes regions of high spatial frequency which often correspond to edges. The input to the operator is a grayscale image the same as to the output is the most common usage for this technique. Pixel values in every point in the output represent the estimated complete magnitude of the spatial gradient of the input image at that point.

3.2.5. Log

The Laplacian of Gaussian (LoG) was proposed by Marr(1982). It has two effects, it smoothes the image and it computes the Laplacian, which yields a double-edge image. Locating edges then consists of finding the zero crossings between the double edges.

4. Applications

4.1. Histogram matching technique

```
clear
clc
close all

pkg load image

aa=imread('girl.jpg');
ref=imread('lena.jpg');
figure(1); imshow(aa); colormap(gray)
figure(2); imshow(ref); colormap(gray)

M = zeros(256,1,'uint8'); % Store mapping - Cast to uint8 to respect data type
hist1 = imhist(aa); % Compute histograms
hist2 = imhist(ref);
cdf1 = cumsum(hist1) / numel(aa); % Compute CDFs
cdf2 = cumsum(hist2) / numel(ref);

% Compute the mapping
for idx = 1 : 256
    [~,ind] = min(abs(cdf1(idx) - cdf2));
    M(idx) = ind-1;
end

% Now apply the mapping to get first image to make
% the image look like the distribution of the second image
out = M(double(aa)+1);

figure(3); imshow(out); colormap(gray)
```

4.2. BPBHE Algorithm in Matlab

```
[filename,pathname] = uigetfile({'*.jpg'; '*.pgm'}, 'Choose the image');
I = imread(fullfile(pathname,filename));

HM = round(mean2(I));
ILE = find(I>=HM);
IHE = find(I<HM);

IL = I;
IL(ILE)=0;
IH = I;
IH(IHE)=0;

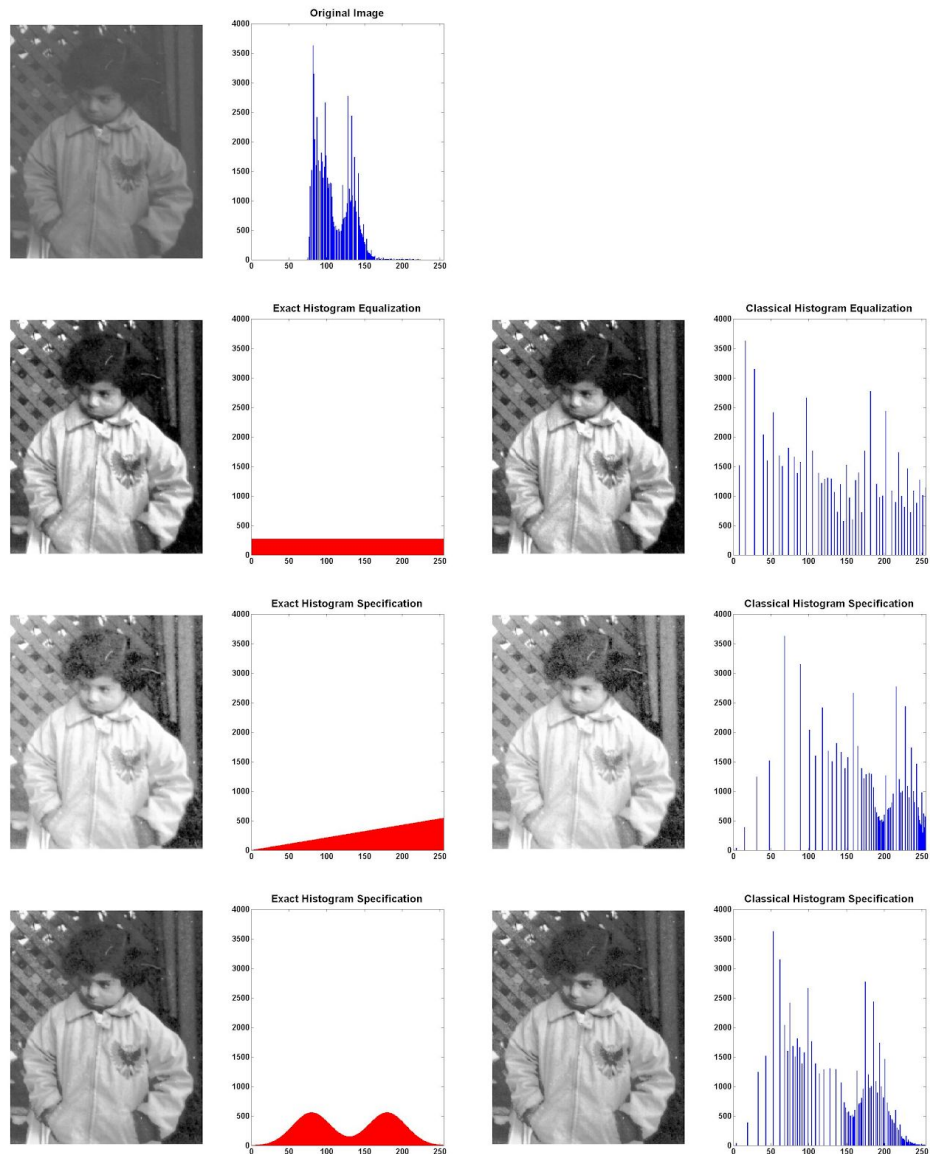
[~, lowMap] = histeq(IL,HM);
[~, highMap] = histeq(IH,256-HM);
lookupTable = uint8([HM*lowMap(1:HM), 256*highMap(HM+1:end)]);
HEI = intlut(I, lookupTable);

MI= mean2(I);
MEI = mean2(HEI);
SI= std2(I);
SIE=std2(HEI);

figure('NumberTitle', 'off', 'Name','Original Image')
subplot(1,2,1)
imshow(I)
title('Original Image','fontsize',22)
subplot(1,2,2)
imhist(I)
text(150,4000,{'Mean =', num2str(MI),'Standard Diviation'
='',num2str(SI)},'Color','black','FontSize',13)
figure('NumberTitle', 'off', 'Name','Bi Histogram Equalization Image')
subplot(1,2,1)
imshow(HEI)
title('Bi Histogram Equalization','fontsize',22)
subplot(1,2,2)
imhist(HEI)
text(130,5000,{'Mean =', num2str(MEI),'Standard Diviation =',n
um2str(SIE)},'Color','black','FontSize',13)
```

4.3. Exact histogram equalization and specification [13]

Histogram equalization is a traditional image enhancement technique which aims to improve visual appearance of the image by assigning equal number of pixels to all available intensity values. Histogram specification is a generalization of histogram equalization and is typically used as a standardization technique to normalize image with respect to a desired PDF (probability density function) or properties such as mean intensity, energy and entropy. Unlike classical histogram specification, the exact histogram specification algorithm implemented here is able to modify the histogram of any image almost exactly (see snapshot).



The author of this algorithm (Anton Semachko) was based on this paper [15] by D. Coltuc et al. The algorithm proposed in [13] is implementing `exact_histogram_matching`.

4.4. Contrast Limited Fuzzy Adaptive Histogram Equalization (CLFAHE) [16]

Contrast limited fuzzy adaptive histogram equalization (CLFAHE) is a method that improves the contrast of MRI Brain images. The method consists of three stages. First, the gray level intensities are transformed into membership plane and membership plane is modified with contrast intensification operator. In the second stage, the contrast limited adaptive histogram equalization is applied to the modified membership plane to prevent excessive enhancement in contrast by preserving the original brightness. Finally, membership plane is mapped back to the gray level intensities.

The purpose of this method is to generate an image of higher contrast than the original image. This is achieved by giving the larger weight to the gray levels that are closer to the mean gray level of the image when compared to those that are farther from the mean. The fuzzy image enhancement involves three stages namely image fuzzification, modification of membership values for image enhancement and image defuzzification.

4.5. Three-dimensional contrast limited adaptive histogram equalization (3D-CLAHE) [17]

Three-dimensional contrast limited adaptive histogram equalization (3D-CLAHE) is a method for improving contrast in the context of medical imaging. It differs from the original approach, 2D-CLAHE, in the sense that this method operates directly on the three-dimensional volumes, without requiring the extraction of two-dimensional sections of images.

Several histograms are constructed and modified to redistribute the pixel intensities of the images, significantly improving their local contrast. Experiments are conducted on different medical volumetric data sets to demonstrate the effectiveness of this method. Initially, two-dimensional slices are stacked together to form a volume. The volume is then subdivided in blocks with predefined size. The histogram is calculated for each block and part of histogram is cut according to a predefined value. Finally, the blocks are joined and a trilinear interpolation function is applied to remove artifacts that may occur on the boundaries between blocks.

The following algorithm describes the main steps of 3D CLAHE. Function CDF denotes the Cumulative Distribution Function, whereas functions min and max return the minimum and maximum grayscale values in the given image, respectively. H is a vector of histograms, where each position of the vector stores the histogram of a subblock, CS is an auxiliary vector to store the result of CDF, and MAP is a vector that maps each grayscale value to a new intensity value after the histogram equalization.

```

1 3D_CLAHE (image, size, clip_limit, nbins)
   input : image: volumetric image;
           size: size of subblocks;
           clip_limit: value at which the
           histogram is clipped;
           nbins: number of bins for
           histogram;
   output: image_equalized;
2 Divide image into subblocks with the given
   size;
3 Create image_equalized with same size as
   image;
4 for each subblock  $S_i$  do
5      $H[S_i] \leftarrow \text{histogram}(S_i, \text{nbins})$ ;
6     Clip  $H[S_i]$  according to clip_limit and
       redistribute equally the excess voxels
       across the histogram;
7      $CS \leftarrow \text{CDF}(H[S_i])$ ;
8      $\text{MAP}[S_i] \leftarrow CS \times (\max(\text{image}) -$ 
        $\min(\text{image})) + \min(\text{image})$ ;
9 for each voxel  $V_i$  from image do
10    Find 8 closest neighboring subblocks
       centers;
11    Use the pixel intensity to find the map
       value at the 8 subblocks;
12    Use the 8 mapped values to interpolate
       with trilinear interpolation to obtain
       the  $V_i$  mapped value and assign this
       value to the corresponding voxel in
       image_equalized;
13 return image_equalized;

```

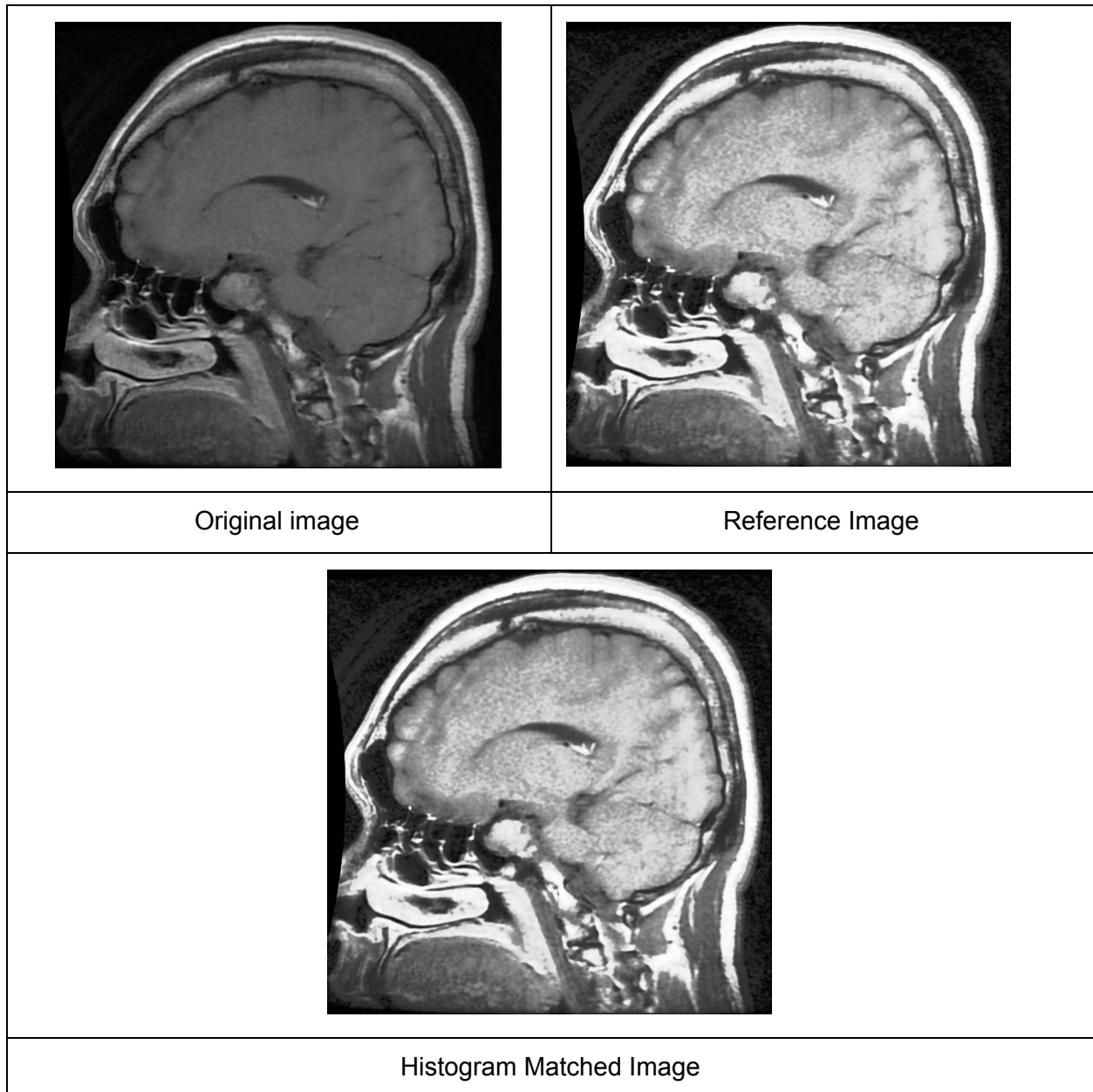
5. Results

By implementing exact_histogram_matching introduced in [13] we applied in the code an error function that give us metrics about Mean Squared Error of the images.

Mean Squared Error is described by the follow formula :

$$MSE = \frac{1}{m \cdot n} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [I(i, j) - K(i, j)]^2$$

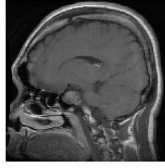
MSE: 4533.24 (between original image and the matched one)



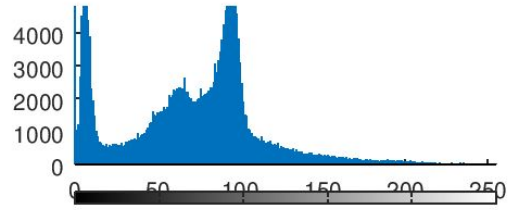
Implementing Histogram matching technique in [6]

MSE: 4758.6 (between original image and the matched one)

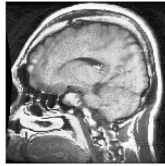
Input Image



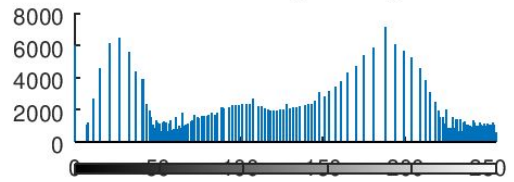
Input image histogram



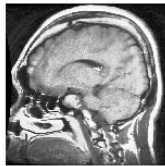
Reference Image



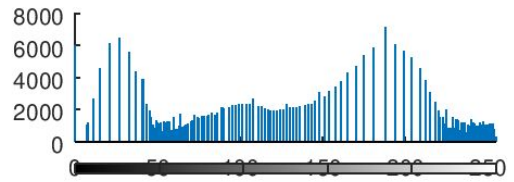
Reference Image Histogram



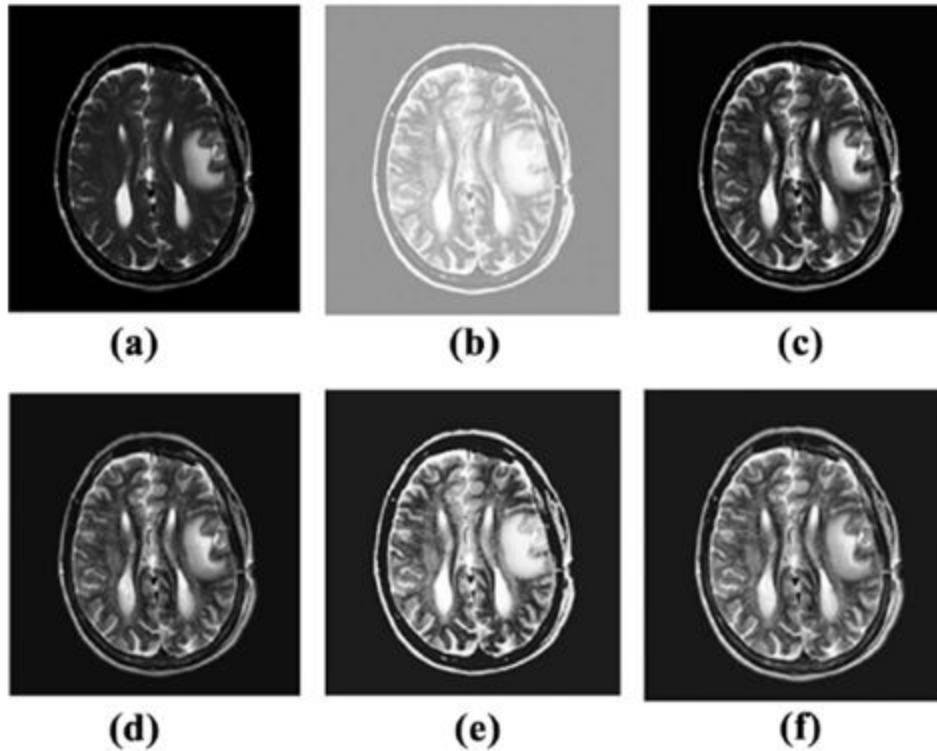
Transformed Image



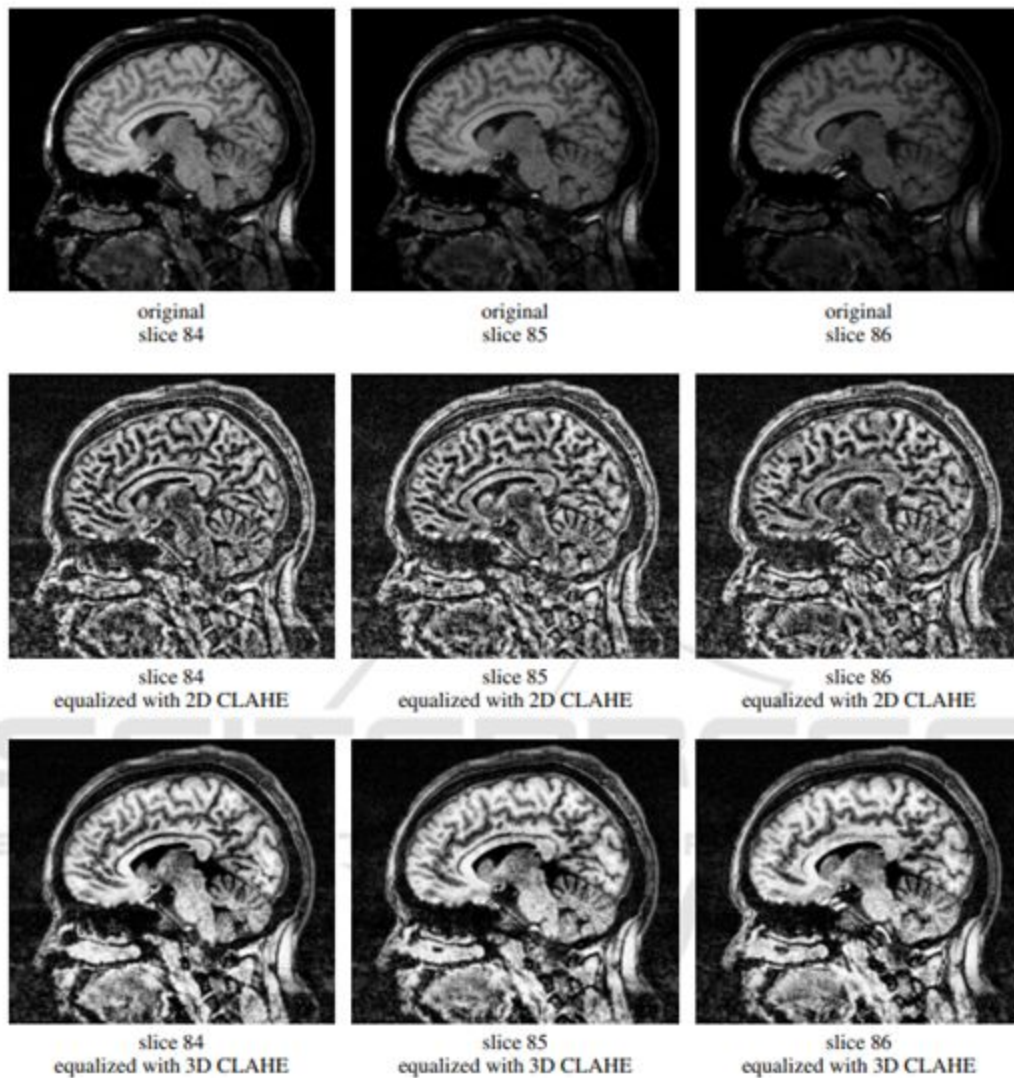
Transformed Input Image Histogram



By implementing Contrast Limited Fuzzy Adaptive Histogram Equalization (CLFAHE), the following MRI brain images obtained by the various existing methods and CLFAHE method. Figure 2a shows the original input image. Figure 2b shows Histogram Equalized image. Figure 2c shows the Adaptive Histogram Equalized image with contrast improvement. Figure 2d shows the resultant image of Contrast Limited Adaptive Histogram Equalization. Bi-histogram equalized image is shown in Figure 2e. Figure 2f shows the output image of the CLFAHE method.



By implementing three-dimensional contrast limited adaptive histogram equalization (3D-CLAHE), at the following figure, in the first row all images are non-equalized. It is possible to observe that these images have different level of contrast. Images in the second row were equalized with the 2D CLAHE technique. They present better contrast when compared to their corresponding original images, however, they have different contrast between the slices which it is not ideal for volume rendering or 3D segmentation purpose. Finally, images in the third row were equalized with 3D CLAHE method, which present uniform contrast.



6. Conclusion - Discussion

Histogram equalizations techniques

Based on the needs there comes a different approach on which technique will be used therefore there is no a specific technique for every occasion .

Edge detection techniques

Canny is considered as the ideal edge detection algorithm for images that are corrupted with noise. Canny's aim was to discover the optimal edge detection algorithm which reduces the probability of detecting false edges, and gives sharp edges.

Exact histogram equalization and specification

The exact histogram specification algorithm is able to modify the histogram of any image almost exactly.

Contrast Limited Fuzzy Adaptive Histogram Equalization (CLFAHE)

A simple contrast limited fuzzy adaptive histogram equalization is presented for image contrast enhancement. The conventional contrast enhancement methods cause significant change in brightness and may bring undesired artifacts and unnatural look image. Hence, CLFAHE method can preserve naturalness of an image and prevent significant change in brightness.

Three-dimensional contrast limited adaptive histogram equalization (3D-CLAHE)

Three-dimensional contrast limited adaptive histogram equalization (3D-CLAHE) method improves contrast in the context of medical imaging. It differs from the original approach, 2D-CLAHE, because it operates directly on the three-dimensional volumes, without requiring the extraction of two-dimensional sections of images.

7. References

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<https://github.com/bemoregt/octaveHistogramMatching/blob/master/histogramMatching.m>
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<https://github.com/MaRK0960/Bi-Histogram-Equalization-Matlab/blob/master/testBi.m>
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8. Glossary Links

HM : Histogram matching

HE : Histogram equalization

AHE : Adaptive Histogram equalization

CLAHE : Contrast Limited Adaptive Histogram equalization

BPBHE : Brightness Preserving Bi-Histogram Equalization

BBPHE : Background Brightness Preserving Histogram Equalization

RMSHE: Recursive Mean Separated Histogram Equalization

BPDHE: Brightness Preserving Dynamic Histogram Equalization

DSIHE: Dualistic Sub-Image Histogram Equalization

MMBEBHE: Minimum Mean Brightness Error Bi-HE Method

BPDFHE: Brightness Preserving Dynamic Fuzzy Histogram equalization

PDF : Probability density function

CDF: Cumulative distribution function

CLFAHE: Contrast Limited Fuzzy Adaptive Histogram Equalization

3D-CLAHE: Three-dimensional contrast limited adaptive histogram equalization